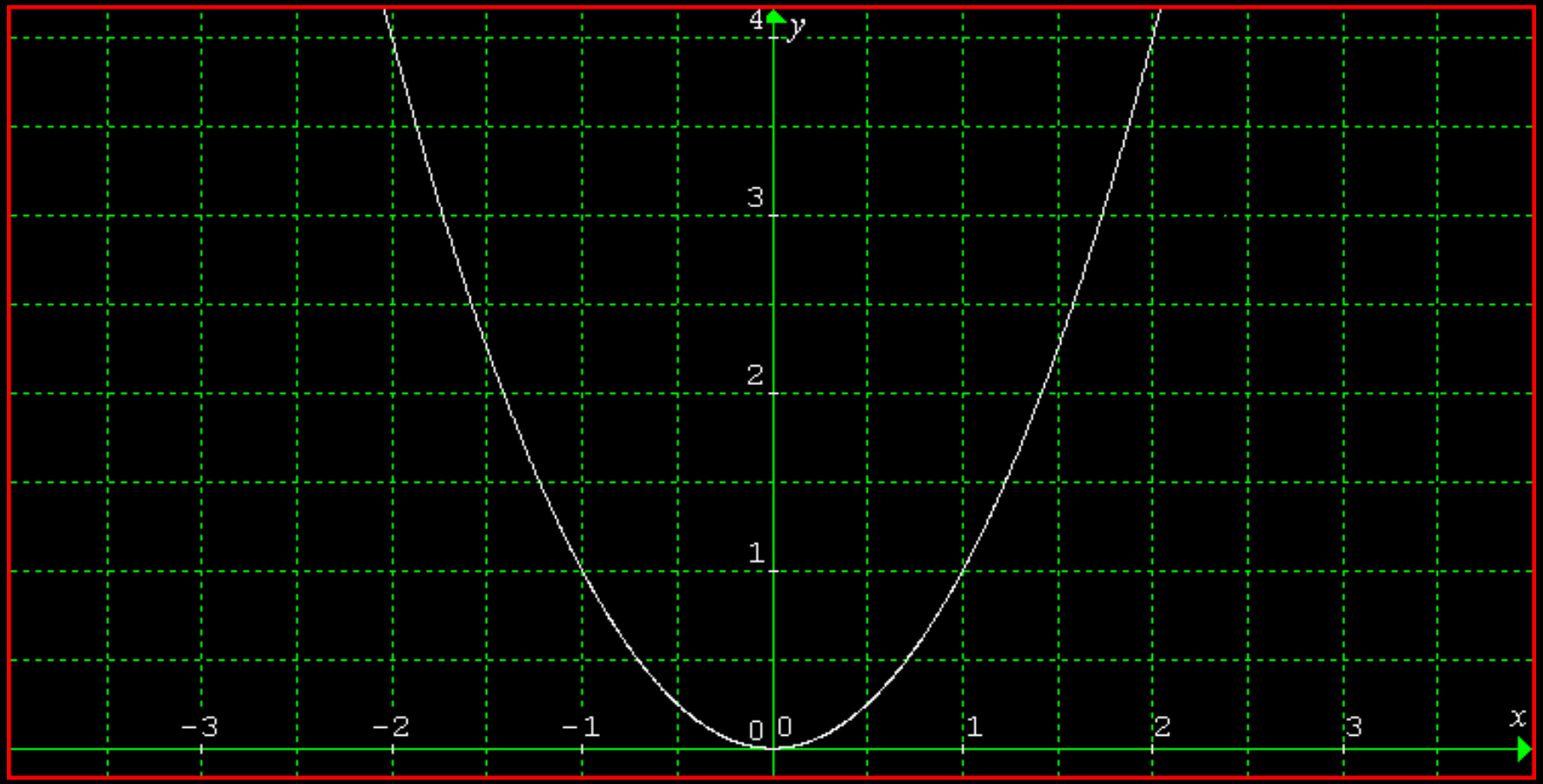
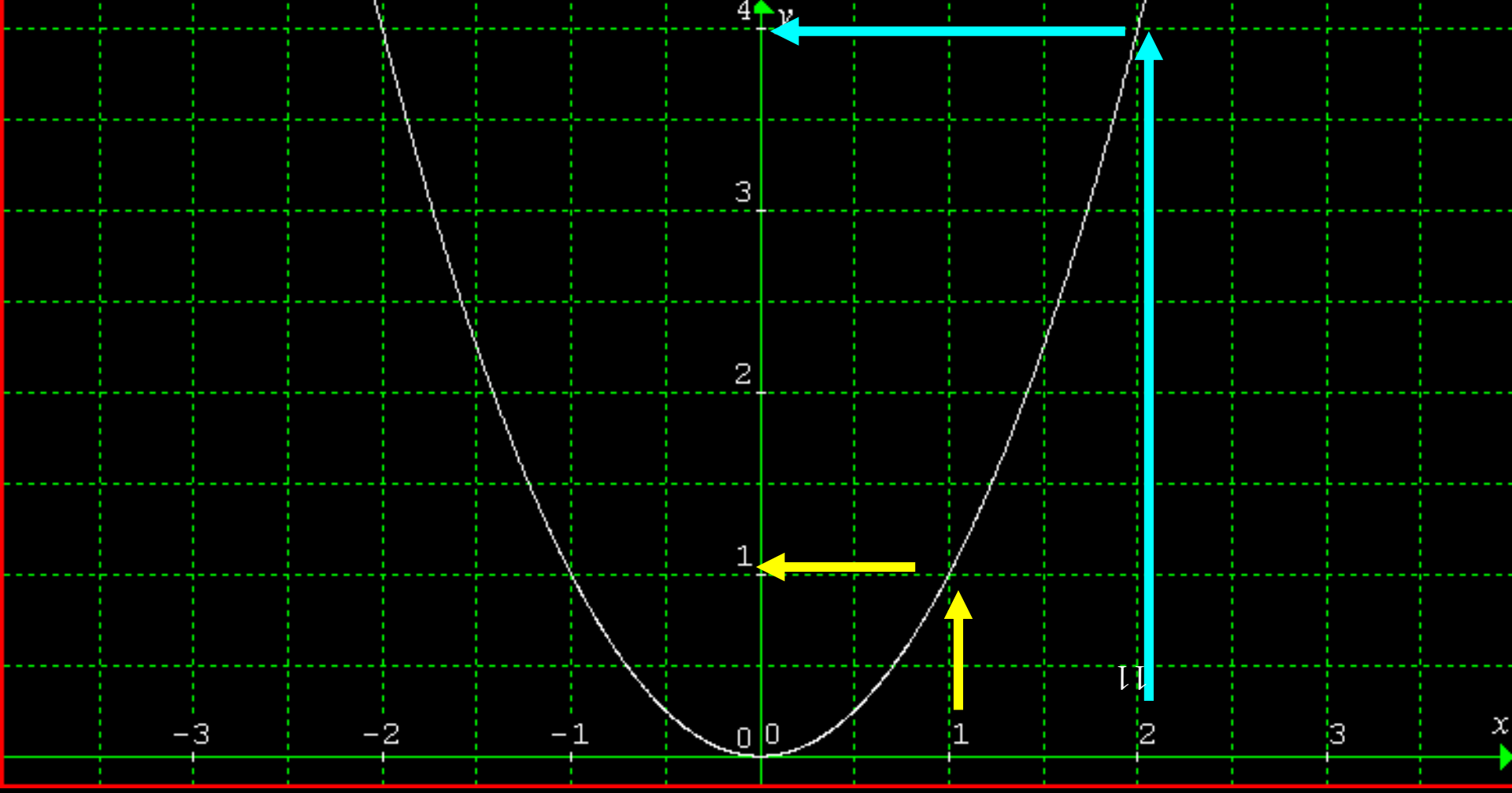
DIFFERENTIAL CALCULUS

AVERAGE GRADIENT



Find the average gradient between x = 1 & x = 2



If x = 1, f(1) = 1.

When x = 2 , f(2) = 4

$$Average\ gradient = \frac{\Delta y}{\Delta x}$$

$f(x) = x^2$

$f(2)=4$

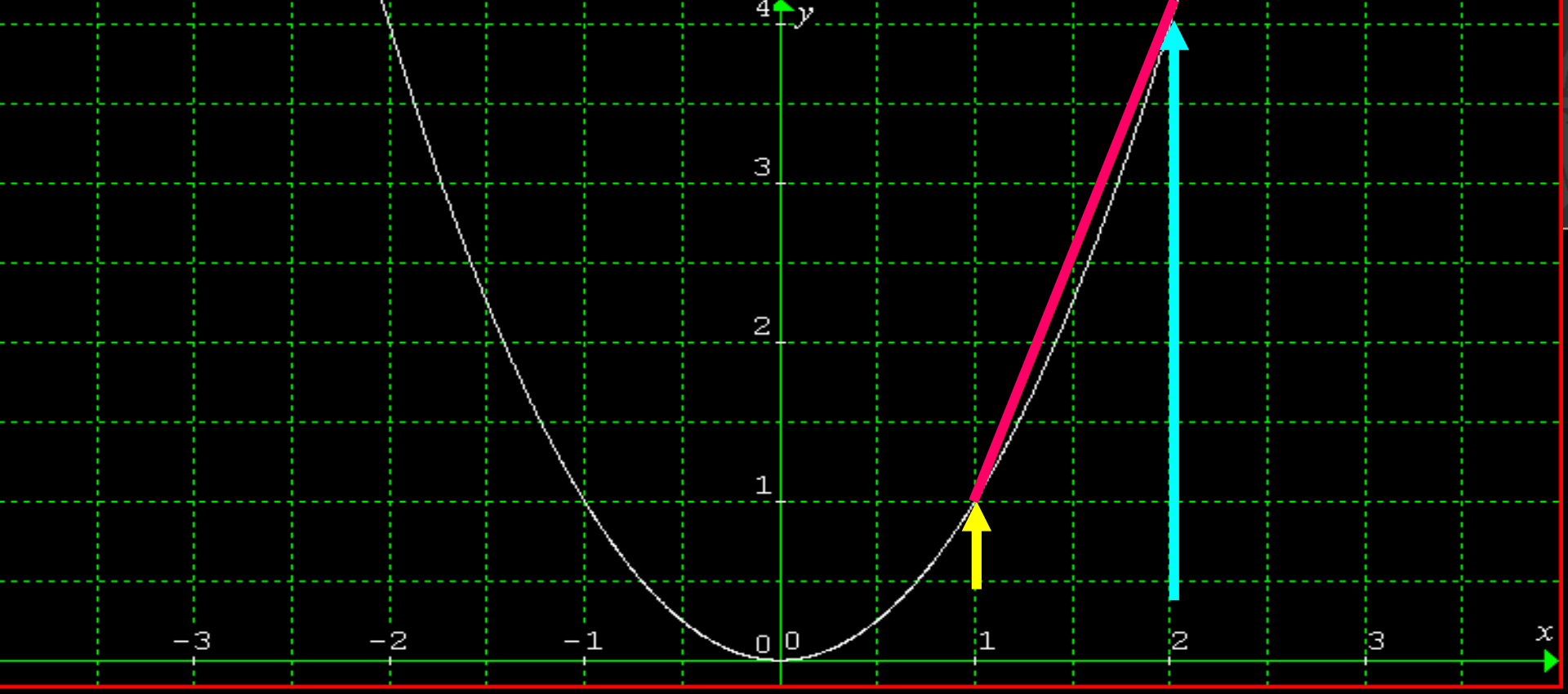
$f(1)=1$

$$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(2) - f(1)}{2 - 1}$$

$$= \frac{4 - 1}{2 - 1}$$

$= 3$



Average Gradient:

- Assumes a straight line is drawn between two points and the average gradient of that straight line is found
- E.g. Average gradient between x = 1 and x = 2 is 3
- Doesn't tell us what the gradient is at a particular point on a curve

Gradient at a Point / Instantaneous Gradient

- Assumes a straight line is drawn between two points on a curve, BUT that these two points are brought closer and closer together, until the distance between the two points tends towards zero (i.e. the the two points "merge" into one point)
- If h = distance between the two points, then as the distance tends towards zero, we write it as $h \to 0$
- In this way, we can find the gradient of a point anywhere along a curve

Average Gradient to Gradient at a Point

LIMITS

- This method of making a variable tend towards a particular number and then evaluating an expression containing that variable is known as "finding the limit"

- This can be written as:

$$\lim_{Variable \to Number} (Expression)$$

Introduction to Limits

Limit Examples

1. $\lim_{h \to 0} (4h - 4)$

$= 4(0) - 4$

$= -4$

2. $\lim_{h \to 0} \left(\frac{5}{h}\right)$

$= \frac{5}{(o)}$

$= limit\ does\ not\ exist$

3. $\lim_{x \to 2} (-5x^2 + x - 1)$

$= -5(2)^2 + (2) - 1$

$= -19$

Since the expression cannot be simplified, substitute straight into the expression

Limit Examples

4. $\lim_{x \to -3} \left(\frac{x^2 - 9}{x+3}\right)$

$$\lim_{x \to -3} \frac{(x-3)(x+3)}{(x+3)}$$

$$\lim_{x \to -3} (x-3)$$

$$= (-3) - 3$$

$$= -6$$

This expression MUST FIRST be simplified before substitution. If not, incorrect answer of a limit that does not exist!

Limit Example 1

Limit Example 2

Using Limits to find the Gradient at a Point

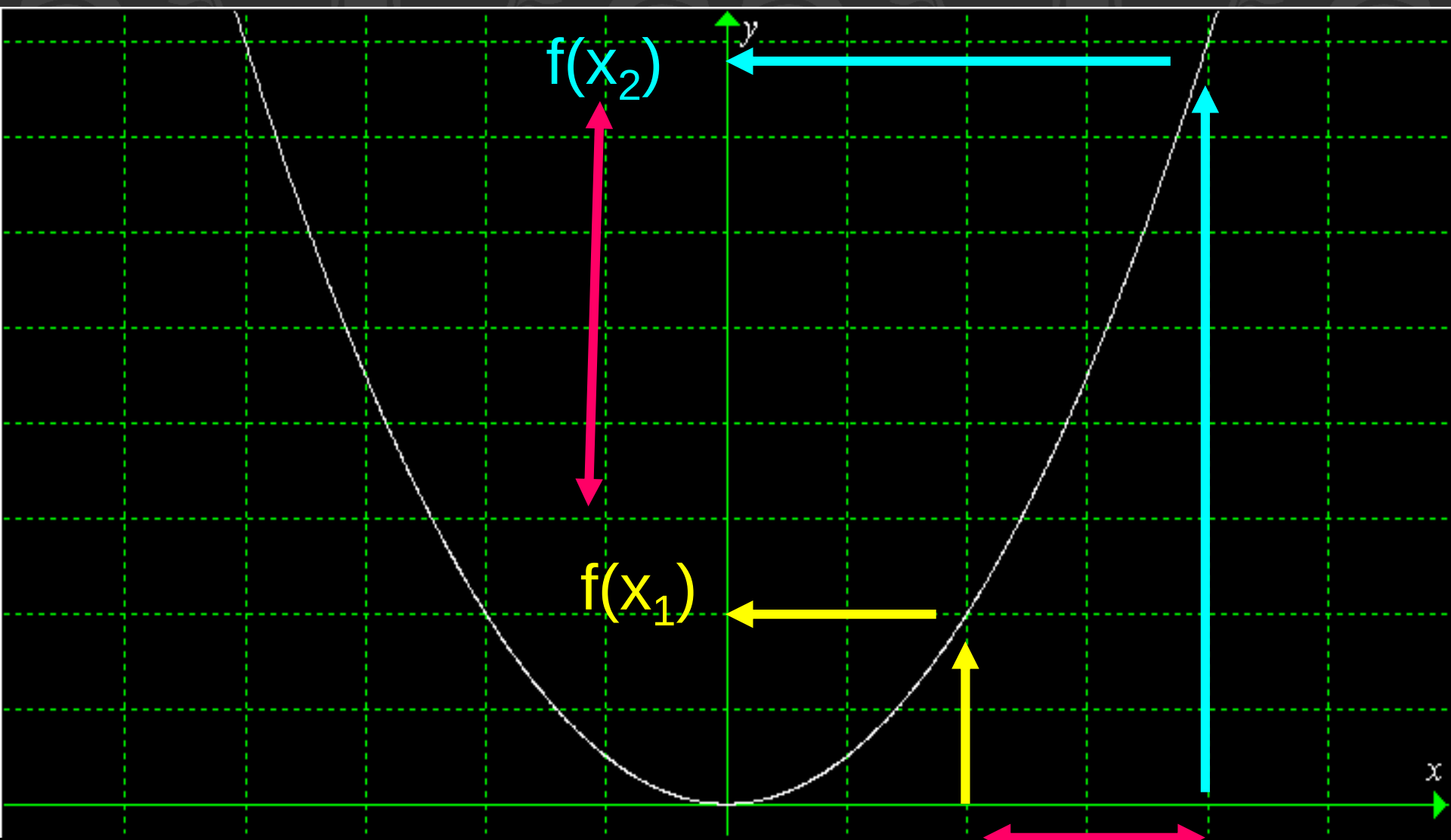
- If we take the average gradient formula:

$$\textit{Average gradient}$$

$$= \frac{\Delta y}{\Delta x}$$

$$= \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

This can be visualized as follows …



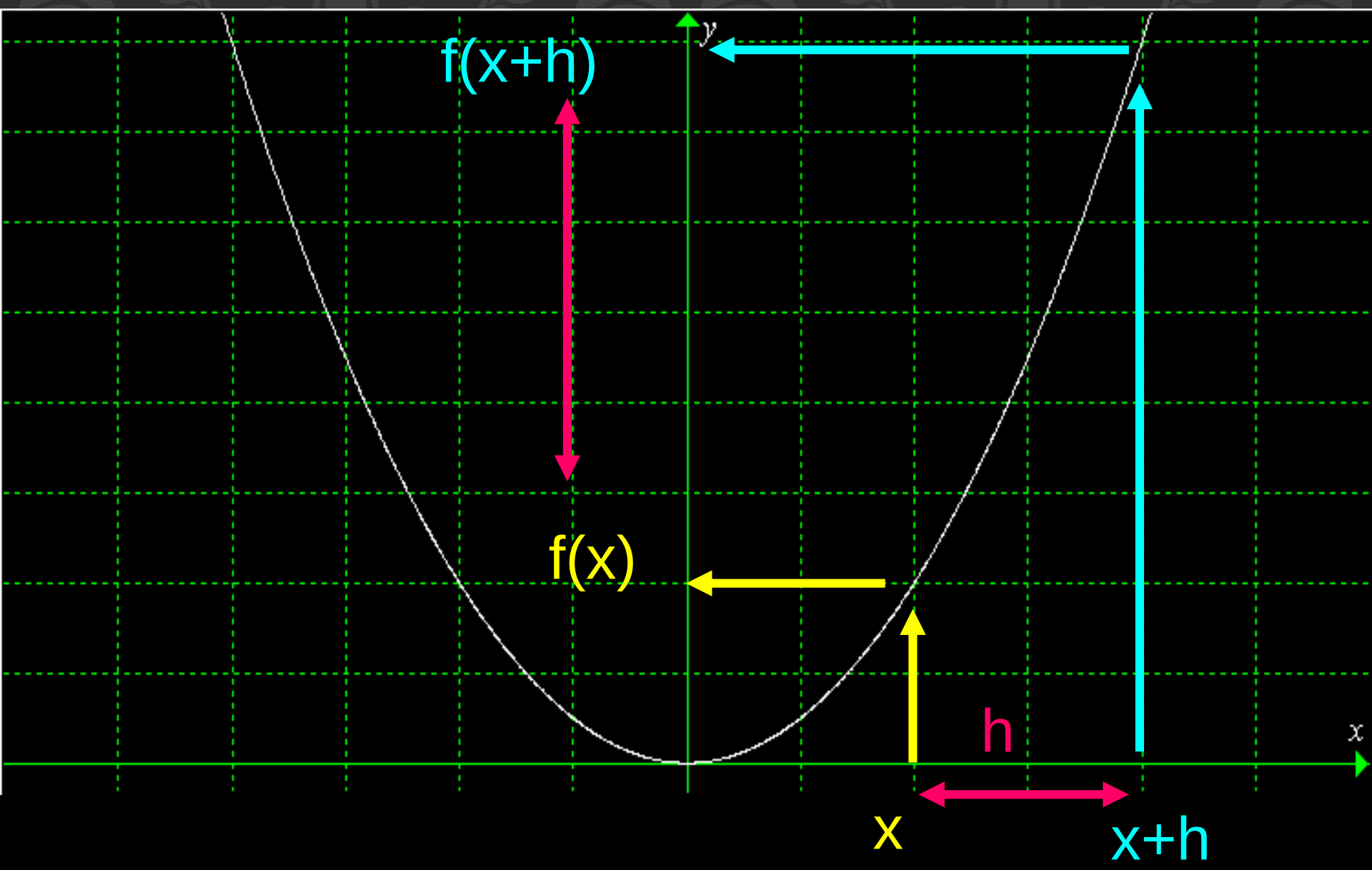
$$Average\ gradient = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

Using Limits to find the Gradient at a Point

- Now, if we take let the first x-coordinate and assume it is h units away, then the second x-coordinate would be $x+h$.
- We can rewrite the average gradient formula as:

$$\textbf{\textit{Average gradient}} = \frac{f(x_1)-f(x_2)}{x_1-x_2} = \frac{f(x+h)-f(x)}{x-(x+h)} = \frac{f(x+h)-f(x)}{h}$$

This can be visualized as follows …



$$Average\ gradient = \frac{f(x+h) - f(x)}{h}$$

Using Limits to find the Gradient at a Point

$$Average\ gradient = \frac{f(x+h) - f(x)}{h}$$

- As the distance between the 2 values of x becomes smaller and smaller so the second value of x gets closer and closer to the first value (x_1) and so the distance between the two points, *h*, becomes very very small and approaches zero.
- We will find the limit as

h tends towards 0 i.e. $\lim_{h \to 0}$

INSTANTANEOUS GRADIENT

- Instantaneous Gradient = Gradient at a Point
  = the Derivative

- The derivative is indicated by $f'(x)$

- $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

- This formula is known as The First Principles Formula or Newton's Quotient

FIRST PRINCIPLES

Visual representation of 1st Principles

Graphical Representation of Derivatives

- We are going to find the gradient at a point, anywhere along any function, using the First Principles Formula:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

First Principles Example: f(x) = a (where a is a constant)

$f(x) = 3$

$\therefore f(x+h) = 3$ A CONSTANT so does NOT change

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{3-3}{h}$$

$$= \lim_{h \to 0} \frac{0}{h}$$

$= 0$

First Principles Example: f(x) = ax

$f(x) = 2x$

$f(x+h) = 2(x+h) = 2x + 2h$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2x + 2h - 2x}{h}$$

$$= \lim_{h \to 0} \frac{2h}{h}$$

$$= 2$$

First Principles Example:
$f(x) = ax^2$

$f(x) = x^2$

$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - (x^2)}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$= 2x$

First Principles Example:
$f(x) = ax^2$

$f(x) = 2x^2 - 1$

$f(x+h) = 2(x+h)^2 - 1 = 2x^2 + 4hx + 2h^2 - 1$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(2x^2 + 4hx + 2h^2 - 1) - (2x^2 - 1)}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4hx + 2h^2 - 1 - 2x^2 + 1}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h)}{h}$$

$= 4x$

Derivative of a Quadratic Function

First Principles Example:
$f(x) = ax^3$

$f(x) = 2x^3$

$f(x+h) = 2(x+h)^3 = 2(x+h)(x^2+2xh+h^2)$

$= 2x^3 + 6x^2h + 6xh^2 + 2h^3$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(2x^3 + 6x^2h + 6xh^2 + 2h^3) - (2x^3)}{h}$$

$$= \lim_{h\to 0} \frac{(2x^3 + 6x^2h + 6xh^2 + 2h^3) - (2x^3)}{h}$$

$$= \lim_{h\to 0} \frac{h(6x^2 + 6xh + 2h^2)}{h} = 6x^2$$

Derivative of a Cubic Function

First Principles Example:

$$f(x) = \frac{a}{x}$$

$$f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \left(\frac{1}{x+h} - \frac{1}{x}\right) \div \frac{h}{1}$$

$$= \lim_{h \to 0} \frac{x - (x+h)}{x(x+h)} \times \frac{1}{h}$$

Fractions within fractions are a clumsy! So, use division signs

Get an LCD

First Principles Example continued .. $f(x) = \frac{1}{x}$

$$f'(x) = \lim_{h \to 0} \frac{x - (x+h)}{x(x+h)} \times \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{x - x - h}{x(x+h)} \times \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-h}{x(x+h)} \times \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{-1}{x(x+h)}$$

$$= \frac{1}{x[x + (0)]}$$

$$= \frac{1}{x^2}$$

DIFFERENTIATION NOTATION

1. Question: f(x) = …

   Differentiated Answer: $f'(x) =$ …

2. Question: y = …

   Differentiated Answer: $\frac{dy}{dx} =$ …

3. Question: $D(x)$ […]

   Answer: = …

Relationship between f(x) and f ' (x)

RULES OF DIFFERENTIATION

1. $f'(a) = 0$
2. $f'(ax) = a$
3. $f'(ax^n) = n.a\ x^{n-1}$

where a and n are constants

Differentiation Rules Examples

1. $f(x) = -2$

   $f'(x) = 0$

   $f'(a) = 0$

2. $f(x) = 4x$

   $f'(x) = 4$

   $f'(ax) = a$

3. $f(x) = -3x^2 + 6x - 9$

   $f'(x) = -6x + 6$

   $f'(ax^n) = n.ax^{n-1}$

Differentiation Rules Examples

4. $f(x) = (x-1)(x-2)$

$= x^2 - 3x + 2$

$f'(x) = 2x - 3$

First multiply out brackets

5. $f(x) = \frac{1}{x}$

$= x^{-1}$

$f'(x) = -x^{-2}$

Can't differentiate with x in the denominator … so write with negative exponents

6. $f(x) = \frac{x}{3}$ $(= \frac{1}{3}x)$

$f'(x) = \frac{1}{3}$

Careful now! There is no x in the denominator

Differentiation terms in the form ax^n

Differentiation Rules Examples

7. $f(x) = \frac{8x^2 - x}{x}$

$= \frac{8x^2}{x} - \frac{x}{x}$

$= 8x - 1$

$f'(x) = 8$

Simplify by splitting terms!

8. $f(x) = \frac{x^2 - 9}{x - 3}$

$= \frac{(x-3)(x+3)}{x-3}$

$= x + 3$

$f'(x) = 1$

Simplify by factorization!

Differentiating terms with fractions

Differentiation Rules Examples

9. $f(x) = \sqrt{x}$

$= x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

Can't differentiate with roots, so write with rational exponents!

10. $f(x) = 4.\sqrt[3]{x}$

$= 4.x^{\frac{1}{3}}$

$f'(x) = \frac{4}{3}x^{-\frac{2}{3}}$

Careful now with the 4 !

Differentiating terms with roots

Recap of the concept of the derivative

FIND THE GRADIENT AT A POINT

Find the gradient of $f(x) = 5x^2 + 2x$ at x = 6.

- To find the gradient at a point => differentiate!

$$f'(x) = 10\,x + 2$$

- This is an expression for the gradient at any point along the curve

- To find the gradient at the point where x = 6, we substitute into the general gradient expression:

$$f'(6) = 10(6) + 2 = 62$$

Derivative at a Point

FIND THE POINT, GIVEN THE GRADIENT

Find the point along the curve of $f(x) = -3x^2 - 1$ at which the gradient is 4.

- To find the gradient at a point => differentiate!

$$f'(x) = -6x$$

- To find the point at which the gradient is 4, we substitute into $f'(x)$ and solve for x:

$$4 = -6x$$

$$x = -\frac{2}{3}$$

FIND THE EQUATION OF THE TANGENT

Example 1

Determine the equation of the tangent to $f(x) = 2x^2$ at x = -1.

- Tangents are straight lines that touch a curve at one point only!

$$\therefore y = mx + c$$

- To find the gradient of a curve at a point => differentiate!

$$f'(x) = 4x$$

Differentiating a Parabola

Finding the equation of the tangent to $f(x) = 2x^2$ at x = -1 continued …

- To find the gradient at the point where x = 6, we substitute into the general gradient expression:

$$f'(x) = 4x$$
$$f'(-1) = 4(-1) = -4$$
$$\therefore y = -4x + c$$

- To find the y-coordinate, when x = -1, we substitute into the original curve i.e. f(x):

$$f(x) = 2x^2$$
$$f(-1) = 2(-1)^2 = 2$$
$$\therefore Point: (-1; 2)$$

Finding the equation of the tangent to $f(x) = 2x^2$ at x = -1 continued …

- To find the value of c in $y = mx+c$, we substitute the point into the equation of the tangent:

$$y = -4x + c$$
$$\text{Subst: } (-1; 2)$$
$$2 = -4(-1) + c$$
$$c = -2$$

$$\therefore \textit{Equation of tangent: } y = -4x - 2$$

The Equation of a Tangent at a Point

Investigating Tangents

FIND THE EQUATION OF THE TANGENT

Example 2

Determine the equation of the tangent to $f(x) = 2x^3 - 6x$ at x = 2.

$f'(x) = 6x^2 - 6$

$f'(2) = 6(2)^2 - 6 = 18$

$\therefore y = 18x + c$

$f(x) = 2x^3 - 6x$

$f(2) = 2(2)^3 - 6(2) = 4$

$Point: (2; 4)$

$\therefore y = 18x + c$

$Subst: (2; 4)$

$4 = 18(2) + c$

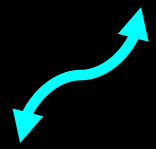
$c = -32$

$Equation\ of\ tangent:$

$y = 18x - 32$

STANDARD FORM OF A CUBIC GRAPH

$$y = ax^3 + bx^2 + cx + d$$

a > 0: increasing function 

a < 0: decreasing function 

d: y-intercept

Investigating the Standard Form of a Cubic Graph

Identifying Graphs from their Equations

ENRICHMENT:
"TURNING POINT" FORM OF A CUBIC GRAPH

$$y = a(x - p)3 + q$$

a > 0: increasing function

a < 0: decreasing function

p > 0: shifts graph to the right

p < 0: shifts graph to the left

q > 0: shifts graph up

q < 0: shifts graph down

Investigating the "Turning Point" Form of a Cubic Graph

SKETCHING CUBIC GRAPHS

Example 1

Sketch the graph of $f(x) = -x^3 - 2x^2 + x + 2$.

Y-int. (x=0)

(0;2)

X-int. (y=0)

$0 = -x^3 - 2x^2 + x + 2$

To factorize a cubic, use the factor theorem!

$f(1) = -(1)^3 - 2(1)^2 + (1) + 2 = 0$ ✓ factor

$\therefore (x - 1)\ is\ a\ factor$

Sketch $f(x) = -x^3 - 2x^2 + x + 2$ *continued* ...

X-int. (y=0)

$0 = -x^3 - 2x^2 + x + 2$

$0 = (x - 1)(-x^2 - 3x - 2)$

$0 = -(x - 1)(x^2 + 3x + 2)$

$0 = -(x - 1)(x + 1)(x + 2)$

$\therefore x = 1 \ \ or \ \ x = -1 \ \ or \ \ x = 2$

We expect 3 roots for a cubic graph!

Stationary points = "Turning Points"

$\therefore m = 0, so \ differentiate!$

$f(x) = -x^3 - 2x^2 + x + 2$

$f'(x) = -3x^2 - 4x + 1$

Can't factorize? Use the Quadratic Formula!

Sketch $f(x) = -x^3 - 2x^2 + x + 2$ *continued ...*

$$0 = -3x^2 - 4x + 1$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-3)(1)}}{2(-3)}$$

$$= \frac{4 \pm \sqrt{28}}{-6}$$

$$x = -1{,}55 \ \ or \ \ x = 0{,}22$$

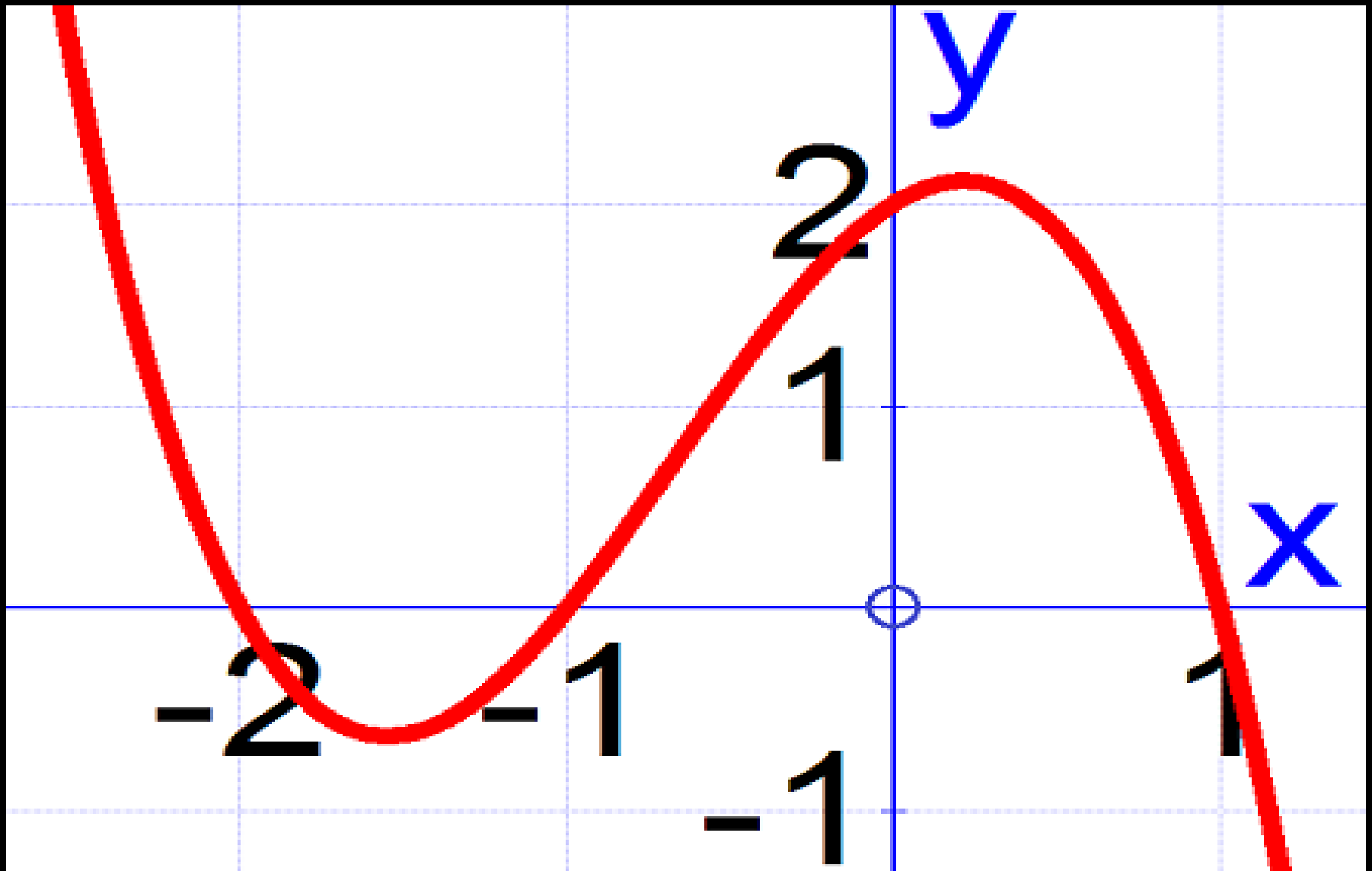
We expected two stationary points!

Finding out the y-coordinates of the stationary points

$$f(-1{,}55) = -(-1{,}55)^3 - 2(-1{,}55)^2 + (-1{,}55) + 2$$
$$= -0{,}63$$

$$f(0{,}22) = -(0{,}22)^3 - 2(0{,}22)^2 + (0{,}22) + 2$$
$$= 2{,}11$$

Cubic Graph: $f(x) = -x^3 - 2x^2 + x + 2$



SKETCHING CUBIC GRAPHS

Example 2

Sketch the graph of $f(x) = x^3 - x^2 - 8x + 12$.

Y-int. (x=0)

(0;12)

X-int. (y=0)

$0 = x^3 - x^2 - 8x + 12$

To factorize a cubic, use the factor theorem!

$f(1) = (1)^3 - (1)^2 - 8(1) + 12 = 4$ ✘ factor

$f(2) = (2)^3 - (2)^2 - 8(2) + 12 = 0$ ✓ factor

$\therefore (x - 2)\ is\ a\ factor$

Sketch $f(x) = x^3 - x^2 - 8x + 12$ *continued* ...

X-int. (y=0)

$0 = x^3 - x^2 - 8x + 12$

$0 = (x - 2)(x^2 + x - 6)$

$0 = (x - 2)(x - 2)(x + 3)$

$\therefore x = 2 \ \ or \ \ x = 2 \ \ or \ \ x = -3$

When there are 2 identical x-intercepts, the stationary point will fall on an x-intercept.

Stationary points = "Turning Points"

$\therefore m = 0, so \ differentiate!$

$f(x) = x^3 - x^2 - 8x + 12$

$f'(x) = 3x^2 - 2x - 8$

Can't factorize? Use the Quadratic Formula!

Sketch $f(x) = x^3 - x^2 - 8x + 12$ *continued* ...

$$0 = 3x^2 - 2x - 8$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-8)}}{2(3)}$$

$$= \frac{2 \pm \sqrt{100}}{6}$$

$$x = 2 \ \ or \ \ x = -\frac{4}{3}$$

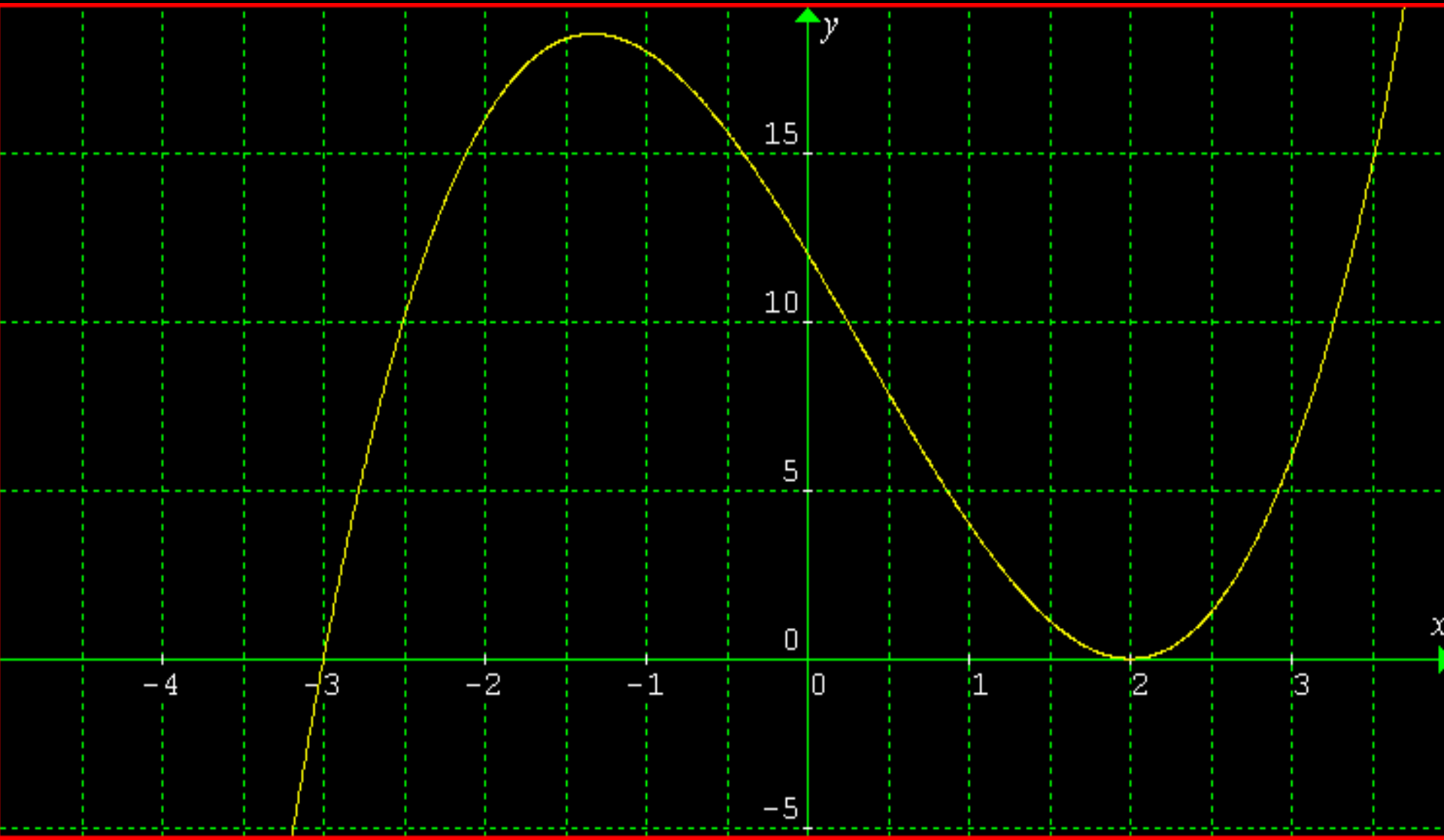
We expected x=2, as the stationary point falls on the x-intercept!

Finding out the y-coordinates of the stationary points

$$f(2) = (2)^3 - (2)^2 - 8(2) + 12 = 0$$

$$f\left(-\frac{4}{3}\right) = \left(-\frac{4}{3}\right)^3 - \left(-\frac{4}{3}\right)^2 - 8\left(-\frac{4}{3}\right) + 12 = 0$$

Cubic Graph: $f(x) = x^3 - x^2 - 8x + 12$



SKETCHING CUBIC GRAPHS

Find the Roots of a Cubic Function

Finding the Stationary Points of the Cubic Function

Sketching a Cubic Graph without an Equation

SKETCHING CUBIC GRAPHS

Example 3

Sketch the graph of $f(x) = x^3$.

Y-int. (x=0)

(0;0)

X-int. (y=0)

$0 = x^3$

$x = 0$

This is odd – as we expect to have 3 real roots??

Sketch $f(x) = x^3$ *continued* ...

Stationary points = "Turning Points"

$\therefore m = 0, so\ differentiate!$

$f(x) = x^3$

$f'(x) = 3x^2$

$0 = 3x^2$

$x = 0$

This is odd – as we expect to have 2 stationary points??

So, we have the following:

Y-int: (0;0)

X-int: (0;0)

Stationary points: (0;0)

How do you plot a cubic graph with only 1 point??

POINT OF INFLECTION

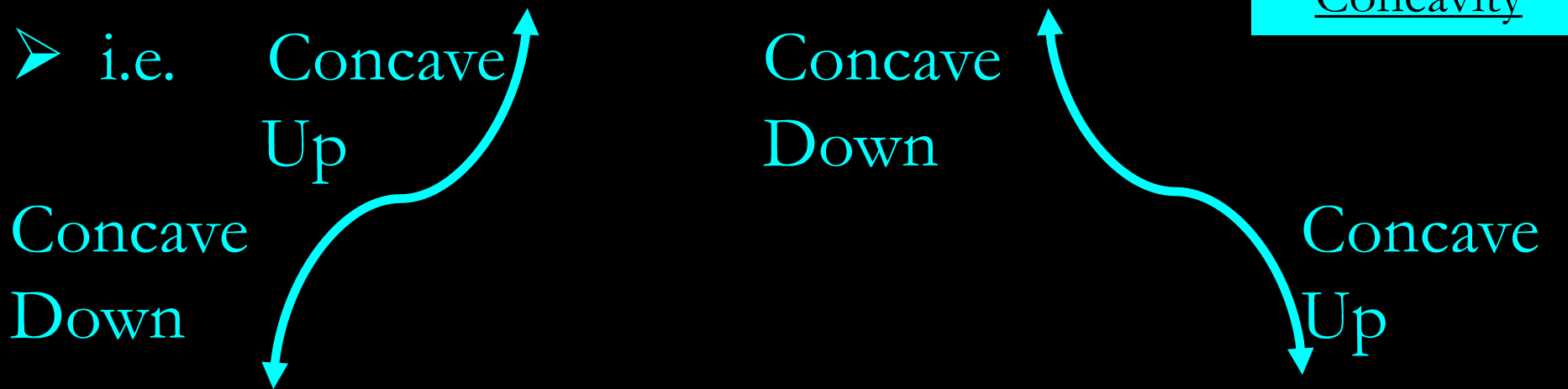
- With stationary points, we have local maximum and local minimum points

Maxima, Minima and Critical points

- However, with the graph of $f(x) = x^3$, we don't have these local maximum and minimum points, but instead, have a change in concavity

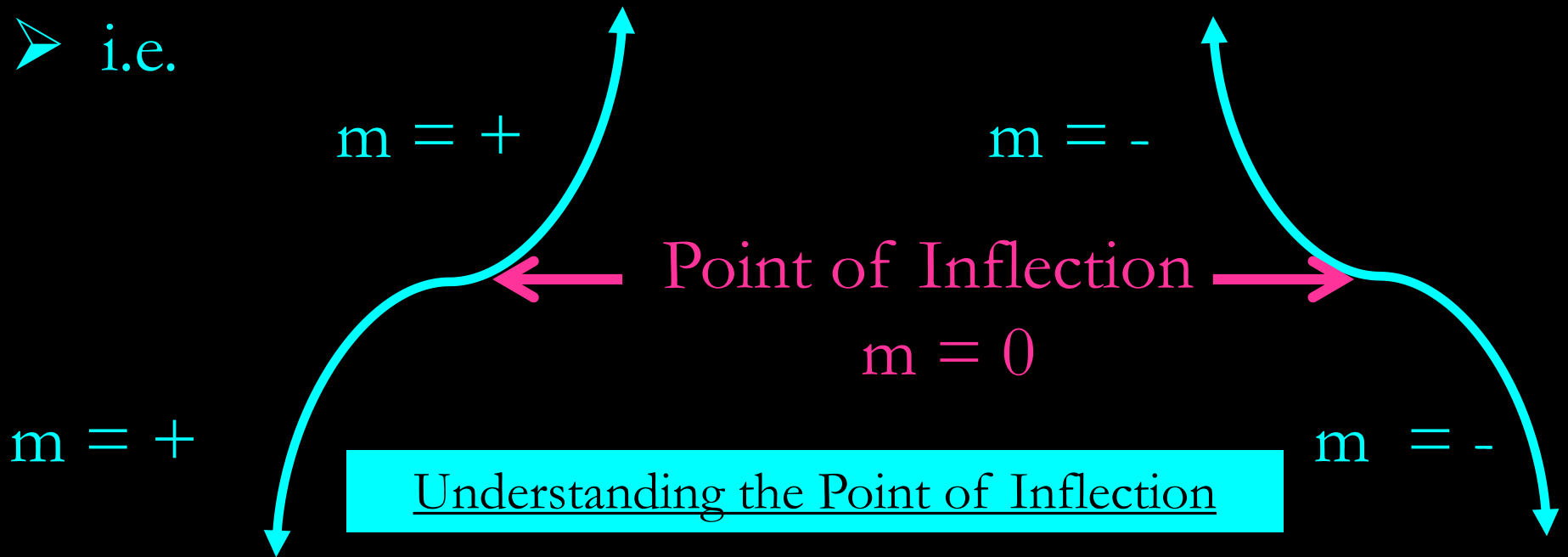
Changes in Concavity

- i.e.



POINT OF INFLECTION

- When there is a change in concavity, we have what we call a point of inflection
- At the point of inflection, the gradient = 0
- i.e.



POINT OF INFLECTION

- To find the point of inflection algebraically, we take the second (or double) derivative and put it equal to 0

$$f(x) = ax^3 + bx^2 + cx + d$$
$$f'(x) = 3ax^2 + 2bx + c$$
$$f''(x) = 6ax + 2b$$

Practicing Second Derivatives

e.g. $f(x) = 3x^3 + x^2 + x + 8$

$$f'(x) = 9x^2 + 2x + 1$$
$$f''(x) = 18x + 2$$
$$0 = 18x + 2$$

First and Second Derivative Graphs of a Cubic Function

Sketch $f(x) = x^3$ *continued* ...

So, back to the sketching the graph of $f(x) = x^3$:

Y-int: (0;0)

X-int: (0;0)

Stationary points: (0;0)

Point of Inflection:

$f(x) = x^3$

$f'(x) = 3x^2$

$f''(x) = 6x$

$0 = 6x$

$x = 0$

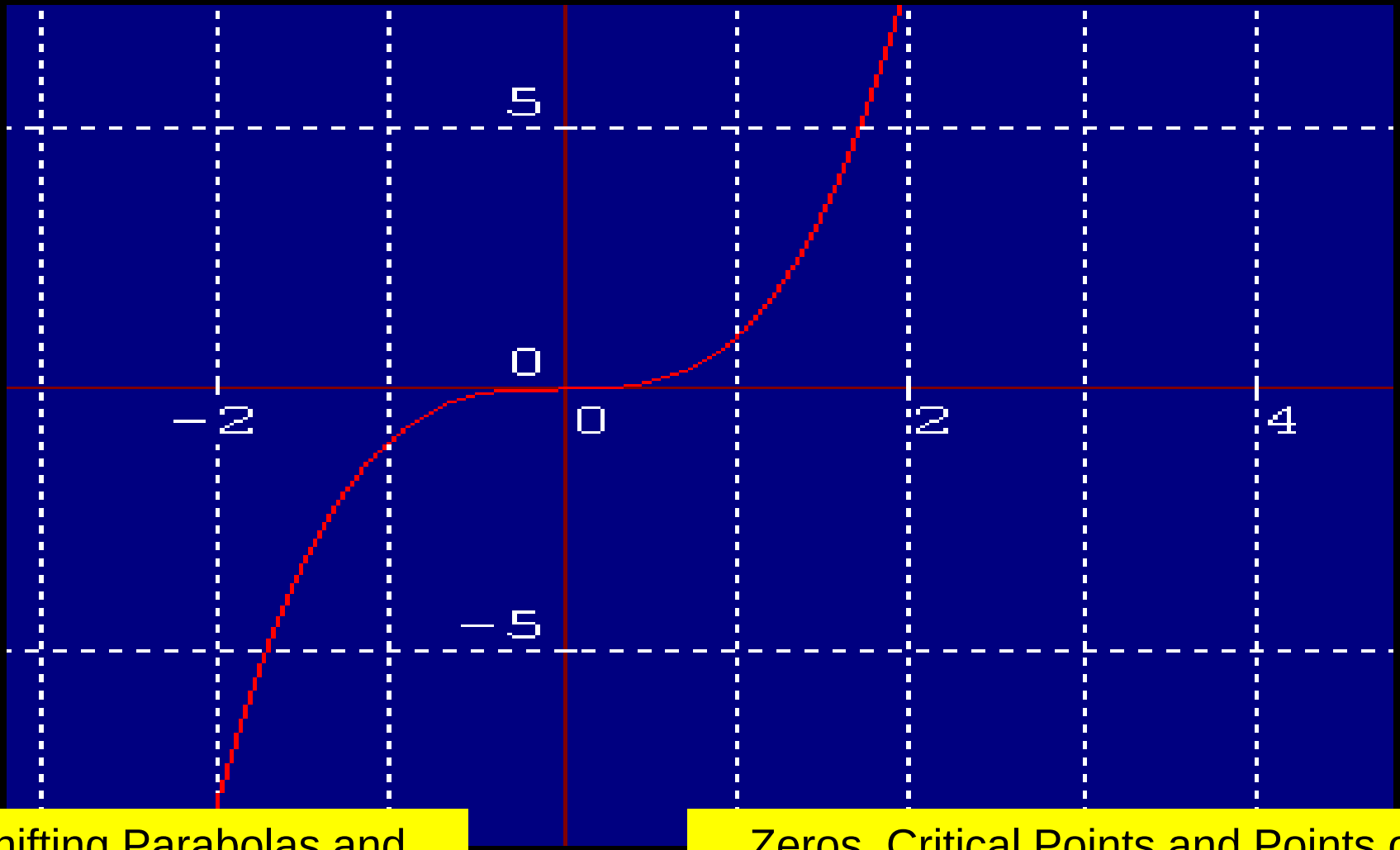
To find y-coordinate of the Point of Inflection:

$f(x) = x^3$

$f(0) = (0)^3 = 0$

$\therefore$ *Point of inflection*: $(0; 0)$

Cubic Graph: $f(x) = x^3$



Shifting Parabolas and Cubic Graphs

Zeros, Critical Points and Points of Infection of a Cubic Function

DERIVATIVE GRAPHS: REVIEW

Derivative Graphs

Relationship between Derivative Graphs

Derivatives of Cubic Graphs

Identifying the Derivative Graphs of Graphs

MAXIMA AND MINIMA

- Calculus can be applied to many real-life situations, in which a set of equations need to be maximized (e.g. to make a profit) or minimized (e.g. to reduce costs)
- You will often need to set up equations – so learn the formulae for perimeter, area, total surface area and volume
- To maximize or minimize an equation, we will differentiate and put the equation equal to 0 … i.e. the maximum or minimum point of an equation / graph is the turning point and the gradient equals 0!

Maxima & Minima Examples

Example 1:

The length of a rectangle is x and its breadth is $(25 - x)$. Determine the dimensions of the rectangle in order for it to have a maximum area.

$$Area = length \times breadth$$
$$= x \times (25 - x)$$
$$= 25x - x^2$$

What do we have to do to maximize or minimize?

Maxima & Minima Examples continued ...

$$Area = 25x - x^2$$

$$\frac{dArea}{dx} = 25 - 2x$$

$$0 = 25 - 2x$$

$$2x = 25$$

$$x = 12{,}5$$

Differentiate and put the equation equal to zero!

But the question asked for the dimensions of the rectangle …

$$Length = x = 12{,}5\ units$$

$$Breadth = 25 - x$$

$$= 25 - 12{,}5$$

$$= 12{,}5\ units$$

Maxima & Minima Examples

Example 2:

A cylinder has a radius of x and a height of $(300 - x)$. Find the value of x in order for the volume of the cylinder to be a maximum.

$$Volume = \pi r^2 h$$
$$= \pi (x)^2 (300 - x)$$
$$= 300\pi x^2 - \pi x^3$$

What do we have to do to maximize or minimize?

Maxima & Minima Examples continued …

$$Volume = 300\pi x^2 - \pi x^3$$

$$\frac{dVolume}{dx} = 300\pi x - 3\pi x^2$$

$$0 = 300\pi x - 3\pi x^2$$

$$0 = 100x - x^2$$

$$0 = x(100 - x)$$

$$x = 0 \ \ or \ x = 100$$

Differentiate and put the equation equal to zero!

Since the question asked for a maximum, x = 100 …

$$Volume = 300\pi x^2 - \pi x^3$$

$$= 300\pi(100)^2 - \pi(100)^3$$

$$= 6\ 283\ 185{,}31$$

Maxima & Minima Examples

Minimizing the cost of a storage container

Minimizing the sum of squares

Maximizing the volume of a box – Part 1

Maximizing the volume of a box – Part 2

SPEED, VELOCITY AND ACCELERATION

- Average speed is the distance travelled divided by the time taken.

$$Average\ speed = \frac{change\ in\ distance}{change\ in\ time}$$

Example 1

I took 4 hours to travel to Johannesburg , a distance of 456km.

$$Average\ speed = \frac{456}{4} = 114\ kph.$$

Average Speed

Speed Example

Example 2

A stone falls at a distance of $s(t) = 2x^2 - 3$ meters, where t is time in seconds. Determine the average speed of the stone as it falls.

Note! s represents distance and not speed!

Distance travelled in …

2 seconds: $s(2) = 2(2)^2 - 3 = 5$ m

3 seconds: $s(3) = 2(3)^2 - 3 = 15$ m

$$Average\ speed = \frac{15 - 5}{3 - 2} = 10\ m\ per\ sec$$

Distance, Speed & Time Modeling of Two Cars

SPEED, VELOCITY AND ACCELERATION

- Instantaneous speed – or VELOCITY - is the speed taken at one particular point in time

$$Velocity = derivative\ of\ speed$$

$$v = d'(Speed)$$

Example 1

Determine the velocity at 3 seconds, if the speed of a particle can be given by: $s(t) = 20 - 5t$ (where t = seconds).

$s'(t) = -5.$

Note! s'(t) is the derivative of speed = velocity!

Instantaneous Speed Graph

Velocity Example

Example 2

Determine the speed I was travelling at the exact time of 4 seconds, when the speed trap camera flashed after I had covered a distance of $120x$ meters.

$$Average\ speed = \frac{120x}{4} = 30x\ meters\ per\ sec$$

$$Velocity = d'(Speed)$$

$$= 30\ meters\ per\ second$$

Average Speed vs Actual Speed

Instantaneous Velocity

SPEED, VELOCITY AND ACCELERATION

- Instantaneous velocity – or ACCELERATION - is the velocity taken at one particular point in time

$$Accerlation = derivative\ of\ velocity$$
$$a = d'(Velocity)$$

Example 1

Determine the acceleration of a particle, if the velocity can be given by v(t) = 6t (where t = seconds).

$v'(t) = 6.$

Note! v'(t) is the derivative of velocity = acceleration!

Acceleration Example

Example 2

If the speed of an object can be represented by the equation: $s(t) = 4t^2 - 6t + 1$, determine the acceleration of the object.

$s(t) = 4t^2 - 6t + 1$ … Speed

$s'(t) = 8t - 6$ … Velocity

$s''(t) = 8$ … Acceleration

The concept of Acceleration