MATHEMATICS

Matric Intervention Programme

2018

CALCULUS II

Optimisation

Mixed Exam Questions

GRADE 12

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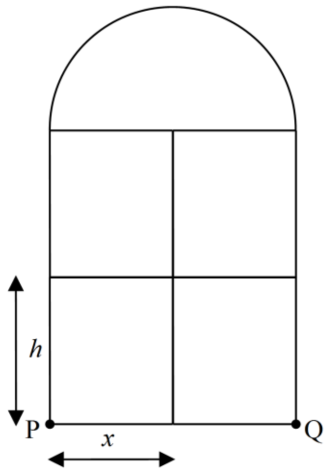
Common Exam Type Questions

Question 1: A chapel window consists of four equal rectangles and a semi-circle.
The length of the metal that is being used for the frame is 36 metres.



1.1 Prove that the area for the frame is given by: $A = 24x - 4x^2 - \frac{\pi x^2}{6}$

SOLUTION:

$$P = 6h + 6x + \frac{1}{2}(2\pi x)✓ \qquad A = 4xh + \frac{1}{2}\pi x^2 ✓$$

$$\therefore 36 = 6h + 6x + \pi x$$

$$\therefore \frac{36 - 6x - \pi x}{6} = h✓$$

$$\therefore A = 4x\left(\frac{36 - 6x - \pi x}{6}\right) + \frac{1}{2}\pi x^2 ✓$$

$$\therefore A = 24x - 4x^2 ✓ - \frac{4\pi x^2}{6} + \frac{1}{2}\pi x^2 ✓$$

$$\therefore A = 24x - 4x^2 - \frac{\pi x^2}{6}$$

1.2 Determine the length of the base PQ for a maximum area of the window.

SOLUTION:

$$A' = 0$$

$$\therefore 24 - 8x - \frac{1}{3}\pi x ✓ = 0 ✓$$

$$\therefore 24 = 8x + \frac{1}{3}\pi x$$

$$\therefore 24 = x\left(8 + \frac{1}{3}\pi\right) ✓$$

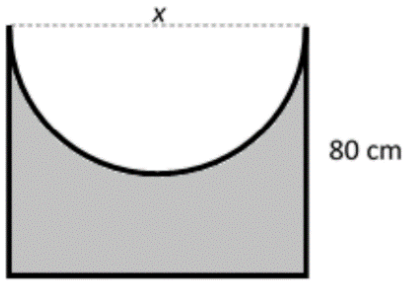
$$\therefore \frac{24}{\left(8 + \frac{1}{3}\pi\right)} = x$$

$$\therefore x = 2{,}65\text{m} ✓$$

$$\therefore \text{PQ} = 2{,}65 \times 2 = 5{,}31\text{m} ✓$$

Question 2

A semi-circle of diameter x is cut from a rectangle with a length of 80 cm.
Calculate the value of x which gives the maximum possible (shaded) area.
Give your answer in terms of π .



SOLUTION:

$$A = 80x - \frac{\pi\left(\frac{x}{2}\right)^2}{2} \checkmark\checkmark\checkmark$$

$$\therefore A = 80x - \frac{\pi x^2}{8}$$

$$\therefore A' = 80 - \frac{\pi x}{4} \checkmark\checkmark$$

$$\therefore x = \frac{320}{\pi} \quad cm \checkmark$$

Question 3

A cannon fires a projectile onto the top of a hill, such that the height s (in metres) reached by the projectile t seconds after it is fired from the cannon is given by the parabola equation, $s = 200t - 5t^2$.

If the velocity of the projectile, when it hits the hill on the way down, is 170 metres per second, what is the height of the hill where the projectile hits?

SOLUTION: $s = 200t - 5t^2$

$s' = 200 - 10t = 170 \therefore t = 3$, using symmetry (AOS: $x = 20$), $t = 37$

or $s' = 200 - 10t = -170 \therefore t = 37$

$s = 200(37) - 5(37)^2 = 555m$

Question 4

The perimeter of a shape is given by: $P = \frac{8}{x} + x\left(\frac{\pi}{2} + 2\right)$

Calculate the value of x that will make the perimeter (P) a minimum.

SOLUTION: $P = 8x^{-1} + \frac{\pi x}{2} + 2x$

$-8x^{-2} + \frac{\pi}{2} + 2 \checkmark = 0 \checkmark$

$\frac{\pi}{2} + 2 = \frac{8}{x^2} \checkmark$

$\pi x^2 + 4x^2 = 16$

$x^2(\pi + 4) = 16$

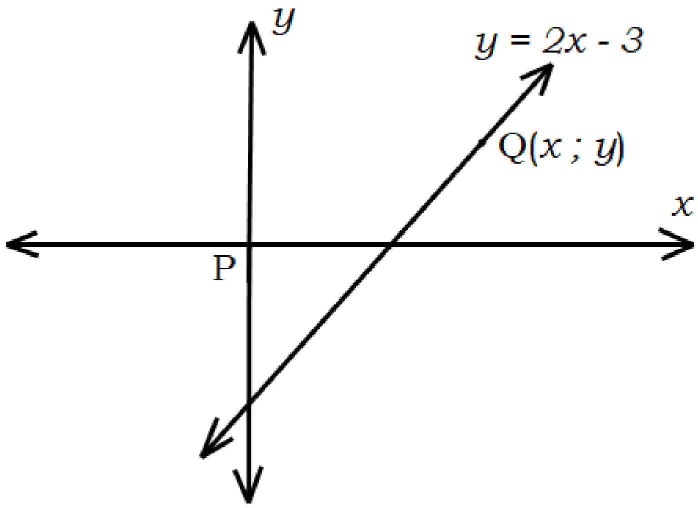
$x^2 = \frac{16}{\pi+4} \checkmark$

$x^2 = 2{,}24$

$x = 1{,}50 \checkmark$

Question 5

In the figure point $P(0;0)$ is the origin and $Q(x;y)$ is any point on the straight line $y = 2x - 3$.



5.1 Show that: $PQ^2 = 5x^2 - 12x + 9$

SOLUTION:

$$P(0\,;0)\ Q(x\,;2x-3)$$
$$\therefore PQ^2 = (x-0)^2 + (2x-3-0)^2$$
$$= x^2 + 4x^2 - 12x + 9$$
$$= 5x^2 - 12x + 9$$

5.2 Now, calculate the shortest distance from the origin to the straight line.

SOLUTION:

$$PQ^2 \text{ min}$$
$$\therefore \frac{dPQ^2}{dx} = 0$$
$$\therefore 10x - 12 = 0$$
$$x = \frac{6}{5}$$
$$\therefore PQ = \sqrt{5\left(\frac{6}{5}\right)^2 - 12\left(\frac{6}{5}\right) + 9}$$
$$\approx 1{,}3 \ units$$

5.3 Find the equation of the line which is the shortest distance from the origin to the given line $y = 2x - 3$

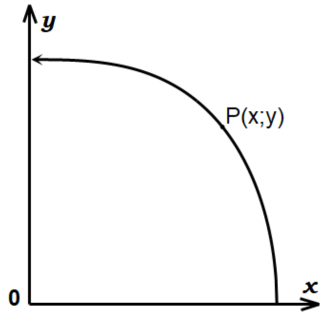
SOLUTION:

$$y = -\frac{1}{2}x$$

Question 6	A bucket has two pipes entering it. One is filling the tank at a variable rate while the other is draining it at a variable rate. The volume of water (in litres) in the tank at time t (in hours) with $t \in [0;3]$ is given by $V = -t^3 - 2t^2 + 15t$.
6.1	What is the average rate of flow in litres/hour in the first hour?
	SOLUTION: $at \quad t=0, \quad V=0$ ✓a $at \quad t=1, \quad V=12$ $average \quad flow = 12 \quad l/hr$ ✓ca
6.2	Did the volume of water increase or decrease over that time?
	SOLUTION: Increase
6.3	What is the instantaneous rate of flow at 2 hours?
	SOLUTION: $V = -t^3 - 2t^2 + 15t$ $\frac{dV}{dt} = -3t^2 - 4t + 15$ ✓m $at \quad t=2, \quad \frac{dV}{dt} = -5 \quad l/hr$ ✓a
6.4	At what point in the three hour time interval was the bucket fullest? Give your answer to the nearest minute.
	SOLUTION: $\frac{dV}{dt} = -3t^2 - 4t + 15 = 0$ ✓a $\therefore t = \frac{5}{3}$ $\therefore 1 \quad hour \quad 40 \quad minutes$ ✓ca
6.5	What is the maximum volume the bucket contained? Give your answer to the nearest litre.
	SOLUTION: $V = -\left(\frac{5}{3}\right)^3 - 2\left(\frac{5}{3}\right)^2 + 15\left(\frac{5}{3}\right) = 15 \quad litres$ ✓ca

QUESTION 1 (DOE Nov '01)

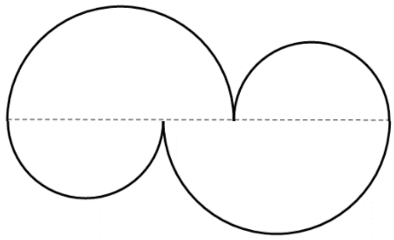
The graph of $x^3 + y^2 = 8$, not drawn to scale, is represented for the interval $x \in [0; 2]$.
$\mathrm{P}(x; y)$ is any point on the graph.



1.1 Determine OP^2 in terms of x and y.
1.2 Show that $OP^2 = -x^3 + x^2 + 8$.
1.3 Determine the maximum value of OP^2.
1.4 Calculate the shortest distance from the origin to the graph.

QUESTION 2 (DOE Nov '02)

An advertising company has asked you to design an advertising disc that consists of four semicircles and has the shape as shown in the figure below. The larger semicircles have radius R and the smaller semicircles have radius r. The values of R and r may vary but $R + r = 200mm$. To minimise costs the company has stated that the area of the shape must be a minimum.



2.1 Show that the area of the figure is given by: $A = 2\pi R^2 - 400\pi R + 4000\pi$
2.2 Use differential calculus to determine the values of R and r if the area, A, of the figure is a minimum.
2.3 Consider your solution in QUESTION 2.2 and explain why the shape suggested by the company is not possible if you want to maintain a minimum area.

QUESTION 3 (DOE Nov '02)

The profit yielded on a taxi is dependent on the average speed at which it is being driven. The profit (P) in rands per hour is calculate from the formula $P = -\frac{3}{80}x^2 + 6x - 180$, where x is the average speed in kilometres per hour, and $x \geq 30$.

Determine:

3.1 the speeds at which no profit is yielded.
3.2 the most economical speed and the corresponding profit per hour.
3.3 the speeds at which the owner will show a loss.

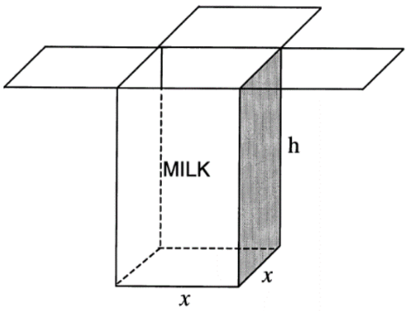
QUESTION 4 (DOE Nov '03)

An environmental study has revealed that the water from a factory has polluted the water in a nearby dam. The pollution had an effect on the water lilies growing on the water surface of the dam. At first the polluted water caused the plants to grow profusely and later on it caused the plants to die and the area covered by the plants to decrease. A mathematician established that the surface area, A (in m^2), covered by the water lilies can be approximated by the formula: $A = -\frac{1}{2}t^3 + 12t^2$, where t is the time in months after the start of the study, with $0 \leq t \leq 24$.

4.1 After how many months did the area covered by the water lilies reach a maximum?
4.2 Calculate the maximum area that was covered by the plants?
4.3 At what rate was the area increasing one month after the study began?
4.4 Sketch the graph of the area covered by the plants versus time.
Show the intercepts and the turning point.

QUESTION 5 (DOE March '03)

A rectangular container has a square base and contains 1 litre of milk. The container has a folding lid that covers 3 times the area of the base. The length of the side of the square base is x cm and the height of the container is h cm. $[1\ litre = 1000cm^3]$.
See the diagram below.



5.1 Show that the area, A, of cardboard used to make the container can be expressed as: $A = 4x^2 + \frac{4000}{x}$.
5.2 Determine the dimensions of the container in cm, so that the quantity of cardboard used is as small as possible.

QUESTION 6 (DOE Nov '04)

The depth (in metres) of water left in a dam t hours after a sluice gate has been opened to allow water to drain from the dam, is given by the formula: $L = 28 - \frac{1}{9}t^2 - \frac{1}{27}t^3$.

6.1 Calculate the average rate at which the depth changes in the first 3 hours.
6.2 Determine the rate at which the depth changes after exactly 2 hours.

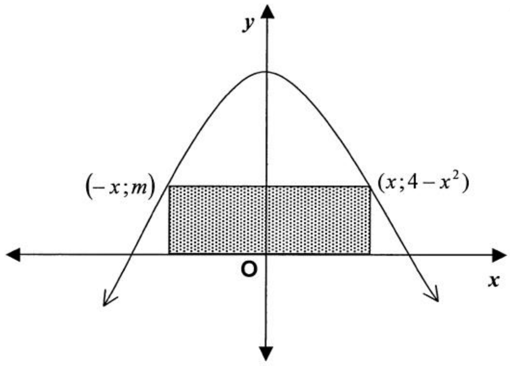
QUESTION 7 (DOE March '05)

During an experiment the temperature T (in degrees Celsius), varies with time t (in hours), according to the formula: $T(t) = 30 + 4t - \frac{1}{2}t^2; t \in [0; 10]$.

7.1 Determine an expression for the rate of change of temperature with time.
7.2 During which time interval was the temperature dropping?

QUESTION 8 (DOE Nov '04)

A rectangle has two vertices on the x-axis and two on the curve with equation: $y = 4 - x^2$, as shown in the sketch below.



8.1 Express m in terms of x.
8.2 Show that the area of the rectangle is given by: $A = 8x - 2x^3$
8.3 Determine the area of the largest rectangle which can be drawn in this manner.

QUESTION 9 (DOE March '05)

After doing some research, a transport company has determine that the rate at which petrol is consumed by one of its large carriers, travelling at an average speed of x km per hour is given by: $P(x) = \frac{55}{2x} + \frac{x}{200}$ litres per kilometre.

9.1 Assume that the petrol costs R4,00 per litre and the driver earns R18,00 per hour (travelling time). Now deduce that the total cost, C, in rands, for a 2000 km trip is given by: $C(x) = \frac{256\,000}{x} + 40x$
9.2 Hence determine the average speed to be maintained to effect a minimum cost for a 2000 km trip.

QUESTION 10 (DOE Nov '06)

10.1 A clothing manufacturer estimates that the cost (in rands) of producing x shirts is given by the function: $C(x) = 10 + 5x + 0{,}001x^3$

Calculate the rate at which the cost is changing when the 100th shirt is being produced.

10.2 There are 40 fruit trees in an orchard. The average yield per tree in a season is 580 fruit. The farmer calculates that for each additional tree planted in the orchard, the yield per tree will drop by 10 fruit. If the number of additional trees planted in the orchard is x and the total yield of the orchard in a season is N, then: $N = (40 + x)(580 - 10x)$.

Calculate how many additional trees must be planted to maximise the total yield of the orchard.

QUESTION 11

V represents the number of litres of water left in a tank, *t* minutes after it begins to drain, where $V = 4(5-t)^2$

11.1 Calculate the rate at which the quantity of water in the tank is decreasing when $t = 3$

11.2 How long does it take for the tank to empty?

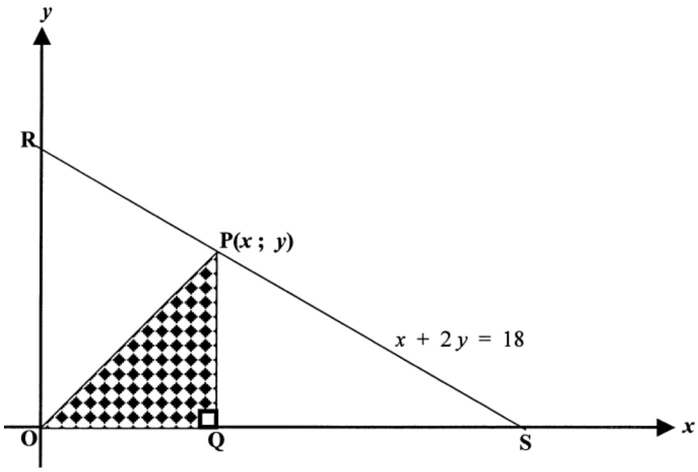
QUESTION 12 (DOE Nov SG '06)

Just after birth the mass of a baby drops for a few days and then starts to increase again. The average mass of a baby in its first 30 days of life can be approximated by the following equation: $m(t) = 0{,}02t^3 - 0{,}2t^2 + 3200;\ \ 0 \leq t \leq 30$ where t is the time in days and $m(t)$ is the mass in grams.

12.1 What is the mass of the baby at birth?
12.2 Calculate on what day the mass reaches a minimum.
12.3 Determine the maximum mass of the baby in the 30-day period.

QUESTION 13 (DOE March '06)

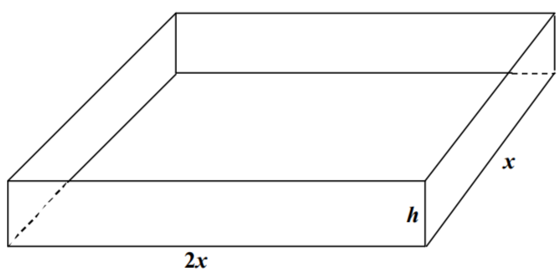
A point P lies on the line segment as shown.



If the equation of RS is given by $x + 2y = 18,\ \ 0 \leq x \leq 18$, and A is the area of the right-angled triangle OPQ, determine the coordinates of P so that the area of ΔOPQ is a large as possible.

QUESTION 14 (DOE Nov '07)

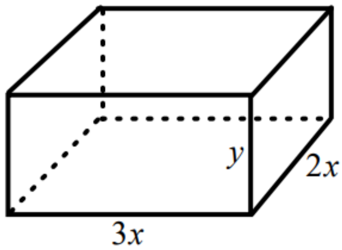
A container has the shape of a rectangular prism with an open top. Its volume is 1 cubic metre. The width of its rectangular base is x metres and the length of the base is $2x$ metres. The height is h metres. The material used for the base costs R5 per m^2 and the material used for the sides costs R180 per m^2.



14.1 Express h in terms of x.
14.2 Show that the cost, C, of material is given by $C = 10x^2 + \frac{540}{x}$.
14.3 Calculate the value of x for which the cost of material is a minimum.

QUESTION 15 (DOE Nov '07)

The sides of the base of a rectangular cardboard box are $3x$ and $2x$ cm respectively.
The height is y cm. The box is open on the top (without a lid).



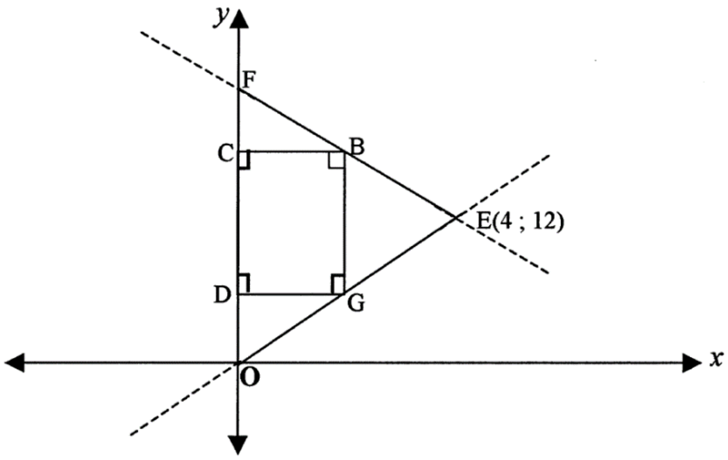
15.1 If the total exterior surface area of the box is $200cm^2$, prove that: $y = \frac{20}{x} - \frac{3x}{5}$

15.2 Express the volume of the box in terms of x.

15.3 Determine the value of x if its volume is to be a maximum.

QUESTION 16 (DOE Nov '08)

The graphs of $y = 3x$ and $y + 2x = 20$ (not drawn to scale) are shown below. $E(4; 12)$ is their point of intersection. The rectangle $GBCD$ is drawn as shown below.



16.1 If the x-coordinate of G is t , show that the area of $GBCD$ is given by: $A(t) = -5t^2 + 20t$.

16.2 Determine the value of t for which the area of $GBCD$ will be a maximum.

QUESTION 17 (DOE Nov '09)

A tourist travels in a car over a mountainous pass during his trip. The height above sea level of the car, after t minutes, is given as metres. The journey lasts 8 minutes.

17.1 How high is the car above sea level when it starts its journey on the mountainous pass?

17.2 Calculate the car's rate of change of height above sea level with respect to time, 4 minutes after starting the journey on the mountainous pass.

17.3 Interpret your answer to QUESTION 17.2.

17.4 How many minutes after the journey has started will the rate of change of height with respect to time be a minimum?

QUESTION 18 (NSC June '09)

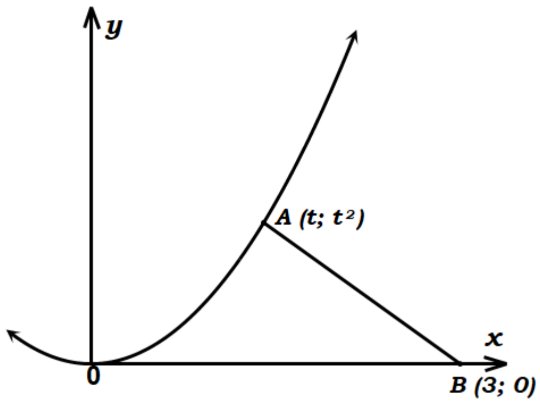
An environmental study showed that the water from a factory polluted the water in a nearby dam. The pollution had an effect on the water lilies growing on the water surface of the dam. At first the polluted water caused the plants to grow in large numbers, and later on it caused some of the plants to die, causing the area covered by the plants to decrease.

A mathematician established that the surface area, $A\ (in\ m^2)$, covered by the water lilies can be approximated by the formula: $A = -\frac{1}{3}t^3 + 10t^2$, where t is the time in months after the start of the study, with $0 \leq t \leq 30$.

18.1 After how many months did the area covered by the water lilies reach a maximum?
18.2 Calculate the maximum area that was covered by the plants.
18.3 At what rate was the area increasing one month after the study began?
18.4 Sketch the graph of the area covered by the plants versus time.
Show the intercepts with the axes and the turning point.

QUESTION 19 (DOE March 2009)

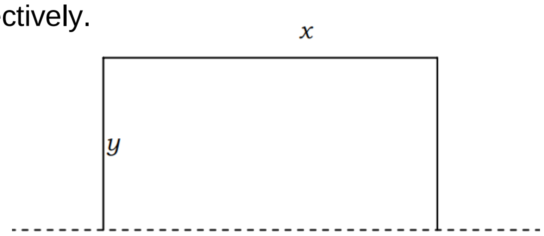
Sketched is the graph of $y = x^2$. $A(t; t^2)$ and $B(3; 0)$ are shown.



19.1 A$(t; t^2)$ is a point on the curve $y = x^2$ and the point $B(3; 0)$ lies on the x –axis.
Show that $AB^2 = t^4 + t^2 - 6t + 9$.
19.2 Hence, determine the value of t which minimises the distance AB.

QUESTION 20 (WCED Sept 2008)

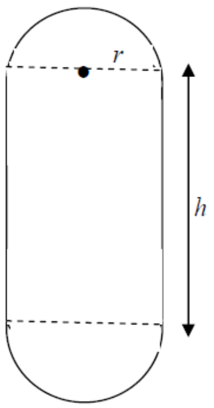
The municipality has decided to build a play park in one of the suburbs. It is to be rectangular with an area of $1800m^2$. Palisade fencing needs to be erected on three sides, since the fourth side already has a brick wall. Let the length and breadth of the play park be x and y respectively.



20.1 Determine an expression for the total length of fencing (L) required in terms of x and y.
20.2 Show that the total length of fencing required can also be written as $L = x + \frac{3600}{x}$.
20.3 Determine the least amount of fencing that will be needed to complete the job.

QUESTION 21 (DOE Nov 2010)

A satellite is to be constructed in the shape of a cylinder with a hemisphere at each end. The radius of the cylinder is r metres and its height is h metres (see diagram below). The outer surface area of the satellite is to be coated with heat-resistant material which is very expensive. The volume of the satellite has to be $\frac{\pi}{6}$ cubic metres.



Outer surface area of a sphere = $4\pi r^2$
Curved surface area of a cylinder = $2\pi rh$
Volume of a sphere = $\frac{4}{3}\pi r^3$
Volume of a cylinder = $\pi r^2 h$

21.1 Show that $h = \frac{1}{6r^2} - \frac{4r}{3}$

21.2 Hence, show that the outer surface area of the satellite can be given as:

$S = \frac{4\pi r^2}{3} + \frac{\pi}{3r}$.

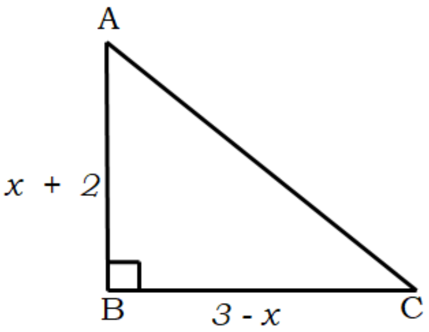
21.3 Calculate the minimum outer surface area of the satellite.

QUESTION 22 (IEB Nov 2009)

A company determines that the profit (in Rands) of producing x units of a certain product is : $P(x) = 50\sqrt{x} - 0{,}5x - 500$

Calculate the number of units that need to be produced so that the maximum profit is generated.

QUESTION 23



23.1 Show that the area of the triangle in the figure above is given by: $A(x) = \frac{x^2}{2} + 4x - \frac{15}{2}$

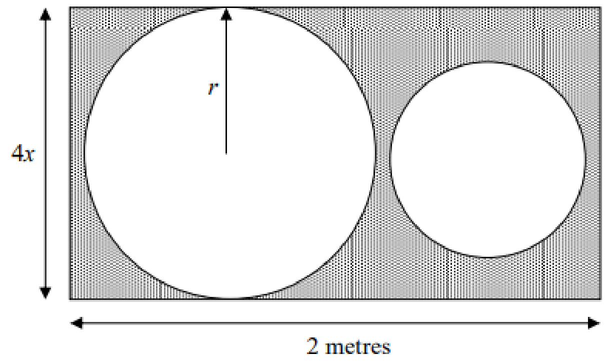
23.2 Calculate the value of x for which the area will be maximum.

23.3 Hence, calculate the maximum area of the given triangle

QUESTION 24 (DOE Nov 2009 V1)

Devan wants to cut two circles out of a rectangular piece of cardboard of 2 metres long and $4x$ metres wide. The radius of the larger circle is half the width of the cardboard and the smaller circle has a radius that is $\frac{2}{3}$ the radius of the bigger circle.

$A = lb$	$A = \pi r^2$	$P = 2(l+b)$	$C = 2\pi r$



24.1 Show that the area of the shaded region is $A(x) = 8x - \frac{52\pi}{9}x^2$.

24.2 Determine the value of x, such that the area of the shaded region is a maximum.

24.3 Calculate the total area of the circles, if the area of the shaded region is to be a maximum.

QUESTION 25 (DOE March 2010)

A wire, 4 metres long, is cut into two pieces. One is bent into the shape of a square and the other into the shape of a circle.

25.1 If the length of wire used to make the circle is x metres, write in terms of x the length of the sides of the square in metres.

25.2 Show that the sum of the areas of the circle and the square is given by:

$f(x) = \left(\frac{1}{16} + \frac{1}{4\pi}\right)x^2 - \frac{x}{2} + 1$ square metres.

25.3 How should the wire be cut so that the sum of the areas of the circle and the square is a minimum?

QUESTION 26 (DOE Nov 2011)

Water is flowing into a tank at a rate of 5 litres per minute. At the same time water flows out of the tank at a rate of k litres per minute. The volume (in litres) of water in the tank at time t (in minutes) is given by the formula $V(t) = 100 - 4t$.

26.1 What is the initial volume of the water in the tank?

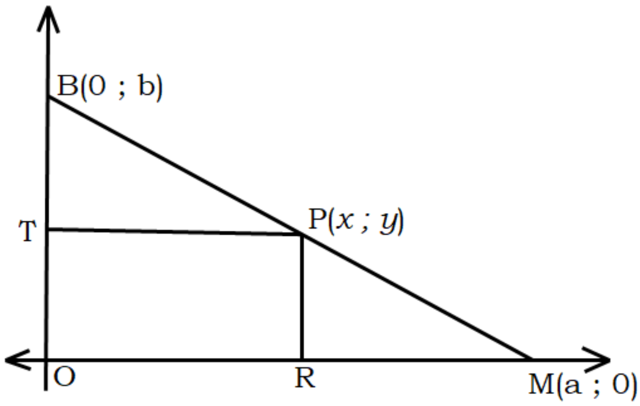
26.2 Write down TWO different expressions for the rate of change of the volume of Water in the tank.

26.3 Determine the value of k (that is, the rate at which water flows out of the tank).

QUESTION 27 (DOE March 2011)

A farmer has a piece of land in the shape of a right-angled triangle OMN, as shown in the figure below. He allocates a rectangular piece of land $PTOR$ to his daughter, giving her the freedom to choose P anywhere along the boundary MN.

Let $OM = a, ON = b$ and $P(x\,;\,y)$ be any point on MN.



27.1 Determine an equation of MN in terms of a and b.
27.2 Prove that the daughter's land will have a maximum area if she chooses P at the midpoint of MN.

QUESTION 28 (DOE Nov 2012)

A particle moves along a straight line. The distance, s, (in metres) of the particle from a fixed point on the line at time t seconds $(0 \geq t)$ is given by $s(t) = 2t^2 - 18t = 45$.

28.1 Calculate the particle's initial velocity. (Velocity is the rate of change of distance.)
28.2 Determine the rate at which the velocity of the particle is changing at t seconds.
28.3 After how many seconds will the particle be closest to the fixed point?

QUESTION 29 (DOE Nov 2013)

Am industrial process requires water to flow through its system as part of the cooling cycle. Water flows continuously through the system for a certain period of time. The relationship between the time (t) from when the water starts flowing and the rate (r) at which the water is flowing through the system is given by the equation: $r = -0{,}2t^2 + 10t$ where t is measured in seconds.

29.1 After how long will the water be flowing at the maximum rate?
29.2 After how many seconds does the water stop flowing?

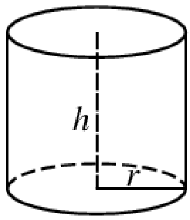
QUESTION 30 (DBE 2015)

The mass of a baby in the first 30 days of life is given by: $M(t) = t^3 - 9t^2 + 3000 \quad ; \quad 0 \leq t \leq 30$.
t is the time in days and M is the mass of the baby in grams.

30.1 Write down the mas of the baby at birth.
30.2 A baby's mass usually decreases in the first few days after birth. On which day will the baby's mass return to normal?
30.3 On which day will this baby have a minimum mass?
30.4 On which day will the baby's mass be decreasing the fastest?

QUESTION 31

A soft-drink company asks you to design a can with a volume of 450 cubic centimetres. The formula for the volume of the can is given by $V = \pi r^2 h$ cubic centimetres, where *r* is the radius of the base, and h is the height of the can.



31.1 Determine h in terms of r.

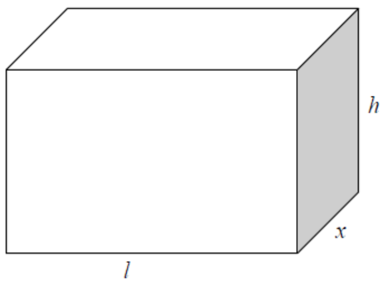
31.2 The total surface area of a can is given by the formula: $A = 2\pi r^2 + 2\pi rh$
Show that the total surface area, in terms of the radius, is given by: $A = 2\pi r^2 + \frac{900}{r}$

31.3 Determine the radius for which the surface area will be a minimum.
(Answer correct to two decimal places.)

31.4 The company wants you to use as little material as possible in designing the can.
What should the height of the can be?
(Answers correct to two decimal places.)

QUESTION 32 (DOE March 2013)

A rectangular box is constructed in such a way that the length (l) of the base is three times as long as its width. The material used to construct the top and the bottom of the box costs R100 per square metre. The material used to construct the sides of the box costs R50 per square metre. The box must have a volume of $9m^3$.
Let the width of the box be x metres.



32.1 Determine an expression for the height (h) of the box in terms of x.

32.2 Show that the cost to construct the box can be expressed as: $C = \frac{1200}{x} + 600x^2$

32.3 Calculate the width of the box (that is the value of x) if the cost is to be a minimum.

QUESTION 33 (GDE Trials 2014)

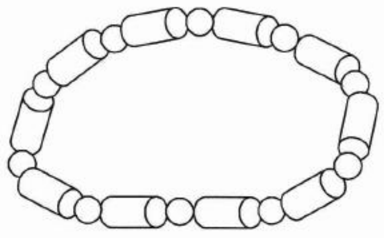
The displacement, (s) (distance from a fixed point), of a car moving towards a fixed point after t seconds, is given by: $s = t^3 - 9t^2 + 24t + 8$.

33.1 Determine an expression for the velocity (v) of the car.
(rate of change of distance with respect to time)

33.2 Calculate the time t for which the car was reversing.

QUESTION 34 (DOE March 2015)

A necklace is made by using 10 wooden spheres and 10 wooden cylinders. The radii, r, of the spheres and the cylinders are exactly the same. The height of each cylinder is h. The wooden spheres and cylinders are to be painted. (Ignore the holes in the spheres and cylinders.)



$V = \pi r^2 h$	$S = 2\pi r^2 + 2\pi r h$
$V = \frac{4}{3}\pi r^3$	$S = 4\pi r^2$

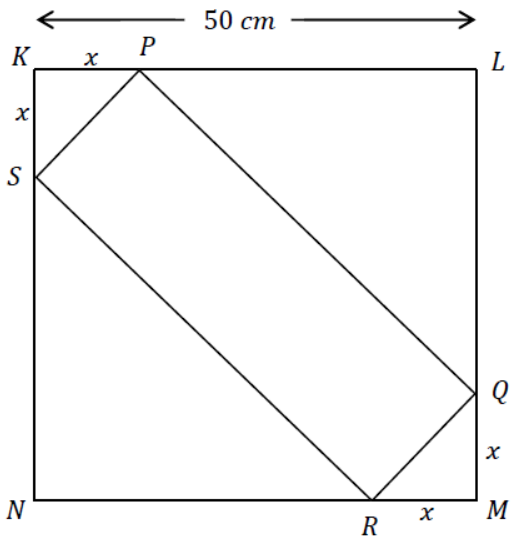
34.1 If the volume of the cylinder is $6cm^3$, write h in terms of r.

34.2 Show that the total surface area (S) of all painted surfaces of the necklace is equal to $S = 60\pi r^2 + \frac{120}{r}$.

34.3 Determine the value of r so that the least amount of paint will be used.

QUESTION 35 (ECED Trials 2014)

$KLMN$ is a square in the figure below. $KL = 50cm$. Rectangle $PQRS$ is drawn inside square $KLMN$ so that $KP = KS = RM = QM = x$.



35.1 Show that the area (A) of rectangle PQRS can be expressed as: $A = 100x - 2x^2$.

35.2 Hence determine the maximum area of PQRS. Show all calculations.

QUESTION 36 (GDE Trials 2015)

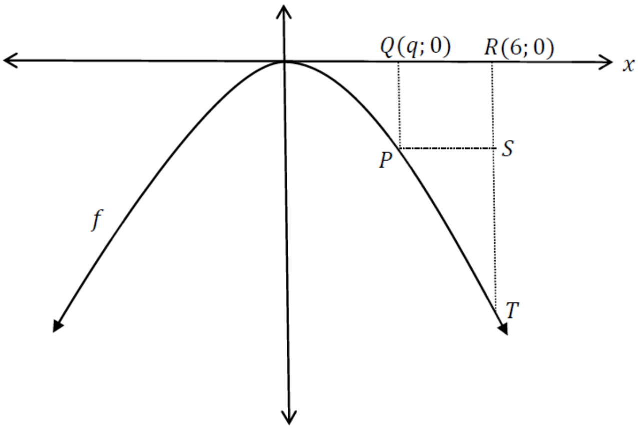
The profit P in rand/day which the owner of a taxi makes when the taxi is driven at an average speed of x km/hour, is given by the formula: $P = x^2\left(40 - \frac{x}{3}\right)$.

36.1 Calculate the average speed at which the owner will suffer a loss in revenue.

36.2 Calculate the most economical speed average speed of the taxi and the corresponding daily profit.

QUESTION 37 (ECED Trials 2015)

The graph below shows the sketch of $f(x) = -2x^2$. R is the point (6;0) and Q is the point $(q;0)$. P and T are points on f. RST is parallel to the y –axis and PS is parallel to the x –axis. $PQRS$ is a rectangle.



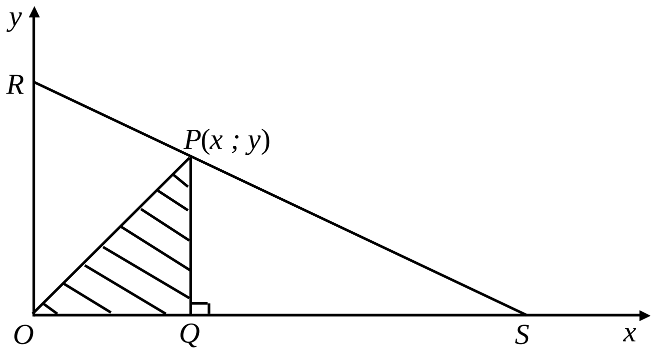
37.1 Write down the coordinates of P in terms of q.

37.2 Show that the area (A) of rectangle $PQRS$ can be expressed as follows: $A = 12q^2 - 2q^3$.

37.3 Determine the maximum area (A) of rectangle $PQRS$.

QUESTION 38 (GDE Trials 2014)

In the diagram below, $P(x;y)$ is a point on line segment PS with equation $x+2y=18$, where $0 \le x \le 18$. Q is a point on the *x*-axis such that ΔPOQ forms a right-angled triangle. $PQ \perp OS$

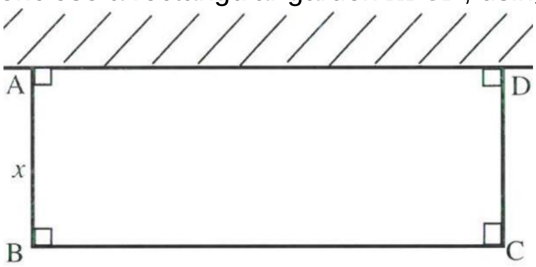


38.1 Write the *y*-coordinate of point P in terms of *x*.

38.2 Calculate the coordinates of P so that the area of ΔOPQ is at its maximum.

QUESTION 39 (GDE Trials 2017)

A $50m$ long fence is used to enclose a rectangular garden $ABCD$, using the wall (AD) as one of its boundaries. Let $AB = x$.

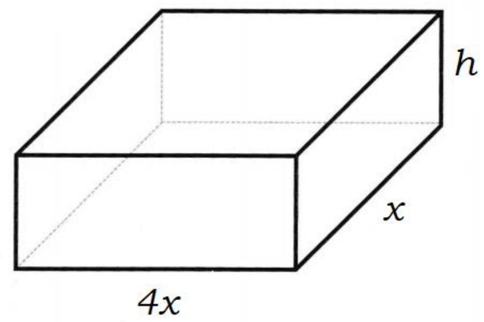


39.1 Express the length of BC in terms of x.

39.2 Hence determine the value of x such that the enclosed area will be a maximum.

QUESTION 40 (GDE Trials 2016)

An open rectangular box is made of a very thin sheet of metal. The volume is $128cm^3$ and the base of the box has a width of x cm and a length of $(4x)$cm .



40.1 Determine an expression for the height of the box in terms of x.

40.2 Show that the total surface area of the box can be written as: $\left(4x^2 + \frac{320}{x}\right) cm^2$.

40.3 Calculate the height of the box for which the surface area is a minimum.

QUESTION 41 (WCED Trials 2016)

A water tank with an inlet and an outlet is used to water a garden.
The equation $D = 3 + \frac{1}{2}t^2 - \frac{1}{4}t^3$ gives the depth of water in metres where t is the time in hours that has elapsed since 09:00.

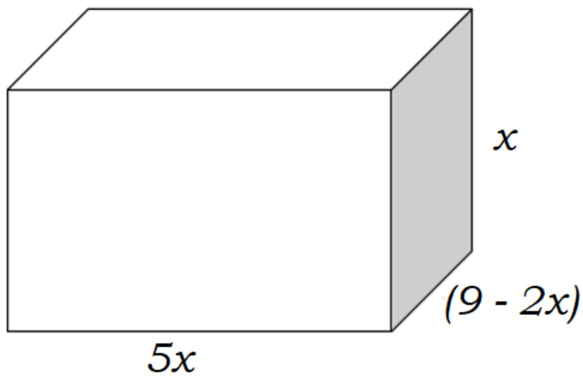
41.1 What is the depth of the water at 11:00 (2 hours later)?

41.2 At what rate does the depth of the water change at 12:00?

41.3 At what time will the inflow of the water be the same as the outflow of water?

QUESTION 42 (KZN Trials 2015)

A rectangular box has a length of $5x$ units, breadth $(9 - 2x)$ units and its height of x units.

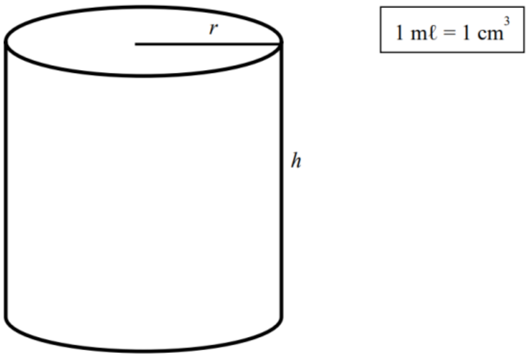


42.1 Show that the volume (V) of the box is given by: $V = 45x^2 - 10x^3$

42.2 Determine the value of x for which box will have maximum volume.

QUESTION 43 (DBE May/June 2016)

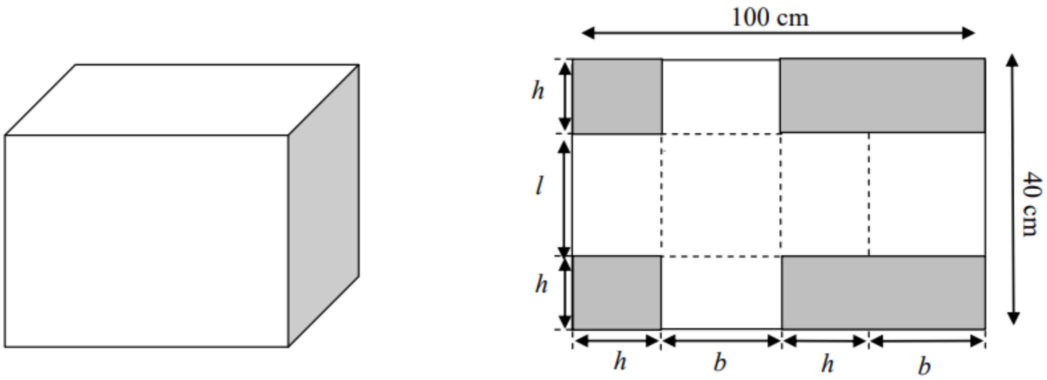
A $340\ m\ell$ can with height h cm and radius r cm is shown below.



43.1 Determine the height of the can in terms of the radius r.
43.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum.

QUESTION 44 (DOE Nov 2014)

A box is made from a rectangular piece of cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram below.



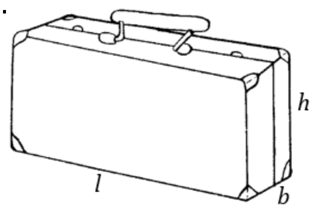
44.1 Express the length l in terms of the height h.

44.2 Hence prove that the volume of the box is given by $V = h(50-h)(40-2h)$

44.3 For which value of h will the volume of the box be a maximum?

QUESTION 45

Airlines have various restrictions on the size of bags that can be taken into the cabin of a plane. Some require that the largest rectangular box shape has a length no longer than 55 cm and the total sum of the length, breadth and height together not being more than 115 cm.

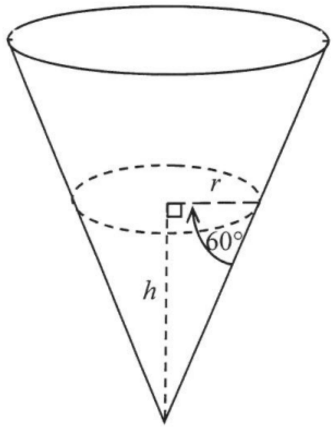


Using these dimensions and the breadth as x :

45.1 Show that the largest volume can be represented by $V = -55x^2 + 3300x$
45.2 Hence, determine the maximum volume of the bag.

QUESTION 46 (DOE Nov 2015)

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is $60°$, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\quad$ $V = \frac{1}{3}\pi r^2 h$

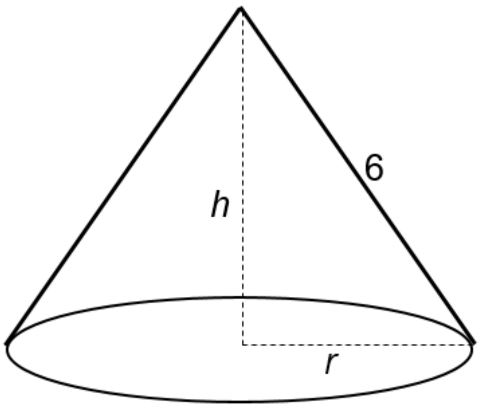
$V = lbh$ $\quad$ $V = \frac{4}{3}\pi r^3$

46.1 Determine r in terms of h. Leave your answer in surd form.

46.2 Determine the derivative of the volume of water with respect to h when h is equal to 9cm.

QUESTION 47

A cone has a slant height of 6 units, as shown in the diagram:



The circular base has a radius of *r* and the height of the cone is *h*.

The volume of the cone is given by the formula: $V = \frac{1}{3}\pi r^2 h$

47.1 Show that the volume can be written as $V = 12\pi h - \frac{\pi}{3}h^3$

47.2 Now calculate the height and radius of the cone that will produce the maximum volume.

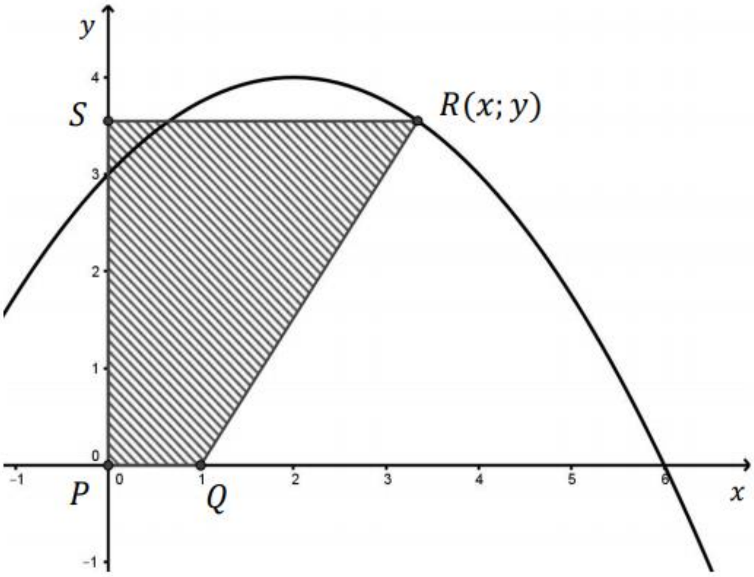
47.3 When $h = 0$ the volume is 0. Show that the volume will again be 0 when $h = 6$.

47.4 Now sketch the graph that shows how the volume of the cone changes as *h* varies between 0 and 6 units.

Clearly indicate on your graph the maximum volume of the cone.

QUESTION 48 (St Marys Trials 2015)

A trapezium is drawn where P is fixed at the origin, Q is fixed at (1;0) and R can move along a path defined by the parabola $y = -\frac{x^2}{4} + x + 3$ for $0 < x < 6$. RS is parallel to PQ.



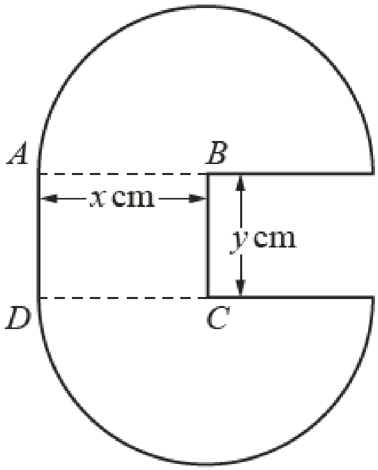
The area of a trapezium is calculated using the formula: $Area = \frac{a + b}{2} \times h$

48.1 Show that the area of PQRS is given by the formula: $A = -\frac{x^3}{8} + \frac{3x^2}{8} + 2x + \frac{3}{2}$

48.2 Calculate the maximum area of $PQRS$.

QUESTION 49 (St Johns Trials 2016)

The diagram shows a badge made of thin sheet metal, consisting of two semi-circular pieces with centres B and C with radius *x*. The semi-circles are connected by a rectangular section, $ABCD$, such that $AB = x$ and $BC = y$.

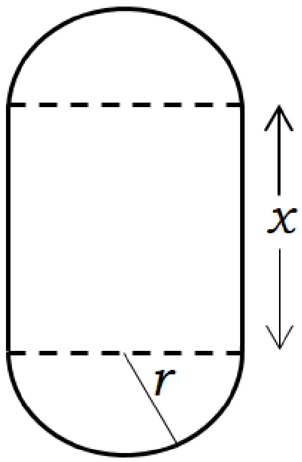


49.1 Given that the area of the badge is 20 cm^2, show that the perimeter, P of the badge is given by: $P = 2x + \frac{40}{x}$

49.2 Hence, determine the minimum value of the perimeter and prove that you have indeed found a minimum.

QUESTION 50

A zoo wishes to build a fence around a crocodile enclosure. The sketch below indicates the shape of the enclosure in terms of x and r, where the straight sections are to be x metres long and the circular end sections are of radius r metres. The zoo requires the area of the enclosure to be $100m^2$.



50.1 Use the area of the enclosure to show that: $x = \frac{100 - \pi r^2}{2r}$

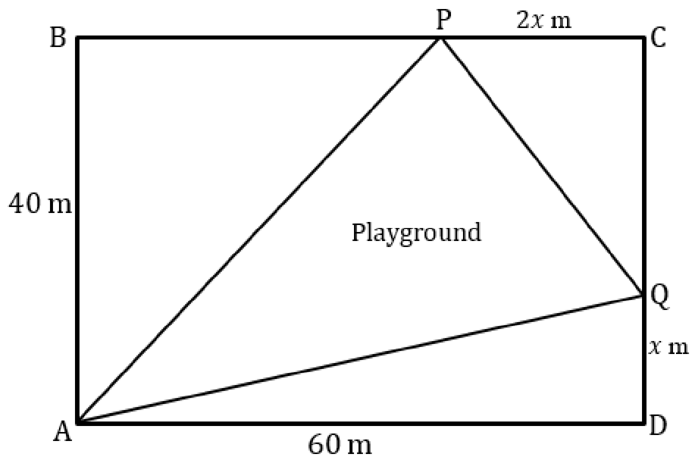
50.2 Show that the length of the fence required is given as: $L = \frac{100}{r} + \pi r$.

50.3 What will be the radius of the circular ends that will ensure the shortest total length L of this fence in order to minimise the building costs?

50.4 Hence find the value of x and describe the shape of the enclosure.

QUESTION 51

The diagram shows a plan for a rectangular park, ABCD, in which AB = 40 metres and AD = 60 metres. Points P and Q lie on BC and CD respectively and AP, PQ and QA are paths that surround a triangular playground. The length of DQ is x metres and the length of PC is $2x$ metres.



51.1 Write an expression for the lengths BP and CQ, in terms of x.

51.2 Show that the area, m^2 , of the triangular playground APQ is given by

$A = x^2 - 30x + 1200$

51.3 Given that x can vary, find the minimum area of the playground.

QUESTION 52

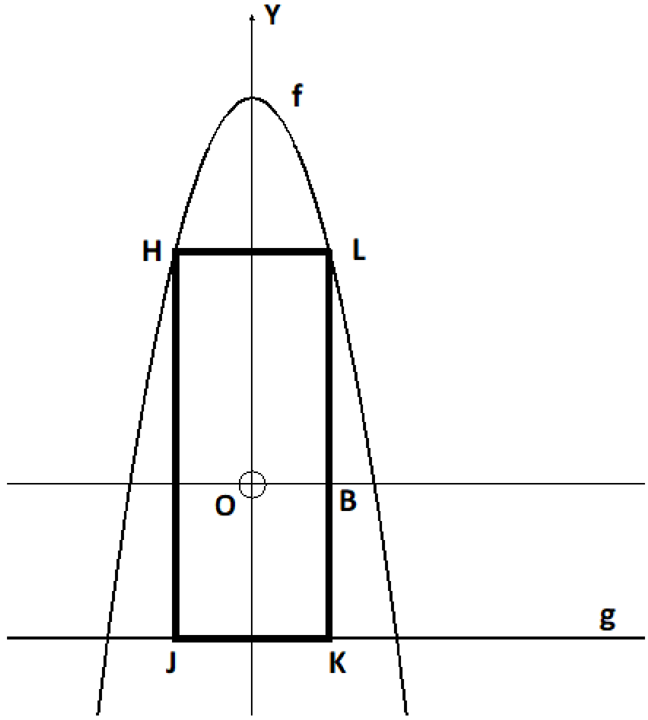
An open box with a square base has a volume of 10 000 cm^3. It is further given that the cardboard, used to make the box, costs 5 cents/cm^2 for the bottom part and 2 cents/cm^2 for the sides.

52.1 Show that the surface area of the box can be expressed as: $SA = x^2 + \dfrac{40000}{x}$

52.2 Determine the dimensions of the base that will result in the cost being a minimum. Round your answer to the nearest *cm*.

QUESTION 53

Sketched below are the graphs of $f(x) = -x^2 + 10$ and $g(x) = -5$. $HJKL$ is a rectangle enclosed by the parabola and the straight line. B is the point of intersection of KL with the x-axis.



53.1 If $OB = x$ units, calculate the lengths of JK and KL in terms of x.

53.2 Show that the area of rectangle $HJKL$ is given by $A = -2x^3 + 30x$.

53.3 Calculate the maximum possible area of rectangle $HJKL$. Leave your answer in surd form.

QUESTION 54

The rabbit population on a small island is observed to be given by the function:
$P(t) = 45t - 0.4t^3 + 500$, where t is the time in months since observations of the island began.

54.1 When is the maximum population attained, and what is that maximum population?

54.2 When does the rabbit population disappear from the island?

54.3 Sketch $P(t)$

QUESTION 55

When a person coughs, the trachea (windpipe) contracts causing air to flow faster through it. According to a mathematical model of coughing, the speed (v) of the airstream through the trachea is related to its radius (r) by the equation: $v = k(n - r)r^2$ provided that $\frac{n}{2} \leq r \leq n$.
In the equation k is a constant and n is the normal radius.

Determine to what fraction of its normal radius the trachea contracts when v is a maximum.

QUESTION 56

A perfectly cylindrical closed cola can has a capacity of 340ml. $[1ml = 1cm^3]$

56.1 Find an expression for the height of the can in terms of the radius of its base, r.
56.2 Hence, or otherwise, show that the total surface area can be expressed as: $2\pi r^2 + \frac{680}{r}$
56.3 The cost of metal sheeting is R8,00 per $m2$.
56.3.1 Find an expression for the cost (C) of the metal used to make the can.
56.3.2 Find the value of the radius which minimises the cost.
56.3.3 Determine the minimum cost of one can.

QUESTION 57

A soccer ball is kicked in the air. Its height h in metres, t seconds after it has been kicked, is given by the formula: $h = 20(t^2 - \frac{1}{3}t^3)$.

57.1 What is the velocity of the ball after 1,5 seconds?
57.2 For what value of t is the **velocity** a maximum?
57.3 For what value of t is the height of the ball a maximum? Calculate the greatest height from the ground.

QUESTION 58

A cylindrical box with height (h) and radius (r), will be tied up with ribbon as shown in the diagram alongside. The longest piece of ribbon available is 150cm of which 30cm is required for the bow.


TOP VIEW

58.1 Show that $h = 30 - 2r$.
58.2 Calculate the largest possible volume of the box (in terms of π).

QUESTION 59

The circulation of the local newspaper is given by the equation (t in years) $N(t) = 50t^2 - 200t + 3000$

59.1 What was the circulation on the first release of the paper?
59.2 At what rate will the circulation change after 3 years?