WTS TUTORING

WTS

TRIGONOMETRY

GRADE : 12

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2019 WTS TUTORING

TERM 1 CAMP

SUBJECTS : MATHS, MATHS LIT, PHYSICS & ACCOUNTING

VENUE : MZINGAZI PRIMARY SCH

GRADE : 08 TO 12

DATE : 03 TO 07 JANUARY 2019

TIME : 08:00 TO 20:00

WE ALSO PROVIDE ONE ON ONE, CROSSNIGHTS AND WEEKEND LESSONS

FOR MORE INFO. CONTACT: 082 6727928/076 091 1957

➢ TRIGONOMETRY

- ✓ It is the study of the relationships between lines and triangles
- ✓ The word trigonometry means 'measurement of triangles'.

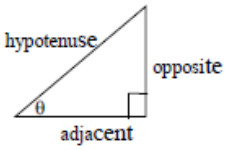
A. PYTHAGORAS THEOREM

- In a right-angled Δ

$\text{Sin}\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\text{Cos}\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{Tan}\theta = \frac{\text{opposite}}{\text{adjacent}}$

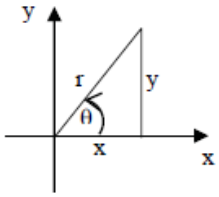


- On a Cartesian Plane

$\text{Sin}\theta = \frac{y}{r}$

$\text{Cos}\theta = \frac{x}{r}$

$\text{Tan}\theta = \frac{y}{x}$



- ➢ It deals with the length and angles
- ➢ It's given by the equation : $x^2 + y^2 = r^2$
- ➢ The Pythagoras theorem can be used anytime if we have a right angled triangle with two sides and want to find a third side.
- ➢ r is always a longest positive side and opposite to 90º

B. TRIGONOMETRIC RATIOS

NOTE: SOHCAHTOA

- ❖ $sin\theta = \frac{opposite}{hypotenuse}$ [SOH]

  $= \frac{y}{r}$

- ❖ $cos\theta = \frac{adjacent}{hypotenuse}$ [CAH]

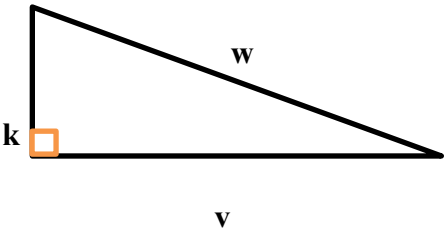
  $= \frac{x}{r}$

- ❖ $tan\theta = \frac{opposite}{adjacent}$ [TOA]

  $= \frac{y}{x}$

NB: Opposite, Adjacent and Hypotenuse are all sides not an angle.

KWV 1



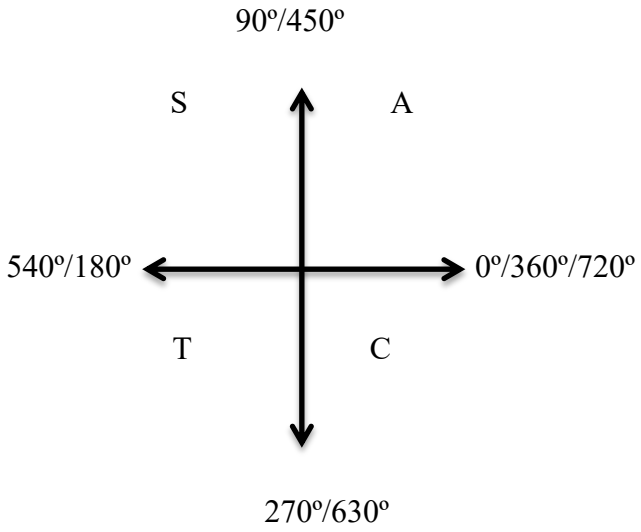
Use the sketch to state the followings NB: indicate β *and* θ on the diagram

a) $\sin\theta$ b) $\cos\beta$

c) $\sin\beta$ d) $\cos\theta$

e) $\tan\theta$ f) $\tan\beta$

C. ASTC DIAGRAM {All Sbiya's Take Coffee}



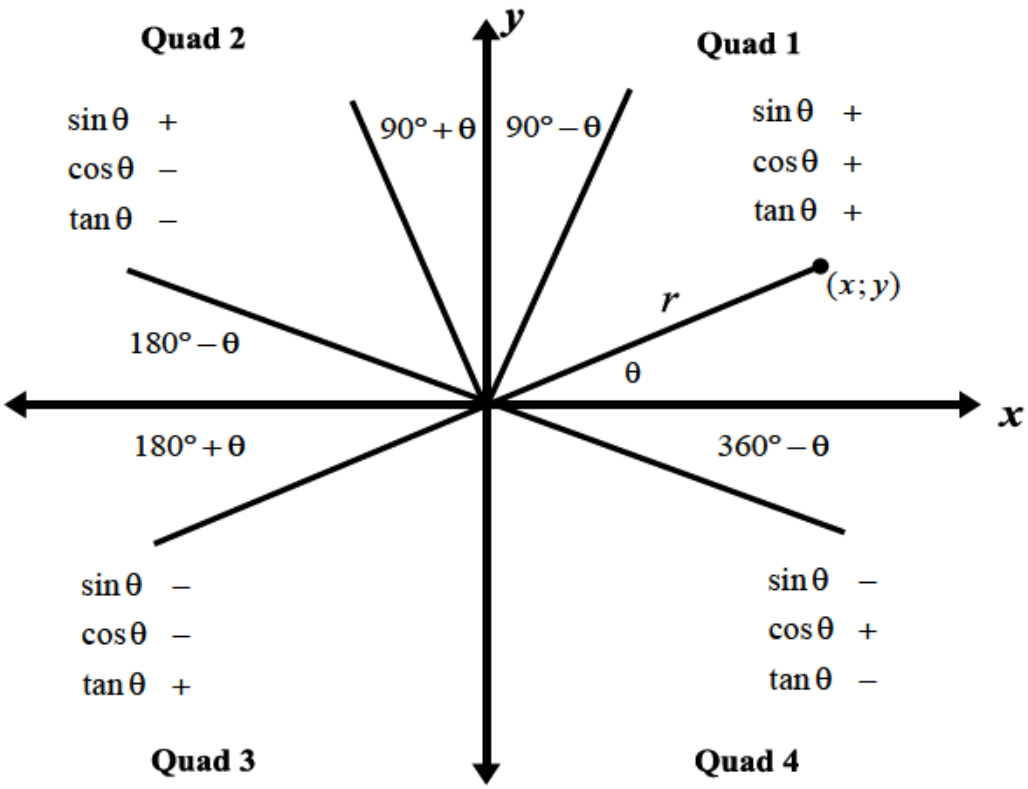
- It's clearly determines a status of each trigonometric quadrant
- A [FIRST QUAD AND ALL TRIG RATIOS ARE POSITUVE]
- S [SECOND QUAD AND ONLY SINE POSITIVE]
- T [THIRD QUAD AND ONLY TANGENT POSITIVE]
- C [FOURTH QUAD AND ONLY COSINE POSITIVE]

- **CONSIDER A DEMONSTRATION BELOW**

TRIG RATIOS	1ST	2ND	3RD	4TH
$\sin\theta$	+	+	-	-
$\cos\theta$	+	-	-	+
$\tan\theta$	+	-	+	-

- **KUTHI HUUUUUU!!!!!!**

$$\sin\theta=\frac{y}{r} \qquad \cos\theta=\frac{x}{r} \qquad \tan\theta=\frac{y}{x}$$



KWV 1

In each case, say which quadrant x lies in if:

a) $sinx > 0$ and $x\epsilon$ [90° ; 270°]

b) $cosx < 0$ and $x\epsilon$ [180° ; 360°]

c) $tanx < 0$ and $90^\circ < x < 270^\circ$

d) $cosx = -\frac{5}{13}$ and $0^\circ < x < 180^\circ$

D. TRIGONOMETRIC IDENTITIES

- It is where we are saying left hand side is equal to right hand side.
- Simplify the complicated side
- Simplify both if all sides are complicated
- Always remember to convert tangent to cosine and sine
- If there is a fraction simple look for LCD
- Check the common factor
- Factorise if there's a quadratic express or equation
- If there nothing to change just simple rationalizes the numerator or denominator.

  e.g. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} x \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

- β / α / θ: usual represent any angle not a side

i. QUOTIENT IDENTITY

$tan\alpha = \frac{sin\alpha}{cos\alpha}$

Proof: $tan\alpha = \frac{sin\alpha}{cos\alpha}$

$= \frac{\frac{y}{r}}{\frac{x}{r}}$

$= \frac{y}{x}$

$= tan\alpha$

ii. SQUARE IDENTITY

✓ $\cos^2\alpha + sin^2\alpha = 1$

- $\sin^2\alpha = 1 - \cos^2\alpha$
- $= (1 - \cos\alpha)\ (1 + cos\alpha\ )$

- $\sin\alpha = \sqrt{1 - cos^2\alpha}$

- $\cos^2\alpha = 1 - sin^2\alpha$
- $= (1 - \sin\alpha)\ (1 + sin\alpha\ )$
- $\cos\alpha = \sqrt{1 - sin^2\alpha}$

KWV 1

Prove that:

$\cos^2\alpha + sin^2\alpha = 1$, using diagram, x, y and r

KWV 2

Simplify the following:

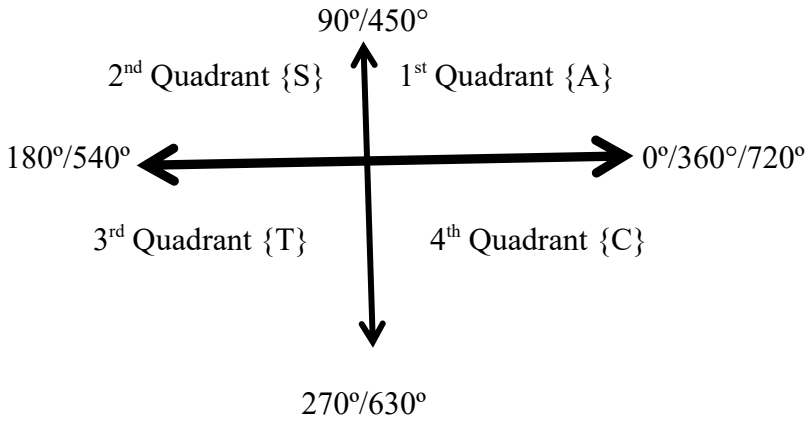
a. $sin^2 200° + cos^2 200°$

b. $\frac{\boldsymbol{sin^2\alpha}}{(\boldsymbol{1+cos\alpha})}$

c. $\frac{1-sin^2\alpha}{1-sin\alpha}$

E. REDUCTION FORMULAR

- It is a process of reducing at 360^0, we are completing the first revolution. Therefore we can complete more than one revolution
- Here we just considered 0^0 to 720^0 ,which are two revolutions
- Take note: negative angle (-α), found in the 4th



NOTE

- MOVING FORWARD YOU MUST ADD
- MOVING BACK YOU MUST SUBTRACT

i. **ANTCLOCKWISE DIRECTION**

- Reading the angles from right to left
- Positive angles
- Give us the angle of inclination

KWV 1

Simplify the following

a. $\cos(180° - \alpha)$ c. $\sin(180° - \alpha)$ e. $\sin(720° - \alpha)$

b. $\tan(360° - \alpha)$ d. $\cos(\alpha + 540°)$ f. $\sin(-\alpha)$

g. $\cos(-135°)$ h. $\sin(-150°)$ i. $\sin(\alpha + 720°)$

ii. **CLOCKWISE DIRECTION**

- Reading the angles from left to right
- Negative angles
- Angle of inclination will be: $180° - ref\ angle$

METHOD 1

❖ Simply add 360° up until the angle becomes positive

METHOD 2

- Take out the negative as a common factor e.g. : $b - a = -(a - b)$
- And then remember that (-α) is in the 4th quadrant

KWV 1

Simplify the following:

1. $\cos(x - 360°)$ 2. $\sin(x - 180°)$
3. $\tan(x - 360°)$ 4. $\cos(-x - 720°)$

F. CO-FUNCTIONS

- Re taking place at vertical axis of ASTC diagram y - axis.
- Co functions of sine is cosine OR cosine is sine.
- It includes angles such as 90°, 270°, 450°, 630°and 810°.

KWV 1

Simplify the following

a. $\sin(90° - \alpha)$

b. $\cos(90° + \alpha)$

c. $\cos(450° + \alpha)$

d. $\sin(630° - \alpha)$

e. $\sin(\alpha - 90°)$

f. $\cos(\alpha - 90°)$

g. $\sin(\alpha - 450°)$

h. $\tan(\alpha - 90°)$

i. $\sin(\alpha + 180°)$

G. COMPLEMENTARY ANGLES

- These are angles that add up to 90°, complementary angles of sinα and cosine, vice versa

KWV 1

Simplify the following:

a. $\frac{\sin 70°}{cos20°}$

b. $\frac{\sin 40°}{cos50°}$

c. $\frac{\sin 10°}{cos80°}$

H. TRIG RATIOS WITH A SQUARE

- Use the closed bracket to include square and simplify inside and then consider the square
- Conclusion: if there is a square the answer will be always positive.

KWV 1

Simplify the following

a) $\sin^2(180° - \beta)$

b) $\cos^2(360° - \beta)$

c) $\tan(90° - \beta)$

d) $\tan 2\beta$

I. ANGLES GREATER THAN 360°

- ✓ Simplify subtract 360° up until the angle becomes less than 360°

KWV 1

Simplify the following:

a. $\sin 930°$

b. $\tan 870°$

c. $\cos 1485°$

d. $\sin(1440° - \beta)$

J. ANGLES

- It includes all angles on the x and y axis such as 0°, 90°,180°,270°,360°,450°,630° and 720° etc.
- If given, simple press the calculator

KWV 1

Simplify the following

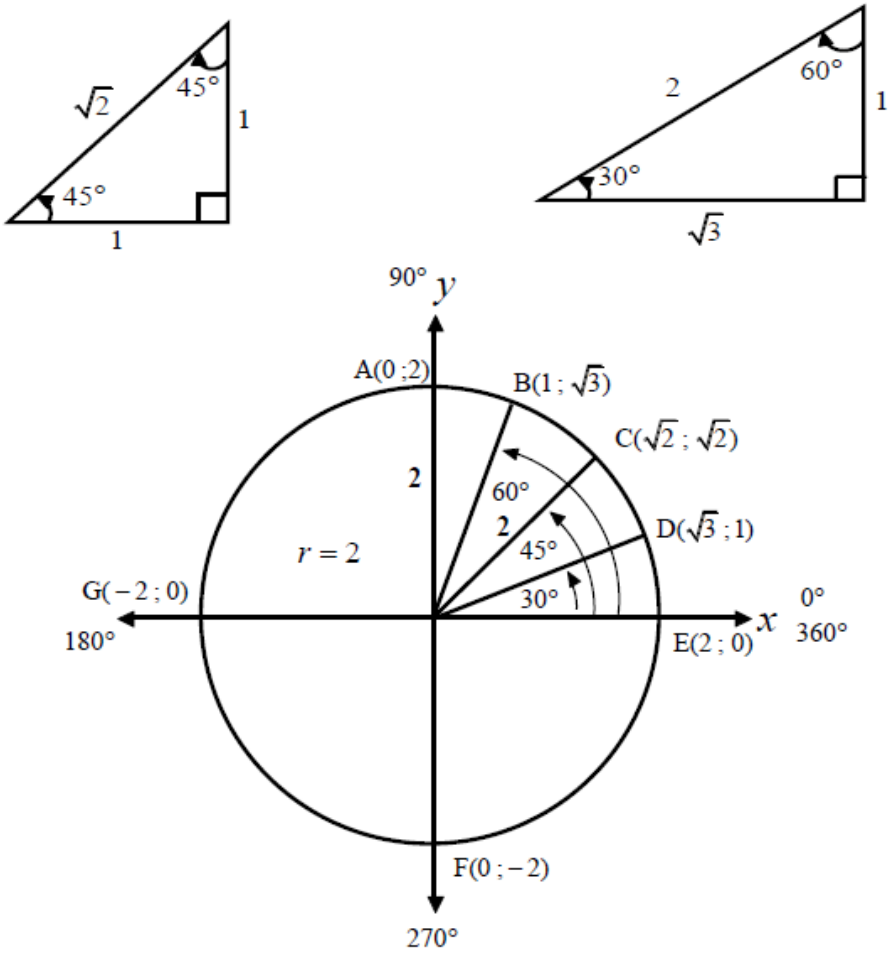
a. $\cos(180°)$ b. $\sin(360°)$

c. $\tan(270°)$ d. $\cos(-180°)$

K. SPECIAL ANGLES

- Includes 30°; 45°; 60°
- If the angle is not an acute angle simple reduce it with special angles.
- If you have a **Casio calculator** simple use it as it gives the correct rationalized values.
- If you are using **old sharp calculator** simple use the triangle method

KUTHI HUUUUUUU!!!!!



KWV 1

Simplify the following

a. Use an equilateral triangle to derive the special angles
b. Use an isosceles right angled triangle to derive the special angle

KWV 2

Simplify the following

a) $\sin 30°$
b. $\cos 45°$
c) $\tan 60°$
d. $\sin 135°$
e. $\sin(-135°)$
f. $\cos(-150°)$

L. RESTRICTIONS VS INTERVALS

- ✓ The closed brackets [ ] are used with ≤ or ≥ {the value is included}
- ✓ The open brackets ( ) are used with < or > {the value is excluded}
- ✓ This can be more useful in general solution and in questions require the use of the diagram

KWV 1

Use the table to derive the first revolution with the closed brackets and intervals.

➢ GRADE 12

A. COMPOUND ANGLES

- ✓ It deals with angles which are not the same e.g. $(\infty + \alpha)$
- ✓ It deals with a relationship between sine and cosine, will be given in the formula sheet
- ✓ **In sine compound**: it keeps the original sign between the terms.
- ✓ **In cosine compound**: it changes the sign between the terms.
- ✓ It will be given in the formula sheet.

➢ **THESE ARE EXAMINABLE, LEARN THEM WELL.**

PROOF:

$$\cos(\alpha + \beta) = \cos[\alpha - (-\beta)] = \cos\alpha.\cos(-\beta) + \sin\alpha.\sin(-\beta)$$

$$= \cos\alpha.\cos\beta + \sin\alpha.(-\sin\beta)$$

$$= \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$$

PROOF:

$$\sin(\alpha + \beta) = \cos[90° - (\alpha + \beta)] = \cos[90° - \alpha - \beta] = \cos[(90° - \alpha) - \beta]$$

$$= \cos(90° - \alpha).\cos(\beta) + \sin(90° - \alpha).\sin(\beta)$$

$$= \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$$

PROOF:

$$\sin(\alpha - \beta) = \cos[90° - (\alpha - \beta)] = \cos[90° - \alpha + \beta] = \cos[(90° + \beta) - \alpha]$$

$$= \cos(90° + \beta).\cos\alpha + \sin(90° + \beta).\sin\alpha$$

$$= -\sin\beta.\cos\alpha + \cos\beta.\sin\alpha$$

$$= \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

KUTHI HUUUUUU!!!!!!!!!

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ …… think "cos cos sin sin" ☺

Note the opposite signs

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ …… think "sin cos cos sin" ☺

Note the same signs

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

KWV 1

Use $\cos(A - B) = cosA.cosB + sinAsinB$, to prove the ff:

a. $\sin(A - B) = sinA.cosB - cosAsinB$

b. $\sin(A + B) = sinA.cosB + cosAsinB$

KWV 2

Expand the following compound:

a. $\cos(\alpha - 50°)$
b. $\sin(62°)$ if $\sin 2° = k$
c. $\cos 25°$ if $\cos 5° = p$
d. $\sin(30° + 40°)$

KWV 3

Simplify the following:

a. $\cos 75°$
b. $sin15°$

c. $\cos 105°$

d. $cos35°.cos20° - sin35°.sin20°$

e. $sin140°.cos80° - cos40°cos10°$

f. $\cos(x - 300°) - \sin(x - 150°)$

B. DOUBLE ANGLES

- ✓ It deals with angles which are the same e.g. $\theta = \beta$
- ✓ It deals with the relationship between sine and cosine.
- ✓ It will be given in the formula sheet.

KUTHI HUUUUUUUU!!!!!:

$\sin 2\alpha = \sin(\alpha + \alpha)$

$\sin 2\alpha = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$= 2(\sin\alpha)(\cos\alpha)$ … (note that sin α and cos α are numbers, or factors in this case)

$\cos 2\alpha = \cos(\alpha + \alpha)$

$\cos 2\alpha = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = (1 - \sin^2\alpha) - \sin^2\alpha$ OR $\cos 2\alpha = \cos^2\alpha - (1 - \cos^2\alpha)$

$\cos 2\alpha = 1 - 2\sin^2\alpha$ $\cos 2\alpha = 2\cos^2\alpha - 1$

KWV 1

Consider:$\sin(A + B) = sinA.cosB + cosA.sinB$, let B = A, and then simplify

Note: $sin2\alpha = 2sin\alpha cos\alpha$

KWV 2

Consider: $cos(A + B) = cosA.cosB - sinA.sinB$, let B = A and then simplify

Note: $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

KWV 3

Use $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$, Prove the following:

a. $\cos 2\alpha = 2\cos^2 \alpha - 1$

b. $\cos 2\alpha = 1 - 2\sin^2 \alpha$

KWV 4

Use double angles to expand the following

a. $\sin 20°$

b. $\cos 40°$

KWV 5

Simplify the following:

a) $1 - 2\sin^2 15°$

b. $2\cos^2 20° - 1$

c. $2\cos^2 75° - 1$

d. $sin15°.cos15°$

e. $\cos^2 22.5° - \sin^2 22.5°$

g. $2\sin^2 15° - 1$

h. $1 - 2\cos^2 20°$

i. $\sin^2 5° - \cos^2 5°$

j. $(\sin\theta - cos\theta)^2$

f. $\sin^2 10° + \cos^2 10°$

C. WITH THE AID OF A DIAGRAM

- ✓ Firstly make the trig ratios be the subject of the formula
- ✓ Check the quadrant
- ✓ Draw the right angled triangle and always the line must lie on the x-axis
- ✓ Use the Pythagoras to calculate the unknown side
- ✓ And then simplify any question, take note of double and compound angles
- ✓ Remember to convert the given angle with special angles
- ✓ And then simplify any question, take note of double and compound angles

- **QUESTIONS**

KWV 1

Given that: $17\sin x - 8 = 0$, where $90° \leq x \leq 270°$

a. $sin2x$

b. $\cos 2x$

c. $\tan 2x$

d. $\cos(90° - x)$

f. $\sin(x - 45°)$

g. $\cos(x + 60)$

h. $\tan(90° + x)$

KWV 2

It is known that $13\sin\alpha - 5 = 0$ and $4\tan\beta + 3 = 0$ where $\alpha\epsilon[90°; 270°]$ and $\beta \in [90°; 270°]$, without using a calculator ,determine the value:

a. $\cos(\alpha + \beta)$

b. $\sin(\beta + \alpha)$

c. $\cos(\alpha - \beta)$

d. $\sin(\beta - \alpha)$

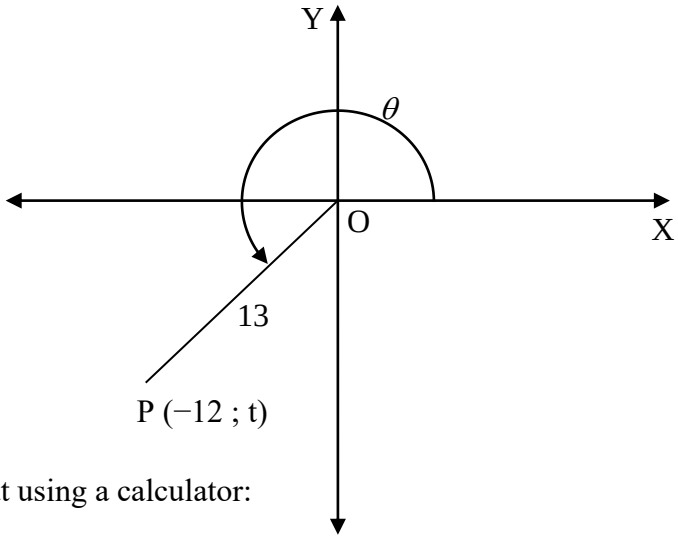
e. $\tan(\beta - \alpha)$

f. $\sin(\alpha - 300°) - \cos(\beta - 150°)$

KWV 3

In the diagram below, reflex angle $X\hat{O}P = \theta$. P is the point (−12 ; t) and

OP = 13 units.



Calculate, without using a calculator:

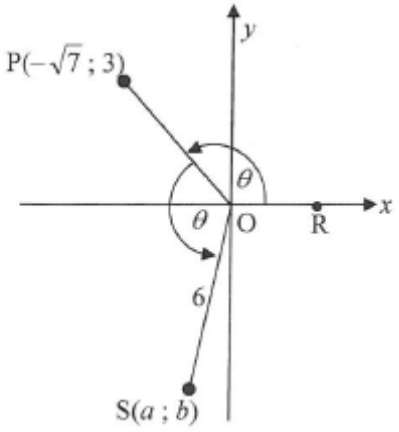
a. the value of t.

b. $\sin 2\theta$

c. $\dfrac{1-\cos 2\theta}{\tan 2\theta}$

KWV 4

P $(-\sqrt{7}\,;\ 3)$ and $s(\ a\ ;\ b)$ are points on the Cartesian plane, as show in the diagram below. $P\hat{O}R = P\hat{O}S = \theta$ and OS = 6.



Determine, without using a calculator, the value of:

a. $tan\ \theta$

b. $sin(-\theta)$

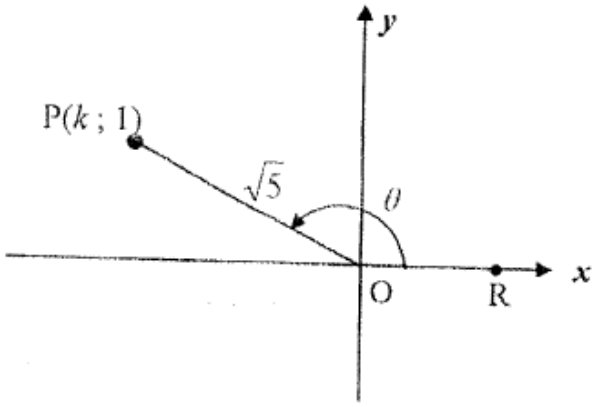
c. a

d. b

e. θ

KWV 5

In the diagram, P(k ; 1) is a point in the 2nd quadrant and is $\sqrt{5}$ units from the origin. R is a point on the positive x-axis and obtuse $R\hat{O}P = \theta$.



1 Calculate the value of k.

2 **Without using a calculator**, calculate the value of:

(a) $\tan\theta$

(b) $\cos(180° + \theta)$

(c) $\sin(\theta + 60°)$ in the form $\frac{a+b}{\sqrt{20}}$

3 **Use a calculator** to calculate the value of $\tan(2\theta - 40°)$ correct to ONE decimal place.

D. EXPRESSING IN TERMS VARIABLES

- ✓ Always draw the diagram on the first quadrant
- ✓ Use the Pythagoras to calculate the unknown side
- ✓ And then simplify any question ,take note of double and compound angles
- ✓ Remember to convert the given angle with special angles

➢ QUESTIONS

KWV 1

If $cos26° = k$, determine the following in terms of k, in its simplest form:

a) Calculate the value $x, y\ and\ r$

b) Indicate all angles of a triangle

c) $cos154°$

d) $cos206°$

e) $sin334°$

f) $sin386°$

g) $tan514°$

h) $tan566°$

i) $sin26°$

j) $sin64°$

k) $tan64°$

l) $cos64°$

m) $\sin(-26°)$

n) $\cos(-26°)$

o) $\tan(-64°)$

p) $\cos(-154°)$

q) $sin52°$

r) $cos52°$

s) $tan52°$

t) $2\cos^2 13° - 1$

u) $2\cos^2 26° - 1$

v) $1 - 2\cos^2 13°$

w) $1 - 2\sin^2 13°$

x) $2\sin^2 13° - 1$

y) $\cos^2 13° - sin^2 13°$

z) $cos56°$

aa) $sin71°$

bb) $cos86°$

cc) $sin13°.cos13°$

dd) $cos13°$

ee) $sin13°$

KWV 2

Repeat all above questions considering the following:

a. $sin154° = k$

b. $sin26° = \sqrt{k}$

KWV 3

Given that $tanx = p$, in any right-angled triangle:

Show that $sin2x = \frac{2p}{p^2+1}$

KWV 4

If $\sin 26° \cos 16° = k$ and $\cos 26° \sin 16° = w$, determine interms of p and q the value of

a. $\sin 42°$

b. $\sin 10°$

c. $\cos 10°$

KWV 5

If $cos2A = k \; and \; sin2B = w$ and then determine the following in terms of k and w

i. $cosA$

ii. $sinA$

iii. $sinBcosB$

KWV 6

Without using a calculator, determine the following in terms of $sin36°$;

a. $sin\ 324°$

b. $cos\ 72°$

KWV 7

If $cos73°cos31° + sin73°sin31°k$ then determine the value of the following in terms of k

(a) $\cos^2 21 - \sin^2 21 + 7$

(b) $\sin 42°$

KWV 8

If$sin31°cos22° + sin22°cos31° = k$ then determine the value of the following in terms of k

(a) $\sin 53°$

(b) $\cos 143°$

(c) $\sin 75° \cos 22° + \cos 75° \cos 22°$

KWV 9

If $cosA + sinA = k$, express the following in terms of k:

(a) $\cos(A - 45°)$

(b) $1 + sin2A$

E. REDUCTIONS AND IDENTITIES

- ✓ Use ASTC diagram and identities
- ✓ In any calculation relate to special angles
- ✓ Always try to convert all terms into sine and cosine e.g. $tan\theta = \frac{sin\theta}{cos\theta}$
- ✓ Always try to convert double to single angles
- ✓ Think about double and compound angles.
- ✓ Addition and subtraction of fractions: find LCD then simplify
- ✓ Factories: take out the HCF, difference of two squares, trinomials, grouping, sum or difference of two cubes.
- ✓ If nothing to factorize: simply rationalize
- ✓ Do not cancel over a + and -.
- ✓ Remember a common denominator.
- ✓ Keep looking at the RHS

➢ QUESTIONS

KWV 1

Simplify to a single trigonometric ratio of A:

(a) $\frac{\tan(180°+A)\cos(180°-A)\sin(360°-A)}{\cos(90°-A)}$

(b) $\frac{\tan(180°+A)\cos(360°-A)}{\sin(180°-A).\cos(90°+A)+\cos(540°+A).\cos(-A)}$

(c) $\cos(90° - 2A)\tan(180° + A)sin^2(A - 360°)$

KWV 2

Simplify the following without using a calculator:

(a) $\frac{sin10°.cos120°}{cos80°.sin150°}$

(b) $\frac{\cos(A-360°).\sin(-A)}{tanA.cos180°}$

(c) $\frac{2cos105°.cos15°}{\cos(45°-x).cosx-\sin(45°-x).sinx}$

(d) $\frac{tan480°.sin300°.cos14°.\sin(-135°)}{sin104°.cos225°}$

(e) $\frac{sin104°(2cos^2 15°-1)}{tan38°.sin^2 412°}$

(f) $\frac{\sin(90°+\theta)+cos180°\sin(-\theta)}{sin180°-tan135°}$

(g) $\frac{cos330°sin140°}{\sin(-160°)tan405°.sin290°}$

(h) $\left(\sqrt{2}cos75°-1\right)\left(1+\sqrt{2}cos75°\right)$

(i) $\frac{sin33°}{sin11°}-\frac{cos33°}{cos11°}$

KWV 3

Prove the following:

(a) $\frac{sin2x}{cos2x+sin^2 x}=2tanx$

(b) $\frac{sin\theta}{1-cos\theta}-\frac{cos\theta}{sin\theta}=\frac{1}{sin\theta}$

(c) $\frac{sin2x}{sinx} - \frac{cos2x}{cosx} = \frac{1}{cosx}$

(d) $\frac{1-cos2x-sinx}{sin2x-cosx} = tanx$

(e) $\frac{\sin(180°+x)\tan(x-360°)cos38°}{\tan(360°-x)\sin(x-180°)cos240°tan225sin488°} = -2$

(f) $\frac{cos^2(90°+\theta)}{\cos(-\theta)+\sin(90°-\theta)cos\theta} = \frac{1}{cos\theta} - 1$

(g) $\cos(90° - 2x)\tan(180° + x) + sin^2(360° - x) = 3sin^2x$

(h) $(tanx - 1)(sin2x - 2cos^2x) = 2(1 - 2sinxcosx)$

(i) $\sin(45° + x).\sin(45° - x) = \frac{1}{2}cos2x$

(j) $sin3x = 3sinx - 4sin^3x$

(k) $\frac{cosx}{sin2x} - \frac{cos2x}{2sinx} = sinx$

(l) 1. Prove the following identity:

$\frac{sin3x}{sinx} = 3 - 4sin^2x$

2. Hence determine the general solution to:

$\frac{sin3x}{sinx} = 2$

KWV 4

Simplify the following without using a calculator:

(a) $\cos 75°$

(b) $\sin 15°$

(c) $cos105°$

(d) $tan15°$

(e) $cos25°.cos20° - sin25°.sin20°$

(f) $sin140°.cos80° - cos40°cos10°$

(g) $\cos(x - 300°) - \sin(x - 150°)$

(h) $\frac{-\cos(90°+x) - sin^3 x}{sin2x}$

(i) $\frac{sinx}{1+\sin(90°+x)} - \frac{\sin(-x)}{1-\cos(-x)}$

(j) $\frac{\tan(180°-x).\cos(-x).sin390°}{(cos300°.sinx)-cos450°}$

(k) $\sin(180° - x) . \cos(-x) + \cos(90° + x) . \cos(x - 180°)$

KWV 5

It is given that $cos2\alpha = cos^2\alpha - sin^2\alpha$:

(a) Similarly, write down a formula for $cos\alpha$ ...

(b) Hence show that $\cos\alpha = \frac{1 - \tan^2 \frac{1}{2}\alpha}{1 + \tan^2 \frac{1}{2}\alpha}$.

KWV 6

Use $\cos(A - B) = cosA.cosB + sinAsinB$, to prove the ff:

(a) $\sin(A - B) = sinA.cosB - cosAsinB$

(b) $\sin(A + B) = sinA.cosB + cosAsinB$

KWV 7

(a) Simplify the following expression as far as possible.

(1) $\dfrac{\sin(180°-\theta).\cos(90°-\theta)-1}{\cos(-\theta)}$

(2) Hence determine for which value(s) of $\theta \in [0°;360°]$, $\sqrt{\dfrac{\sin(180°-\theta).\cos(90°-\theta)-1}{\cos(-\theta)}}$ will be real.

(b) Prove the following identity:

(1) $\tan\theta\sin\theta+\cos\theta=\dfrac{1}{\cos\theta}$

(2) Hence; determine the general solution to:
$\tan\theta\sin\theta+\cos\theta=\dfrac{3}{\sin\theta}$

(c) Prove the following identity:

$$\frac{\cos y+\sin y}{\cos y-\sin y}-\frac{\cos y-\sin y}{\cos y+\sin y}=2\tan 2y$$

KWV 8

(a) Consider:$\sin(A+B)=sinA.cosB+cosA.sinB$, let B = A, and then simplify

Note: $sin2\alpha = 2sinacos\alpha$

(b) Consider: $cos(A+B)=cosA.cosB-sinA.sinB$, let B = A and then simplify

Note: $\cos2\alpha = \cos^2\alpha - \sin^2\alpha$

(c) Use $\cos2\alpha = \cos^2\alpha - \sin^2\alpha$, Prove the following:

i. $\cos2\alpha = 2\cos^2\alpha - 1$

ii. $\cos2\alpha = 1 - 2\sin^2\alpha$

F. ALGEBRAICALLY TRIGONOMETRY

- ✓ Try to manipulate the, if part or the given part into a required one.
- ✓ The skills of squaring both sides and then simplifying are needed.
- ✓ The skill of working from double angles into single angles is required.

- ➢ **QUESTIONS**

KWV 1

Simplify the following:

(a) If $sinx - cosx = \frac{3}{4}$, calculate the value of $sin2x$ without using a calculator.

(b) Given that $= \frac{1}{3}$, calculate the numerical value of $sin3\alpha$, without using a calculator.

(c) If $x = 3\ sin\alpha$ and $y = 3cos\alpha$, determine the value of $x^2 + y^2$

(d) if $sinx = \frac{2n}{n^2+1}$, n > 1 and $0° < x < 90°$, prove that $\frac{1+sinx}{cosx} = \frac{n+1}{n-1}$

(e) Determine the value of $sin3x\ .cosy + cos3x\ .siny$ if 3x + y = 270°

KWV 2

Evaluate, **without using a calculator**: $\sum_{A=38°}^{52°} \cos^2 A$

G. INVALID / UNDEFINED VALUES

- ✓ Let denominator equal to zero
- ✓ If tangent is give simple consider the asymptotes.
- ✓ Solve as like general solution
- ✓ Substitute with integers
- ✓ And then consider the domain

- **QUESTIONS**

KWV 1

a. For which value(s) of x will : $\frac{2tanx-sin2x}{\sin^2 x}$ be undefined in the interval $0° \leq x \leq 180°$

b. Consider the identity: $\frac{8\sin(180°-x)\cos(x-360°)}{\sin^2 x-\sin^2(90°+x)}$

i. Simplify the identity.

ii. For which value(s) of x in the interval $0° < x < 180°$ will the identity be undefined?

KWV 2

Consider the following identity:

$$\frac{1-2\sin A.\cos A}{\sin A-\cos A} = \sin A - \cos A$$

For which values of *A* will the identity not be defined?

KWV 3

Consider the following identity:

$$\frac{1-\tan A}{1+\tan A} = \frac{\cos 2A}{1+\sin 2A}.$$

i. Calculate the values of *A* for which $\frac{1-\tan A}{1+\tan A}$ is undefined for $A \in [0°; 360°]$.

ii. Hence or otherwise, calculate the value of $\frac{1-\tan 22,5^\circ}{1+\tan 22,5^\circ}$.

KWV 4

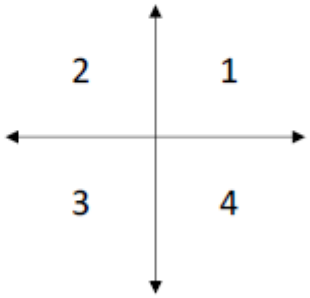
If $M = \dfrac{2\sin^2(180° - \theta) + \sin 2\theta}{\cos 2\theta}$ and $P = \dfrac{2\sin\theta}{\cos\theta - \sin\theta}$

(a) (1) Prove that $M = P$.

(2) For what value(s) of θ will P be undefined if $\theta \in [-180°; 360°]$?

(b) If $\sin\beta = \dfrac{\sqrt{2}}{\sqrt{11}}$ and $\cos\beta < 0$ then

(1) In which quadrant is angle β? (Circle the correct number)



(2) Without the use of a calculator, determine the value of $\tan\beta$.

(c) (1) Prove that $\cos(\alpha - 30°) - \cos(\alpha + 30°) = \sin\alpha$.

(2) Hence, determine the general solution to the equation:
$\cos(\alpha - 30°) - \cos(\alpha + 30°) = 2\sin^2\alpha$.

(d) Prove:

1 $\sin(A + B) - \sin(A - B) = 2\cos A.\sin B$

2 **Without using a calculator,** that $\sin 77° - \sin 43° = \sin 17°$

(e)

1 Show that $\cos(P+Q) + \cos(P-Q) = 2\cos P \cos Q$.

2 Hence evaluate $\cos 70° + \cos 50°$ without using a calculator if $\cos 10° = 0{,}984$.

(f)

If $\sin 2\theta = \frac{2\sqrt{6}}{5}$ where $\theta \in [45° ; 360°]$ determine, with the aid of a diagram and without using a calculator the value of $\sin \theta$.

H. MAXIMUM AND MINIMUM VALUES

- ✓ They are indicated by the amplitude of the trig function.
- ✓ Maximum is the turning point above the horizontal
- ✓ Minimum is the turning point below the horizontal or x axis.
- ✓ Simplify up to the single trig ratio.

➢ QUESTIONS

KWV 1

Write down the maximum and minimum of the following:

(a) $f(x) = sinx$

(b) $f(x) = cos2x$

(c) $f(x) = 2sinx$

(d) $f(x) = 1 + sinx$

(e) $f(x) = sinxcosx$

(f) $8 - 10sinx.cox$

KWV 2

Consider $sinx + cosx = \sqrt{1+k}$

(a) Determine the value of k

(b) Hence, calculate maximum value of k

KWV 3

Consider: $g(x) = -4\cos(x+30°)$

1 Write down the maximum value of $g(x)$.

2 Determine the range of $g(x) + 1$.

3 The graph of g is shifted $60°$ to the left and then reflected about the x-axis to form a new graph h. Determine the equation of h in its simplest form.

I. TRIGONOMETRIC EQUATIONS AND GENERAL SOLUTIONS

- ✓ Indicate all angles including negative ones
- ✓ Take note of the reference angle :acute angle
- ✓ Use k.360° for sine and cosine due to their period
- ✓ K represents elements of integers k€ (-1,0,1)
- ✓ Always make a trig ratio a subject of the formula e.g. a sin b = c
- ✓ Always remember to simplify before considering the quadrant
- ✓ Consider the quadrant/Cartesian plan in the form ASTC(ALL SBIYA'S TAKE COFFIE)

- **QUADRANTS**

- ✓ 1st = reference angle.
- ✓ 2nd = 180° – reference angle.
- ✓ 3rd = 180° + reference angle.
- ✓ 4th = 360° – reference angle.
- ✓ Remember to apply co-functions in case of sine and cosine given.
- ✓ Remember again to convert sine and cosine to tangent

- **QUESTIONS**

KWV 1

Determine the general solution of following:

a) $cosx = 0.5$

b) $sin2x = 0.5$

c) $tan\frac{1}{2}x = 0.5$

d) $2cosx - 1 = 0$

e) $3cosx = \sqrt{2}sinx$

f) $2sinxcosx = 0.5$

g) $sin2xcos2x = \frac{1}{4}$

h) $2cos^2 3x - 1 = 0{,}5$

i) $1 - 2sin^2 2x = 0{,}5$

j) $cos^2 x - sin^2 x = 0{,}5$

k) $1 - 2cos^2(x - 10°) = 0{,}5$

l) $4sinxcosx = 2$

m) $\cos 2x - 4\sin x + 5 = 0.$

n) $sin^2 x + \cos 2x - cosx = 0$

o) $sinx + 2cos^2x = 1$

p) $6cosx - 5 = \frac{4}{cosx}$; $cosx \neq 0$

q) $cos3x = sinx$

r) $\cos 2x = \sin(x - 30^\circ)$.

s) $1 - \frac{tan^2\frac{1}{2}x}{1+tan^2\frac{1}{2}x} = \frac{1}{4}$

t) $\sin(x + 64°)\cos(x + 379°) + \sin(x + 19°)\cos(x + 244°) = \frac{1}{\sqrt{2}}$

u) $2sinx = tan\frac{1}{2}x$

v) $sin^2x - cos^2x + sinx + 1 = 0$

x) $sinx + cosx = 1$

KWV 2

Calculate the values of x where $x \in [-180°; 360°]$

(a) $sinx = cos2x$

(b) $sinx = \cos(x + 30°)$

(c) $2cosx = \sin(x + 30°)$

KWV 3

Given: $2\cos x = 3\tan x$

1 Show that the equation can be rewritten as $2\sin^2 x + 3\sin x - 2 = 0$.

2 Determine the general solution of x if $2\cos x = 3\tan x$.

3 Hence, determine two values of y, $144° \leq y \leq 216°$, that are solutions of $2\cos 5y = 3\tan 5y$.

➢ TRIGONOMETRY SUMMARY

✓ CALCULATE THE VALUE OF A TRIG EXPRESSION WITHOUT USING A CALCULATOR

→ Establish whether you need a rough sketch or special triangles or ASTC rules.

✓ EXPRESS TRIG RATIO IN TERMS OF THE GIVEN VARIABLE.

→Draw a rough sketch with given angle and label 2 of the sides.

→The 3rd side can then be determined using Pythagoras.

→Express each of the angles in question in terms of the angle in the rough sketch.

✓ SIMPLIFY A TRIGONOMETRICALLY EXPRESSION.

→Use the ASTC rule to simplify the given expression if possible.

→See if any of the identities can be used to simplify it, if not see if it can be factorized.
→Check again if any identity can be used.

→This includes using the compound and double angle identities

✓ PROVE A GIVEN IDENTITY.

→Simplify the one side of the equation using reduction formulae and identities until it cannot simplify any further

✓ SOLVE A TRIG EQUATION.

→Find the reference angle by ignoring the "-"sign and write down the two solutions

→Then write down the general solution for the given eq.

→From the general solution you can determine the solution for any specified interval by using various values of k

J. TRIGONOMETRY GRAPHS

i. Consider:$y = asinb(x - p) + q$ or $y = acosb(x - p) + q$

- ✓ a: amplitude $= \frac{1}{2}$(distance between the y − values of T. P)
- ✓ b: co − efficient of an angle used to calculate the period

- ✓ **p affects the horizontal shift of the graph.**

  • If p > 0 (positive), the graph moves p degrees to the left.

  •If p < 0 (negative), the graph moves p degrees to the right.

- ✓ **q affects the horizontal shift of the graph.**

  - If q > 0 (positive), the graph moves upwards
  - If q < 0 (negative), the graphs moves downwards.

ii. Consider:$y = acosbx$ and $y = atanbx$

- ✓ Period(P) $= \frac{360°}{b}$ for sine and cosine
- ✓ Period(P) $= \frac{180°}{b}$ for tangent

- ➢ **Scale** : **with no transformation:$y = acosbx$**
- ✓ $scale = \frac{Period}{4}$

- ➢ **Scale: with transformation:$y = asinb(x - p) + q$ or $y = acosb(x - p) + q$**
- ✓ scale $= (90° - p)$with $p > 0$

- ➢ **Scale: with transformation:$y = atanb(x - p) + q$**
- ✓ scale $= (45° - p)$with $p > 0$

- **Range**

✓ Y values of the turning point.
✓ Use the closed brackets
✓ Start with the minimum to maximum value inside the brackets

- **Domain**

✓ We be always given.
✓ And is not the same as period

- **CALCULATING THE UNKNOWN**

✓ For one variable: use one point along the line
✓ For two variables: use two points along the line and solve the two equations simultaneously

- **POINT OF INTERSECTION**

✓ Means the point where two graphs are equal : $f(x) = g(x)$
✓ Equate the two graphs.
✓ Solve simultaneous.
✓ And the substitute with integers for the size of an angle.
✓ Co-functions can be used in the case of sine and cosine

- **FOR WHICH VALUE(S) OF X:**

a) $f'(x) > 0$ {where the graph is increasing}

b) $f'(x) < 0$ {where the graph is decreasing}

c) $f(x).g(x) > 0$ {where both graphs are above or below}

d) $f(x).g(x) < 0$ {One graph above and other one below the horizontal}

e) $f(x) > 0$ {above the horizontal}

f) $f(x) < 0$ {below the horizontal}

g) $g'(x) = 0$ {x-value of a turning point}

h) $g'(x) > 0 \; and \; f'(x) > 0$ {both graphs increasing}

i) $g'(x) < 0 \; and \; f'(x) < 0$ {both graphs decreasing}

j) $g(x) - f(x) = 1$ {graph $g(x)$ is above $f(x)$ and the distance is 1 unit}

k) $x.f(x) > 0$ {+ve x-value and +ve y-value or –ve x-value and –ve y-value}

l) $f(x).f'(x) < 0$ {+ve y-value and the decreasing curve}

Key note:

- ✓ Remember to manipulate the new given equation into the original one
- ✓ Maximum and minimum values means the y values of the turning point

- **QUESTIONS**

KWV 1

1. On the set of axes provided in the answer book, draw the sketch graphs of $f(x) = \cos 2x$ and $g(x) = \sin(x - 30°)$ for $x \in [-180° ; 90°]$. Clearly indicate the coordinates of the turning points and the intercepts with the axes.

2. Write down range for the following:

(a) $f(x) and\ g(x)$

(b) $h(x) = -f(x)$

(c) $h(x) = f(x) + 2$

3. Write down the period for the following:

(a) $f(x) and\ g(x)$

(b) $h(x) = f\left(\frac{2}{3}x\right)$

4. Write down the amplitude for the following:

(a) $f(x) and\ g(x)$

(b) $h(x) = 2f(x)$

c) $h(x) = -2f(x)$

5. Determine graphically the value(s) of x for which

(a) $f(x) = g(x)$

(b) $f(x) < g(x)$

(c) $f(x) > g(x)$

(d) $f(x) \geq g(x)$

(e) $f(x).g(x) \leq 0$

(f) $f(x).g(x) \geq 0$

(h) $f(x) = 0$

(i) $f(x) \geq 0$

(j) $g(x) = 0$

(k) $g(x) < 0$

(l) $g'(x) > 0$ And $f(x) > 0$

(m) $g(x) - f(x) = 1.5$

(n) the graph of f and g decreasing

(o) $g(x) \geq -1$

(p) the graph of f and g increasing

(q) $\frac{f(x)}{g(x)} > 0$

(r) $\frac{g(x)}{f(x)} \leq 0$

KWV 2

Given: $f(x) = 2cosx$ and $g(x) = tan2x$

a. Sketch the graph of f and g on the same set of axes, for $x \in [-90° ; 90°]$.

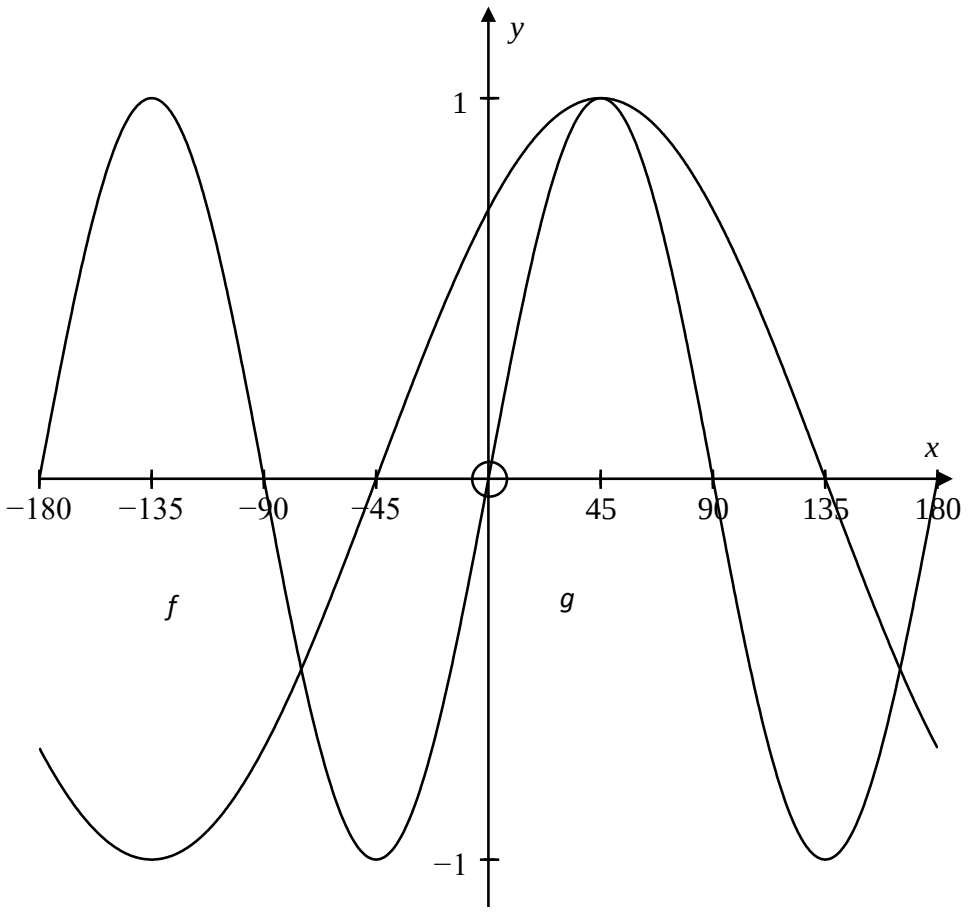
b. Solve for x if $2cosx = tan2x$ and $x \in [-90; 90]$. Show all working details.

c. Use the graph to solve for x: $2cosx.tan2x > 0$.

d. Write down the equation of the asymptotes of $g(x - 25°)$, where $x \in [-90°; 90]$.

KWV 3

The graphs of $f(x) = \{(x; y)/y = sinax\}$and $g(x) = \{(x; y)/y = \cos(x - b)\}$for the domain $x \in [-180°; 180°]$ are shown in the diagram below



(a) Write down the values of a and b.

(b) Write down the period of g.

(c) Determine the equation of h if $h(x) = f(x-30°)$.

(d) Explain how you would use the graphs to solve the equation: $\sqrt{2}\sin 2x = \cos x + \sin x$.

(e) Write down the amplitude and period of :

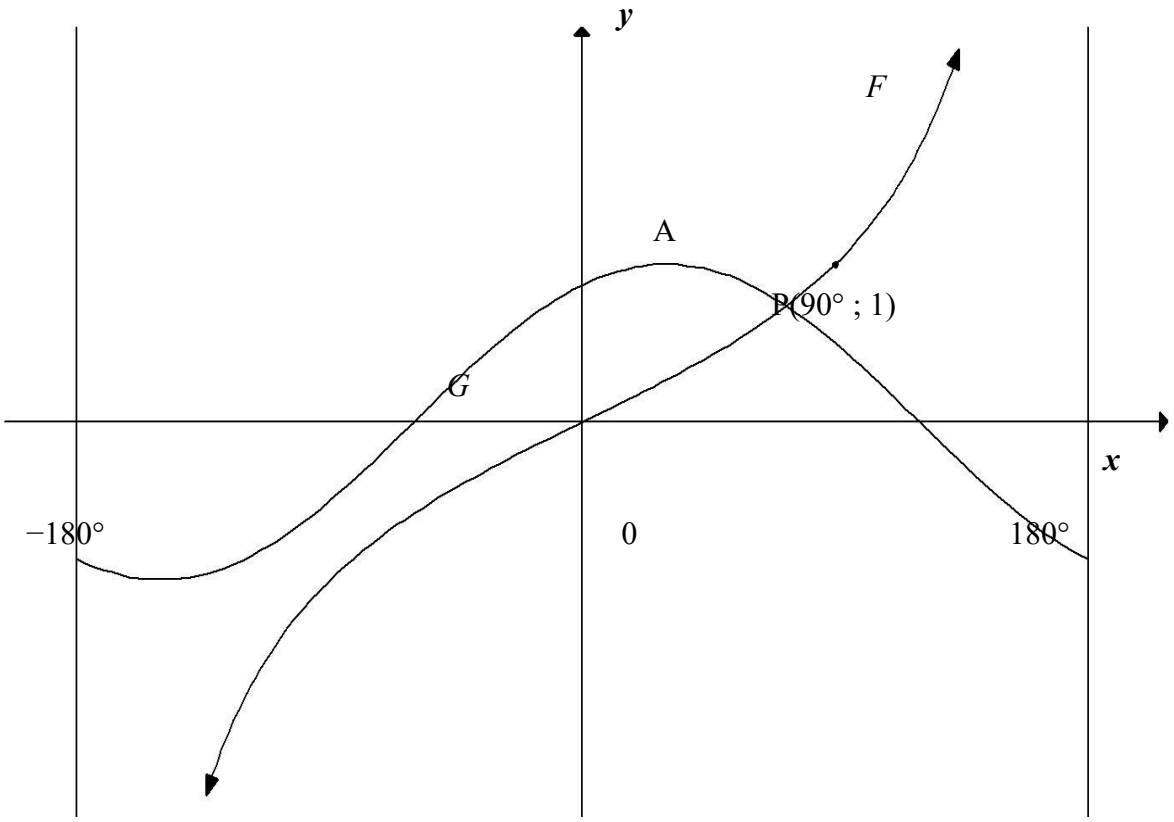
i. f

ii. $h(x) = f(\frac{2}{3}x)$

iii. $k(x) = \frac{f(x)}{2}$

KWV 4

In the diagram below, the graphs of $f(x) = tanbx$ and $g(x) = \cos(x - 30°)$ are drawn on the same system of axes for $-180° \leq x \leq 180°$. The point P (90°; 1) lies on *f*. Use the diagram to answer the following questions.



(a) Determine the value of b.

(b) Write down the coordinates of A a turning point of *g*.

(c) Write down the equation of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°; 180°]$.

(d) Determine the range of *h* if $h(x) = 2g(x) + 1$.

(e) Write down the new equation of g if is shifted 60 units to the right

(f) Write down the new equation of g if is shifted 60 units to the left

(h) Write down the equation of $h(x) = f(x - 30°) + 2$

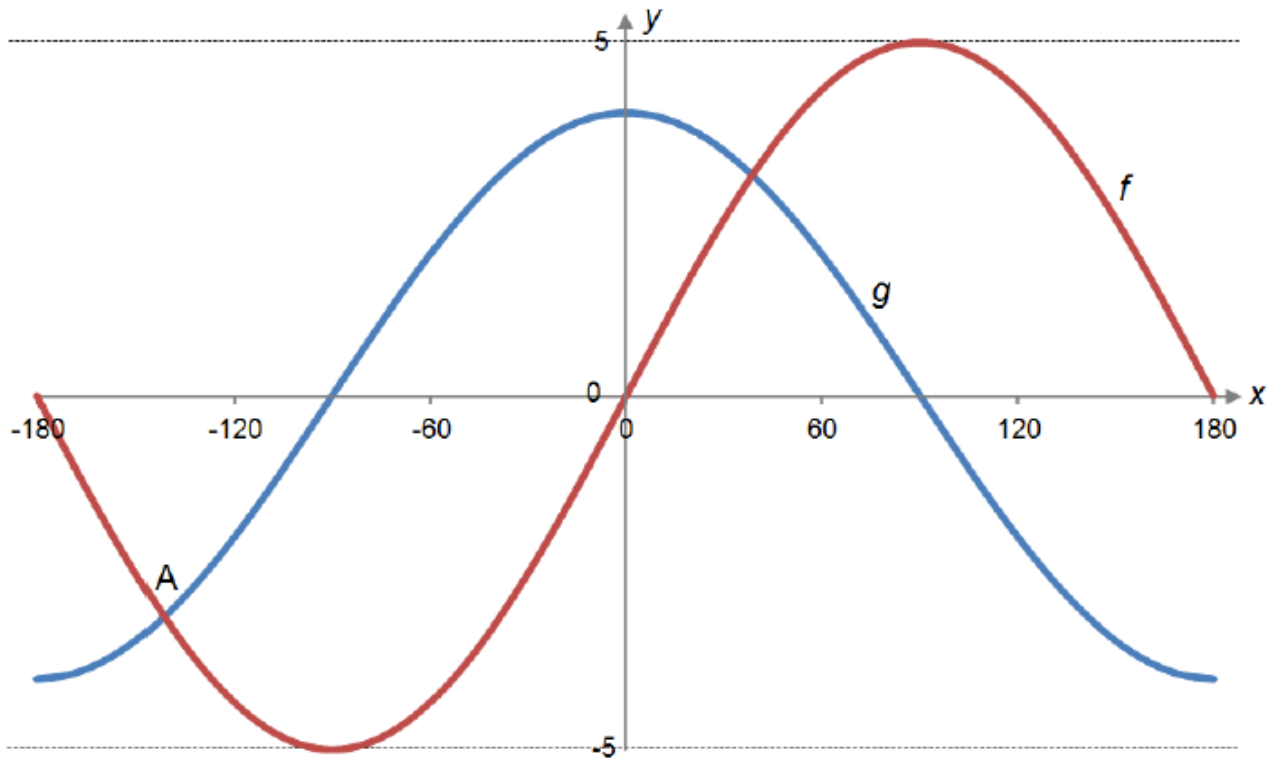
(i) Write down the new asymptote of f if it is shifted 30 units to the right

(j) Write down the new equation for $k(x) = f(x + 60°)$

KWV 5

In the diagram below:

- $f(x) = a\sin x \quad x \in [-180°; 180°]$
- $g(x) = 4\cos bx \quad x \in [-180°; 180°]$
- The graph of f passes through $(90°; 5)$ and the graph of g passes through $(90°; 0)$.



(a) Determine the values of a and b.

(b) Write down the new equation for $f(x)$ if it was shifted $30°$ to the right and two units vertically down.

(c) Calculate the minimum value of $\frac{8}{g(x)}$ if $x \in [-60°; 60°]$.

(d) For what values of k will $f(x) = k$ have no real solutions?

(e) Calculate the coordinates of point A. Give answers correct to one decimal place.

KWV 6

Consider: $f(x) = -2\tan\frac{3}{2}x$

1 Write down the period of f.

2 The point A(t ; 2) lies on the graph. Determine the general solution of t.

3 On the grid provided in the ANSWER BOOK, draw the graph of f for the interval $x \in [-120° ; 180°]$. Clearly show ALL asymptotes, intercepts with the axes and endpoint(s) of the graph.

4 Use the graph to determine for which value(s) of x will $f(x) \geq 2$ for $x \in [-120° ; 180°]$.

5 Describe the transformation of graph f to form the graph of $g(x) = -2\tan\left(\frac{3}{2}x + 60°\right)$.

K. TRIANGLE SOLUTIONS

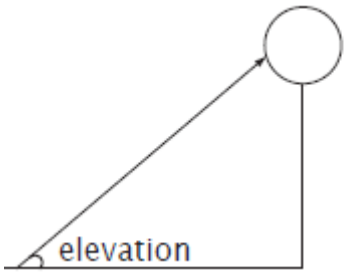
- ✓ Can only be used if the triangle is not a right angled triangle
- ✓ If a triangle is a right angled triangle simply use trig ratios
- ✓ First calculate the unknowns using the information given

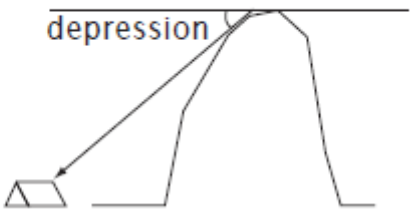
➢ **Take note:**

- ✓ Exterior angles of a triangle
- ✓ Sum of angles of a triangle
- ✓ Conditions of parallel lines
- ✓ Theorem from grade 11 work

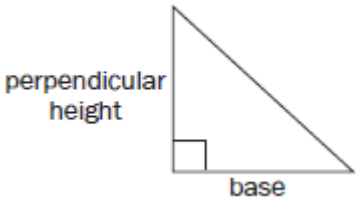
Angles of Elevation and Depression

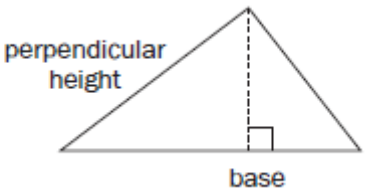
- ✓ angle of elevation measures up
- ✓ angle of depression measures down





➢ **For area rule**



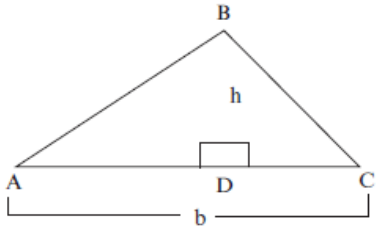
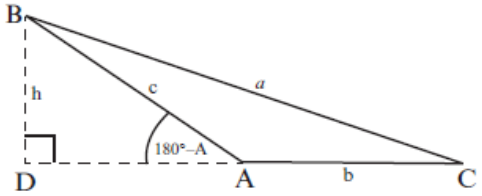


Area of a right-angled triangle

$$\text{Area } \Delta = \frac{1}{2} \text{ base} \times \text{perpendicular height}$$

$$\text{Area } \Delta = \frac{1}{2} bh$$

PROOF:

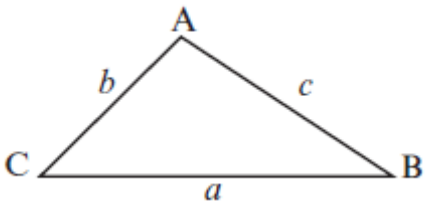
If $\hat{A}$ is acute	If $\hat{A}$ is obtuse
	
Area of $\Delta ABC = \frac{1}{2}bh$.....(1) But $\sin A = \frac{h}{c} \therefore h = c \sin A$ Substituting into (1) Area of $\Delta ABC = \frac{1}{2}bc \sin A$ Similarly it can be shown that Area of $\Delta ABC = \frac{1}{2}ab \sin C$ $= \frac{1}{2}ac \sin B$	Area of $\Delta ABC = \frac{1}{2}bh$.....(1) But $\sin (180° - A) = \frac{h}{c} \therefore h = c \sin A$ Substituting into (1) Area of $\Delta ABC = \frac{1}{2}bc \sin A$ Similarly it can be shown that Area of $\Delta ABC = \frac{1}{2}ab \sin C$ $= \frac{1}{2}ac \sin B$

- **CONDITIONS**

✓ SAS

- Formula:

  The area of any ΔABC is half the product of two sides and sine of the included angle.



So if you choose to use angle A, then

$$\text{Area } \Delta ABC = \frac{1}{2}bc \sin A$$

If you choose to use angle B, then

$$\text{Area } \Delta ABC = \frac{1}{2}ac \sin B$$

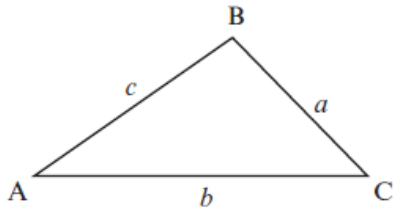
If you choose to use angle C, then

$$\text{Area } \Delta ABC = \frac{1}{2}ab \sin C$$

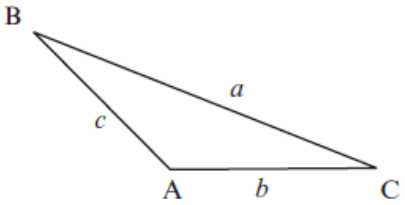
➢ **For sine rule**

PROOF:

If $\hat{A}$ is acute



If $\hat{A}$ is obtuse



Using Area rule for ΔABC:

$\frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B$

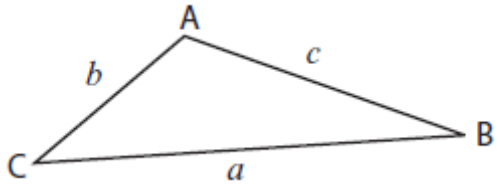
Dividing each by $\frac{1}{2}abc$ results: $\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}$

➢ **CONDITIONS**

✓ SSA

✓ AAS

➢ Formula:



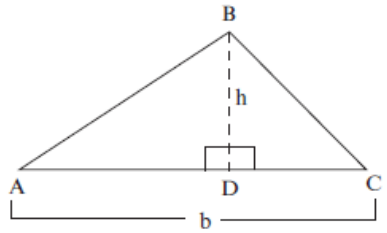
In any triangle ABC:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{OR} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

➢ **For cosine rule**

PROOF:

If $\hat{A}$ is acute



In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)
$= BD^2 + (b - AD)^2$
$= BD^2 + b^2 - 2bAD + AD^2$
But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)

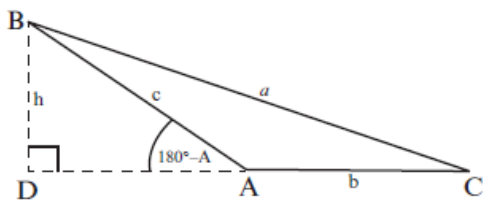
Thus $a^2 = b^2 + c^2 - 2bAD$ .....(1)

In Δ ABD: $\cos A = \frac{AD}{c} \therefore AD = c \cos A$ .....(2)

Substituting (2) into (1)
$\therefore a^2 = b^2 + c^2 - 2bc \cos A$

Similarly it can be shown that:
$b^2 = a^2 + c^2 - 2ac \cos B$ and
$c^2 = a^2 + b^2 - 2ab \cos C$

If $\hat{A}$ is obtuse



In ΔBDC: $a^2 = BD^2 + CD^2$ (Pythagoras Theorem)
$= BD^2 + (b + AD)^2$
$= BD^2 + b^2 + 2bAD + AD^2$
But $BD^2 + AD^2 = c^2$ (Pythagoras Theorem)

Thus $a^2 = b^2 + c^2 + 2bAD$ .....(1)

InΔABD:.
$\cos(180° - A) = \frac{AD}{c} \therefore AD = -c \cos A$.....(2)

Substituting (2) into (1)
$\therefore a^2 = b^2 + c^2 - 2bc \cos A$

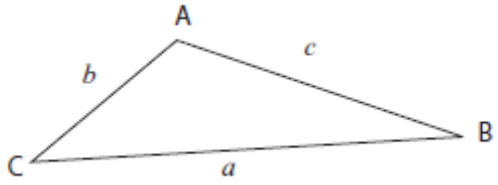
Similarly it can be shown that:
$b^2 = a^2 + c^2 - 2ac \cos B$ and
$c^2 = a^2 + b^2 - 2ab \cos C$

➢ **CONDITIONS**

✓ SAS

✓ SSS

➢ Formula:



In any triangle ABC:

If you choose to use angle A, then → $a^2 = b^2 + c^2 - 2bc \cos A$

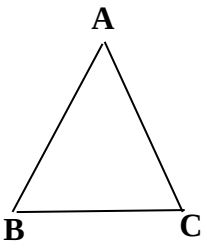
If you choose to use angle B, then → $b^2 = a^2 + c^2 - 2ac \cos B$

If you choose to use angle C, then→ $c^2 = a^2 + b^2 - 2ab \cos C$

- **QUESTIONS**

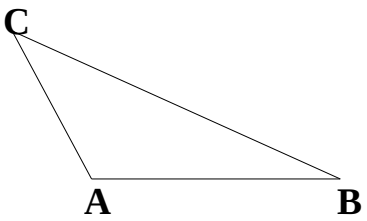
KVW 1

(a) Use the diagram below to prove that in any ΔABC.



Area of ΔABC = ½ a b Sin C

(b) In Δ ABC, Â is obtuse.



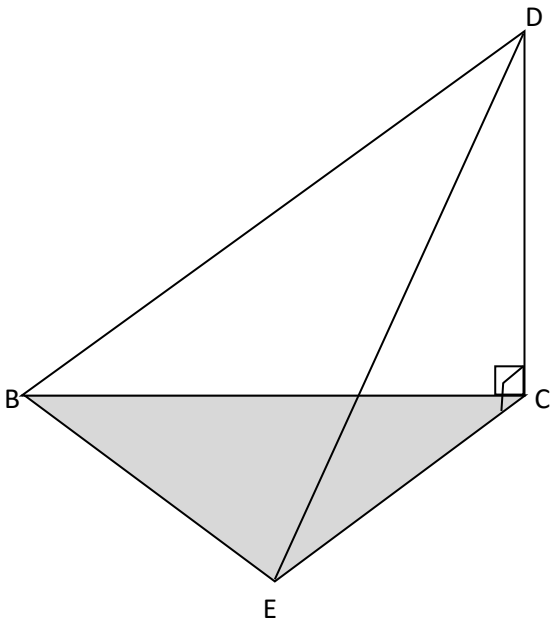
Prove that $\frac{a}{\sin A} = \frac{b}{\sin B}$

(c) Use the diagram in (b) to prove cosine rule

KWV 2

In the figure below, CD is a vertical mast. The points B, C and E are in the same horizontal plane.BD and ED are cables joining the top of the mast to pegs on the ground. DE = 28,1 m and BC = 20,7 m. The angle of elevation of D from B is 43,6°.

$C\hat{B}E = 63°$ and $B\hat{D}E = 35{,}7°$.



Give your answers correct to ONE decimal place in each of the following questions:

(a) Calculate the length of BD.

(b) Calculate the length of BE.

(c) Calculate the area of ΔBEC.

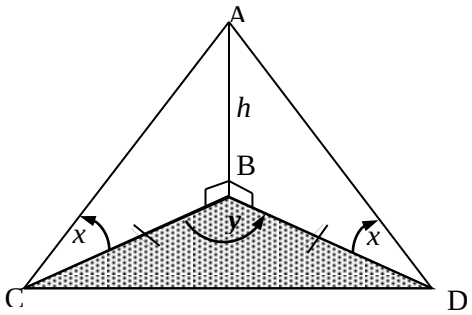
(d) Calculate the length of DC

(e) Hence, calculate angle DEC

(f) Calculate the perimeter of BDCE

KWV 3

In the diagram, C and D represent two ships in the same horizontal plane as B, the base of a lighthouse AB which is *h* metres high. Each ship is the same distance away from the base of the lighthouse. The angles of elevation from C and D to A are both *x*. $C\hat{B}D = y$.



(a) Write down BD in terms of *h* and a trigonometric ratio of *x*.

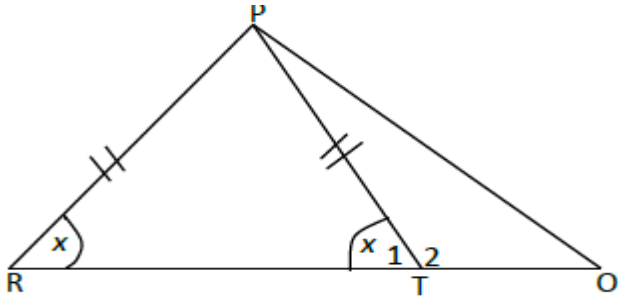
(b) Write down the size of angle CDB

(c) Hence, or otherwise, prove that $CD^2 = \frac{2h^2(1-\cos y)}{\tan^2 x}$

(d) Calculate CD if $h = 23\ cm, y = 150°\ and\ x = 3$

KWV 4

In the figure below



ΔPRT, has PR = PT.

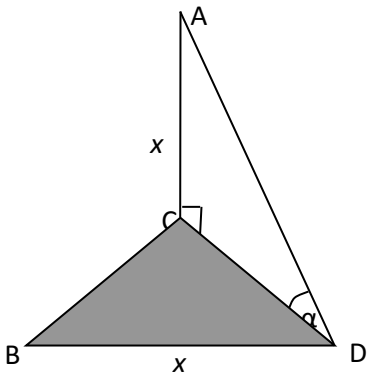
If TO = $\frac{1}{2}$RT = a and $\hat{R} = \hat{T}_1 = x$,

a. Prove that $PT = \frac{a}{cosx}$.

b. Show that the area of ΔPOT = $\frac{1}{2}\ a^2 \tan x$

KWV 5

In the figure B and D are points in the same horizontal plane as C, the foot of a vertical tower AC. If AC = BD = x and $B\hat{D}C = 90° - \alpha$

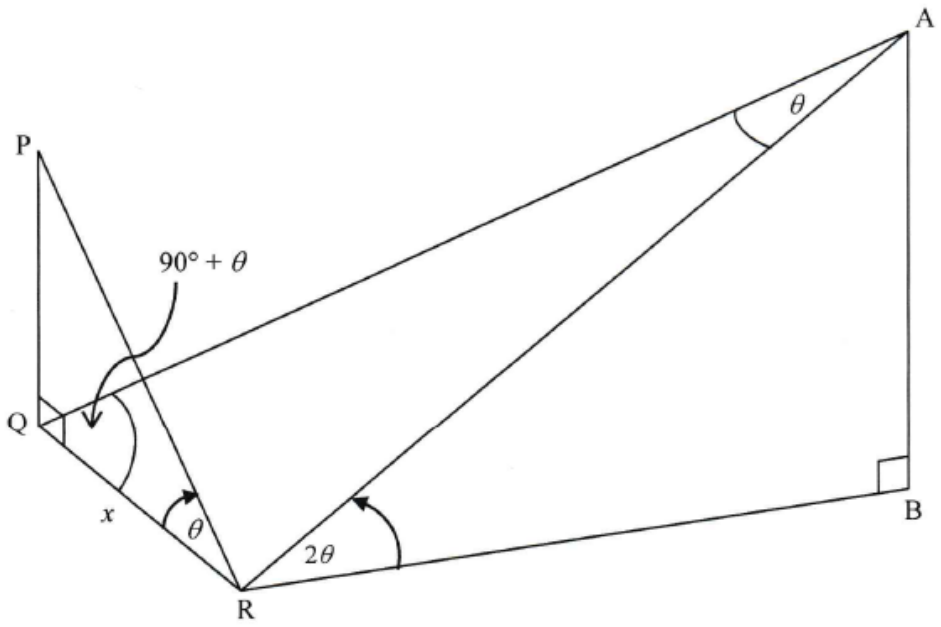


(a) Prove that CD = $\frac{x}{\tan\alpha}$.

(b) Show that BC = $\frac{x}{\tan\alpha}\ \sqrt{\tan^2\alpha - 2\sin\alpha.\tan\alpha + 1}$

KWV 6

PQ and AB are two vertical towers.
From a point R in the same horizontal plane as Q and B, the angles of elevation to P and A are θ and 2θ respectively.
$A\hat{Q}R = 90° + \theta$, $Q\hat{A}R = \theta$ and $QR = x$.



1. Determine in terms of x and θ

   (a) QP

   (b) AR

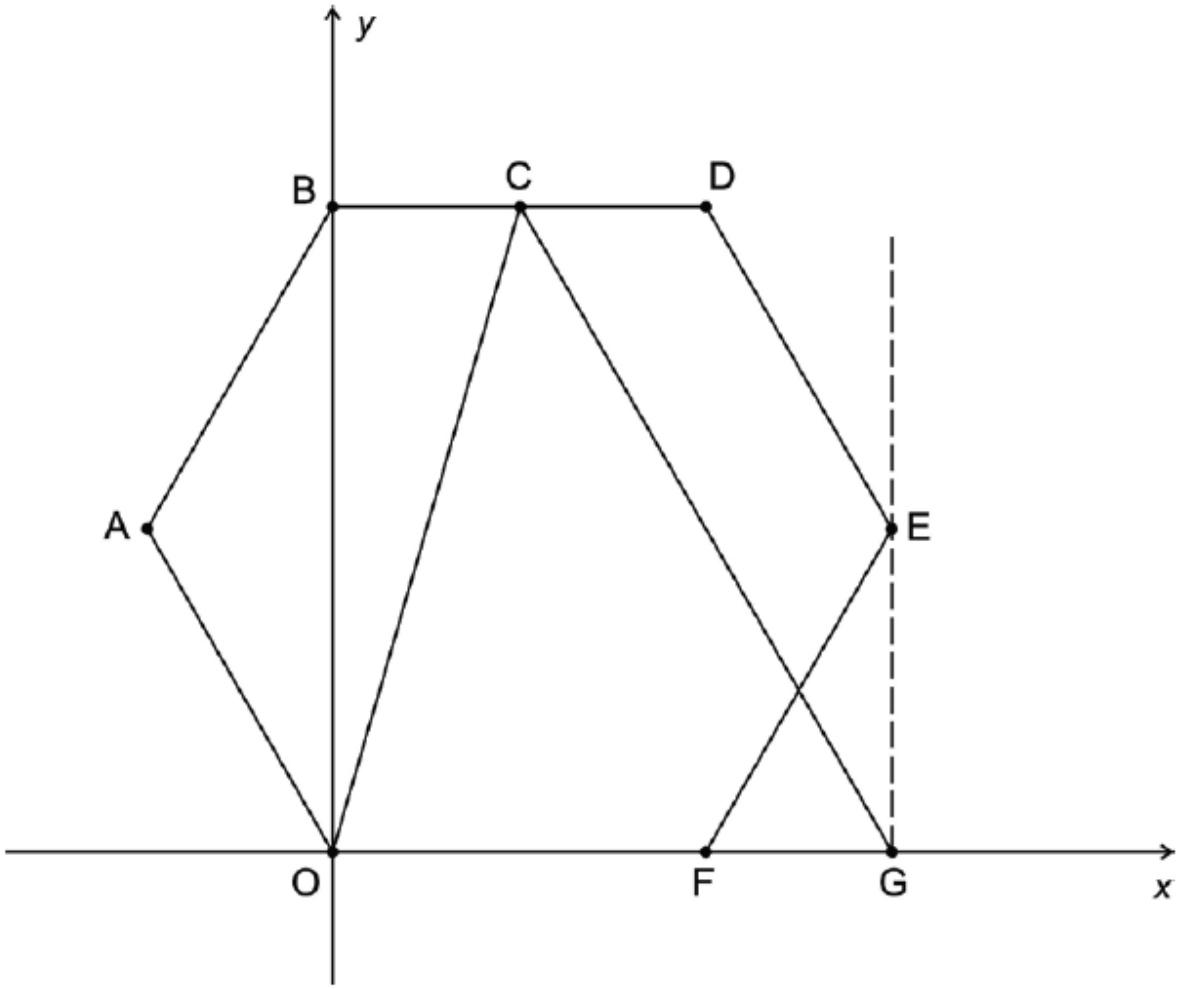
2. Show that $AB = 2x\cos^2\theta$

3. Determine $\frac{AB}{QP}$ IF $\theta = 12°$

KWV 7

In the Cartesian plane below:

- The regular hexagon ABDEFO has side lengths of 4 units.
- EG is perpendicular to the x–axis.
- C is the midpoint of line BD.
- O is the origin and B, F and G are points on the axes.



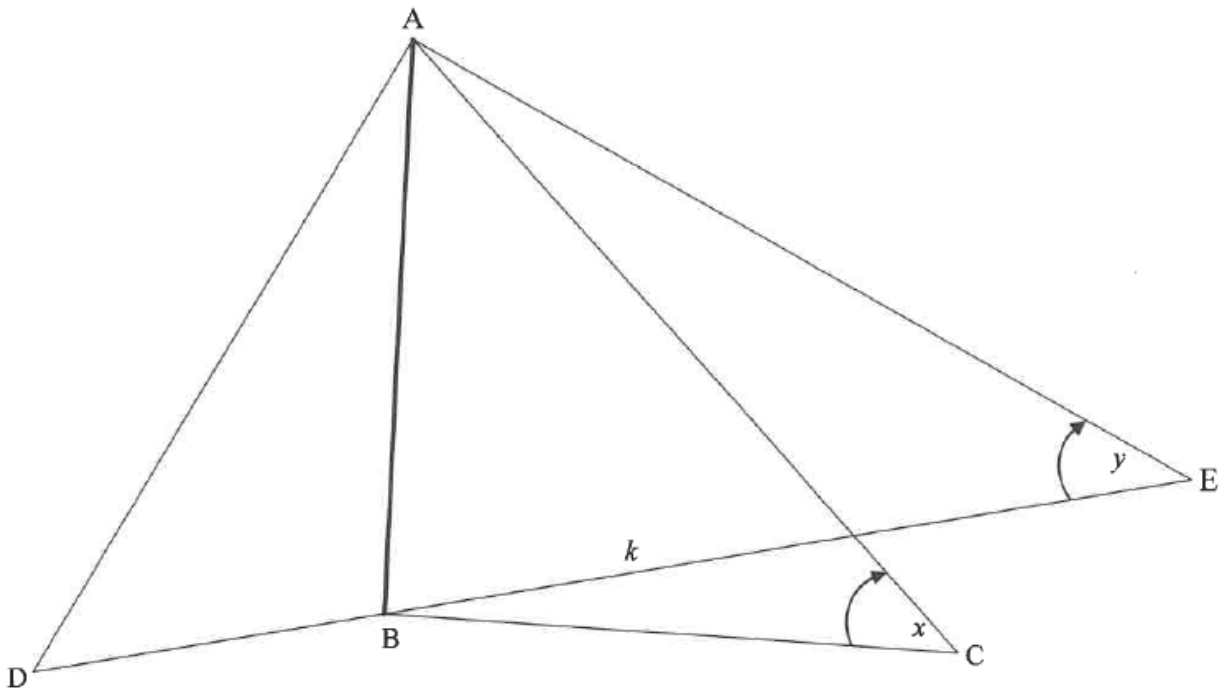
(a) Determine the coordinates of point C. Leave your answers in surd form.

(b) Determine the area of ΔOCG. Leave your answers in surd form.

(c) Determine the size of $O\hat{C}G$. Give your answer correct to one decimal place.

KWV 8

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



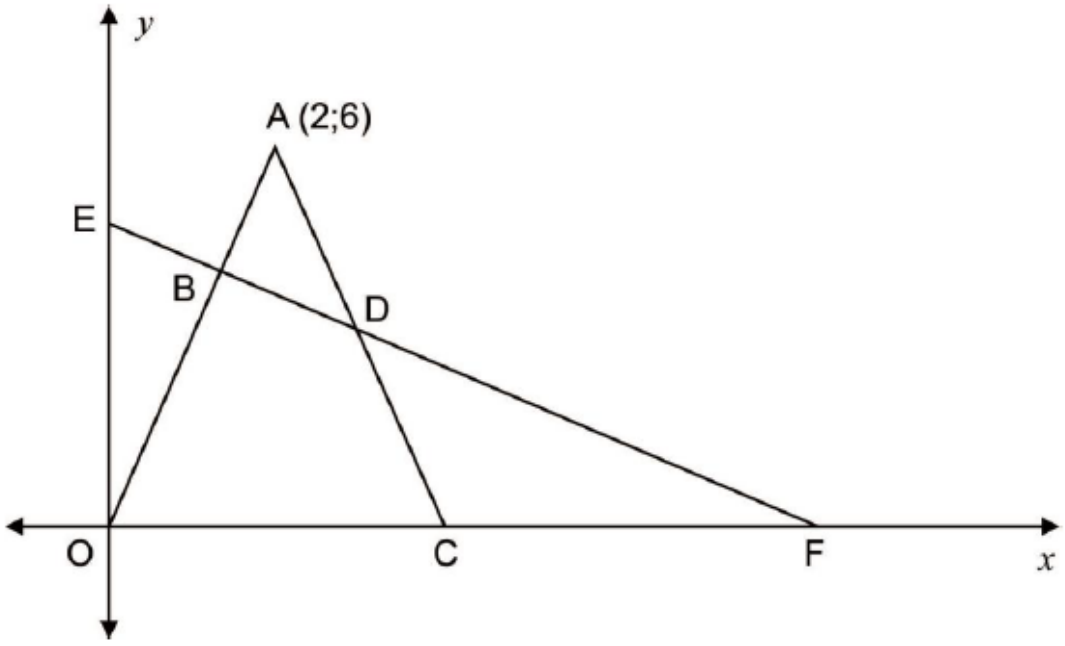
1 Write down the size of $A\hat{B}C$.

2 Show that $AC = \dfrac{k.\tan y}{\sin x}$

3 If it is further given that $D\hat{A}C = 2x$ and AD = AC, show that the distance DC between the players at D and C is $2k\tan y$.

In the diagram below, ΔAOC with A(2;6) and O(0;0) is drawn.

- C is a point on the x-axis.
- AO = AC.
- E lies on the y-axis and F lies on the x-axis.
- Line EF goes through the points B and D on OA and CA respectively.
- The equation of EF is given by $2y + x = 10$.



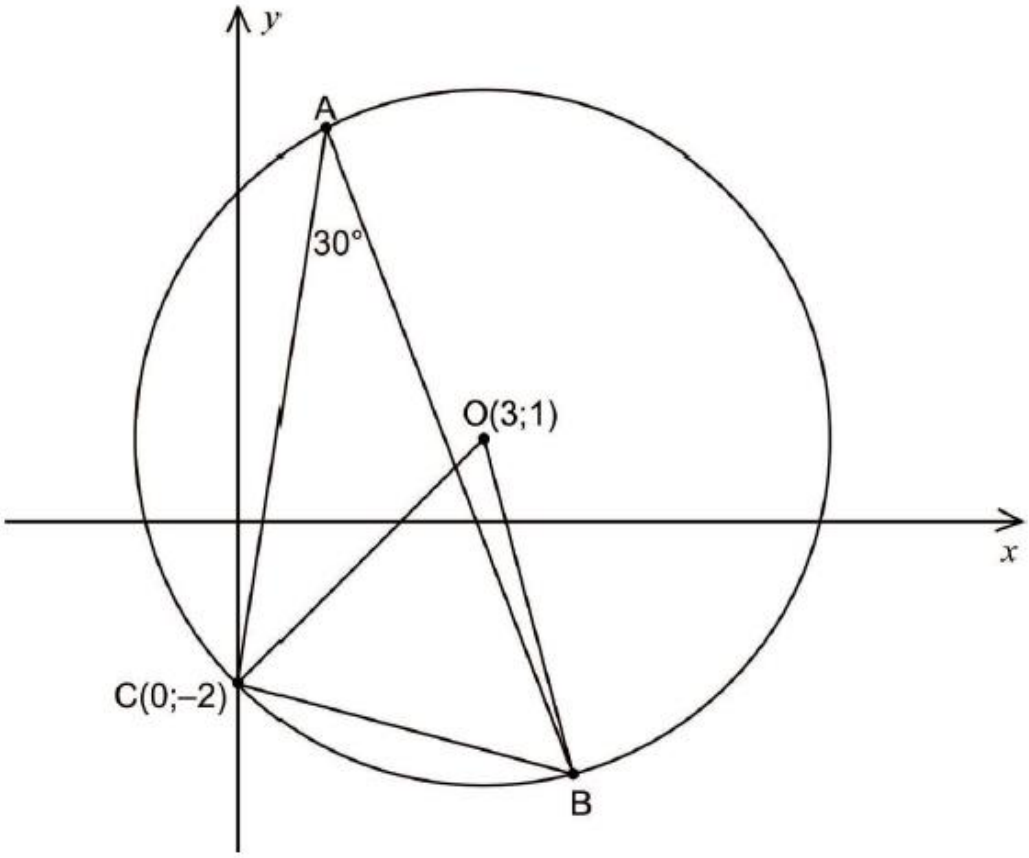
(a) Determine the coordinates of B and hence the area of ΔEBO.

(b) If $D\left(x;\frac{18}{5}\right)$, determine the area of ΔDCF.

KWV 9

In the Cartesian plane below, circle centre O(3;1) is drawn.

- A and C(0;–2) are fixed points on the circle.
- $C\hat{A}B = 30°$.
- B is a variable point on the circle.



(a) (1) Determine the length of OC.

(2) B moves along the circle until BC is parallel to the *x*-axis. Write down the new coordinates of B.

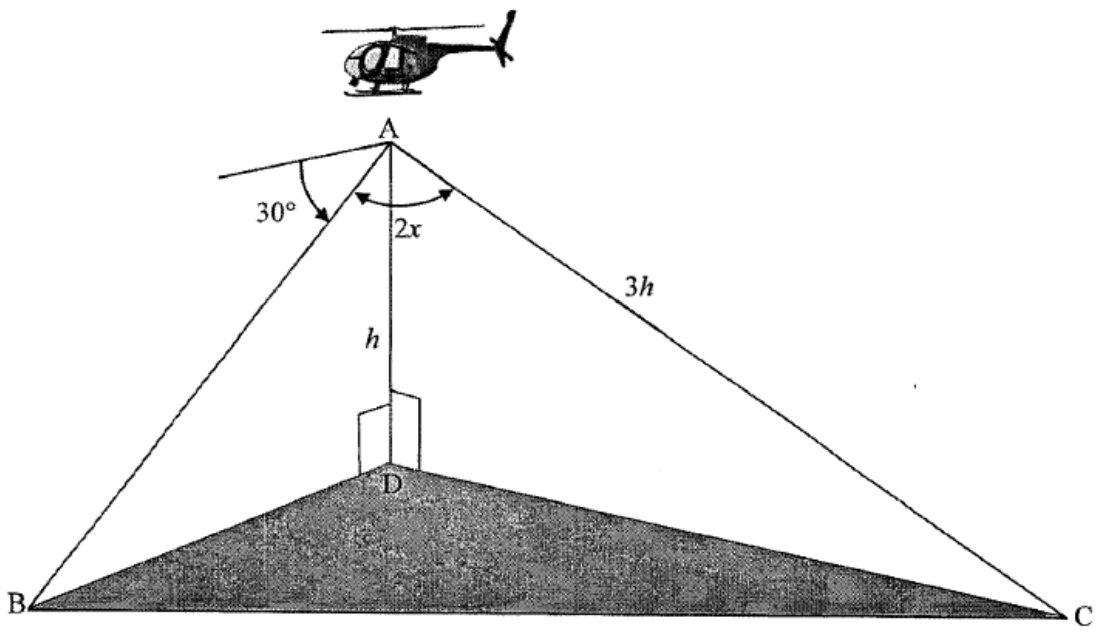
(3) Calculate the size of $C\hat{A}B$ for this new position of B. Give all reasons.

(b) B moves from its **original position** along the circle in an anti-clockwise direction until the area $\Delta OBC = \frac{9}{2}$ square units.

Find the shortest distance that B has to move along the circle for the above to occur.

KWV 10

A pilot is flying in a helicopter. At point A, which is h metres directly above point D on the ground, he notices a strange object at point B. The pilot determines that the angle of depression from A to B is 30°. He also determines that the control room at point C is $3h$ metres from A and $B\hat{A}C = 2x$. Points B, C and D are in the same horizontal plane. This scenario is shown in the diagram below.



1 Determine the distance AB in terms of h.

2 Show that the distance between the strange object at point B and the control room at point C is given by $BC = h\sqrt{25 - 24\cos^2 x}$.

YOUR NOTES:

MERCY!!!!!

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