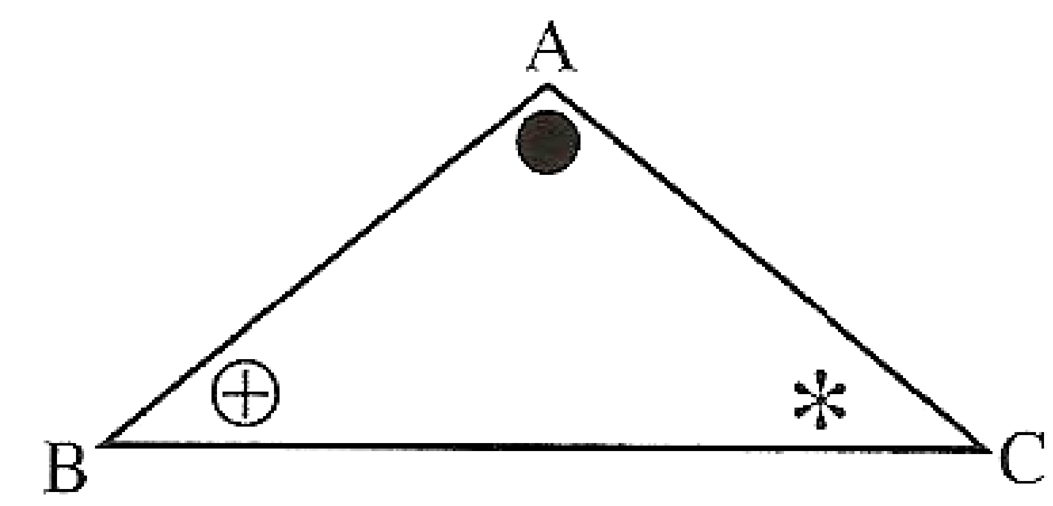
EUCLIDEAN

GEOMETRY

RECAP: SIMILAR TRIANGLES

If two triangles are similar (equiangular), then their corresponding sides are in the same proportion i.e.

If $\Delta ABC \,|||\, \Delta DEF, \quad then \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$



Similar Shapes

RECAP: SIMILAR TRIANGLES

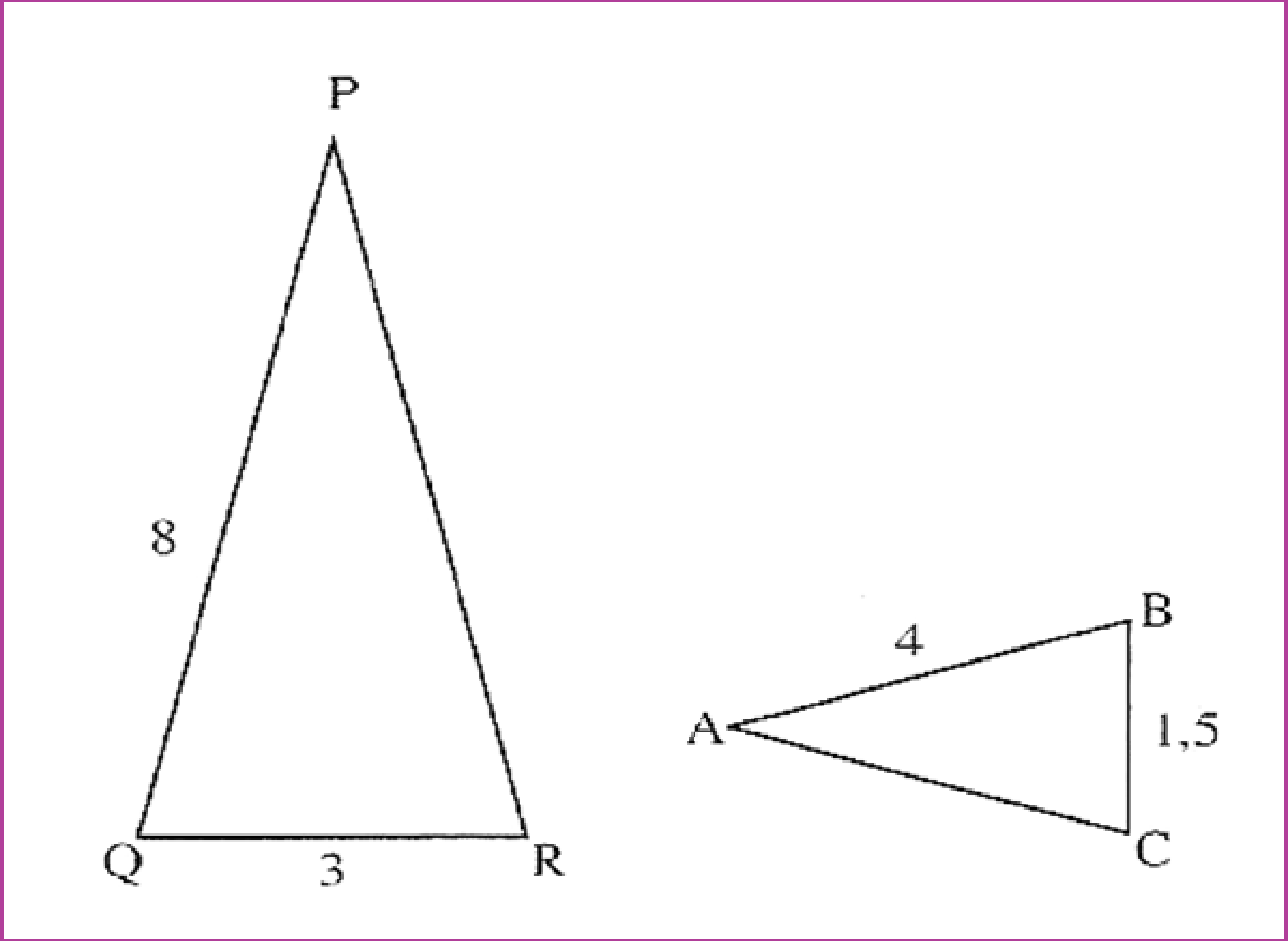
Concept of Similar Triangles

Working with Similar Triangles Part 1

Working with Similar Triangles Part 2

EXERCISE

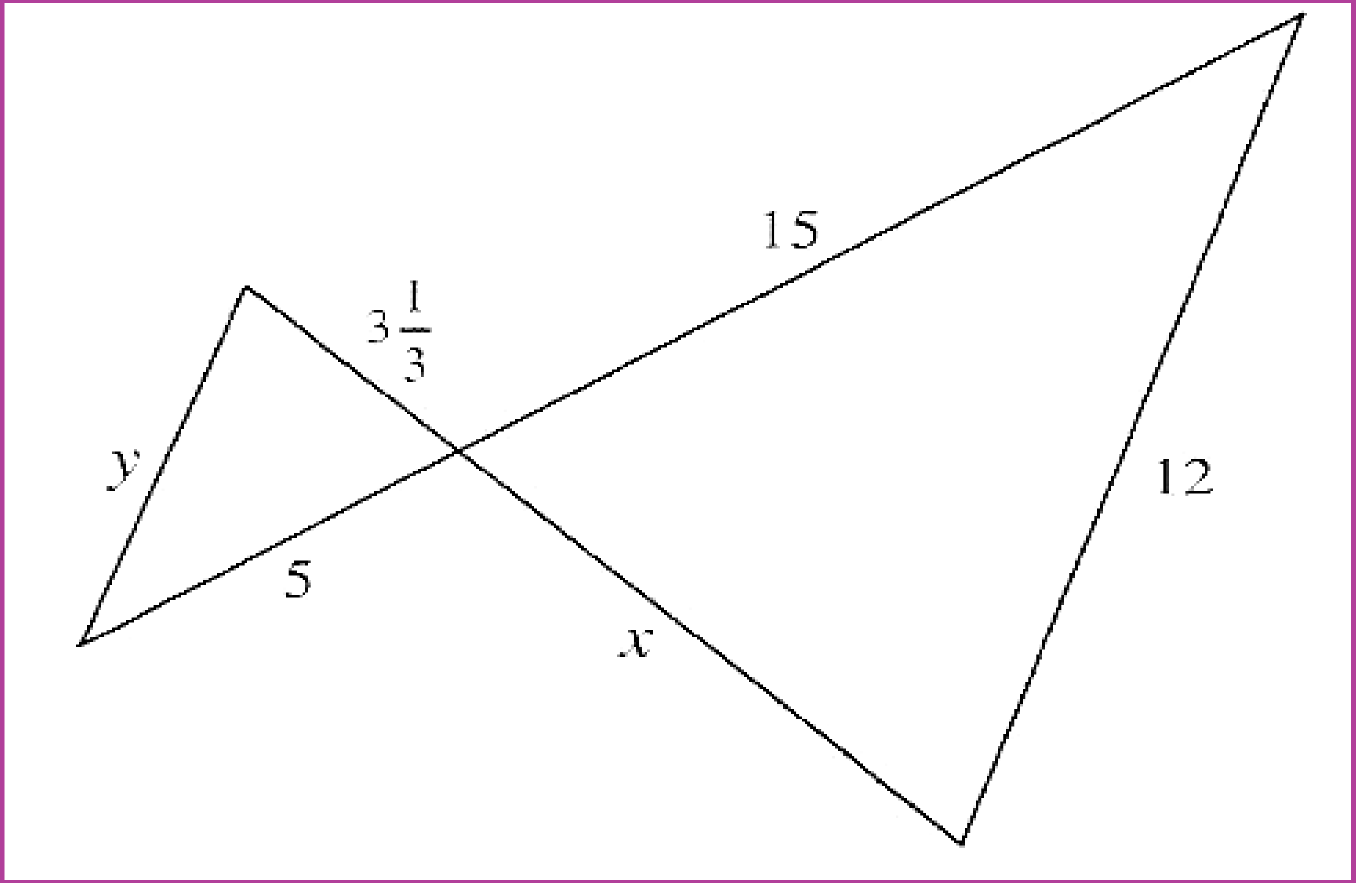
Is ΔPQR /// ΔABC?



EXERCISE

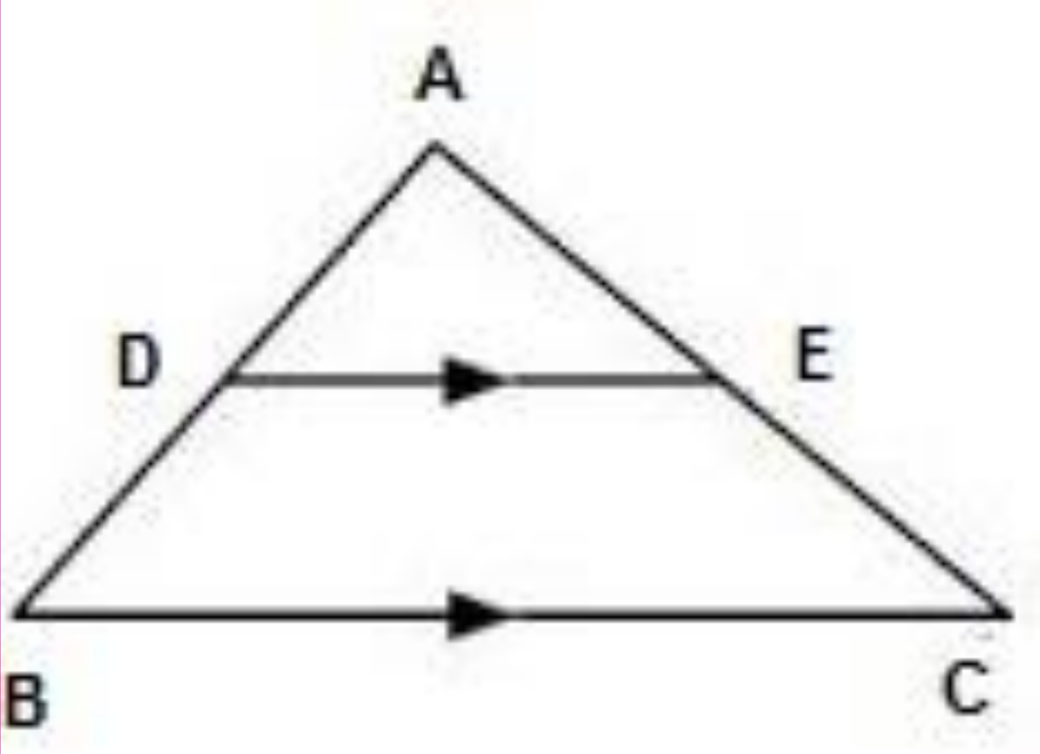
The two triangles below are similar.

Calculate the values of x and y.



THEOREM: A line parallel to one side of a triangle divides the other two sides proportionally

Given: DE // BC



Then:

$$\frac{AD}{DB} = \frac{AE}{EC} \quad and$$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad and$$

$$\frac{AB}{DB} = \frac{AC}{EC}$$

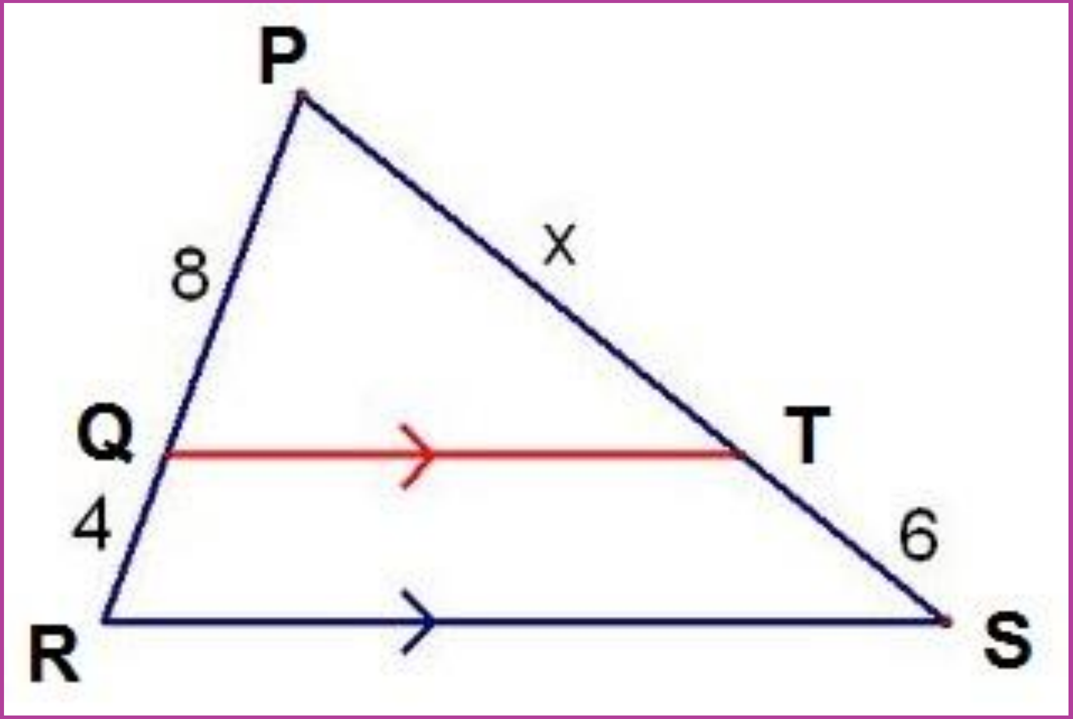
Example 1: Solve for x

$\frac{PQ}{QR} = \frac{PT}{TS}$ (QT //RS)

$$\frac{8}{4} = \frac{x}{6}$$

$$4x = 481$$

$$x = 12$$



Similarity in triangles with parallel lines (4:15 - 7:37)

Example 2: Solve for x

$$\frac{DE}{DF} = \frac{DH}{DG}$$ (QT //RS)

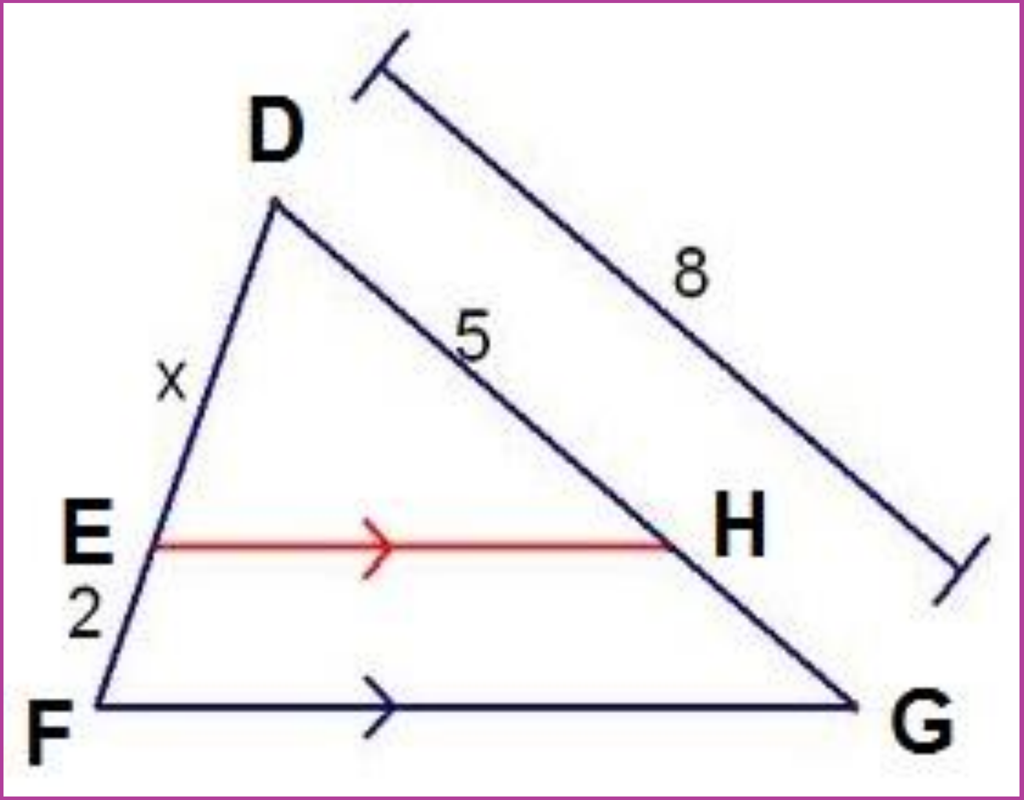
$$\frac{x}{x+2} = \frac{5}{8}$$

$$5(x+2) = 8x$$

$$5x + 10 = 8x$$

$$-3x = -10$$

$$\therefore\ x = \frac{10}{3}$$



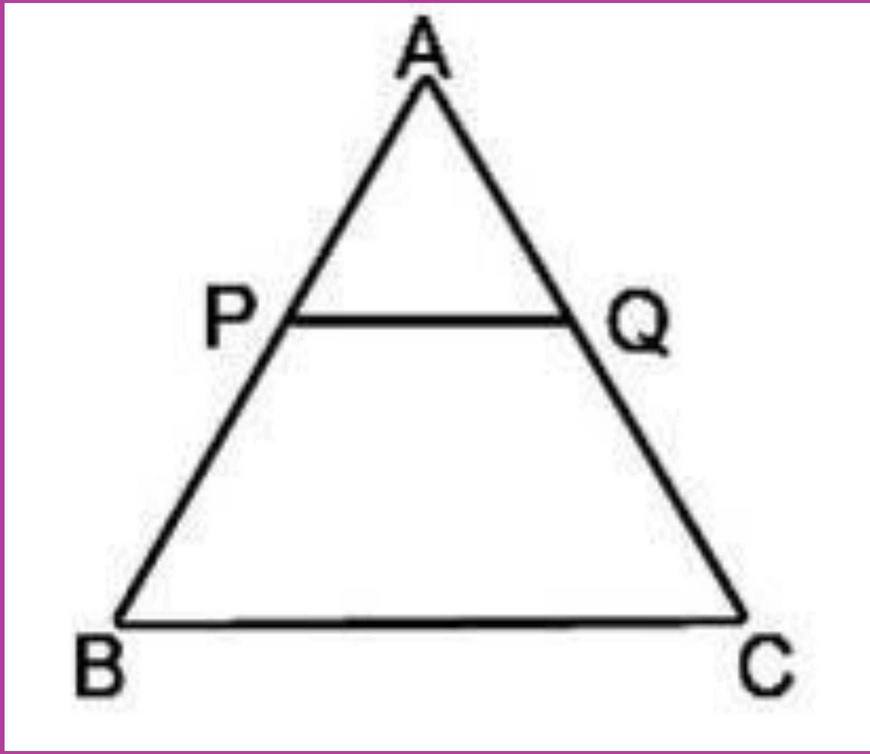
CONVERSE: If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

Given:

$$\frac{AP}{PB} = \frac{AQ}{QC} \ or$$

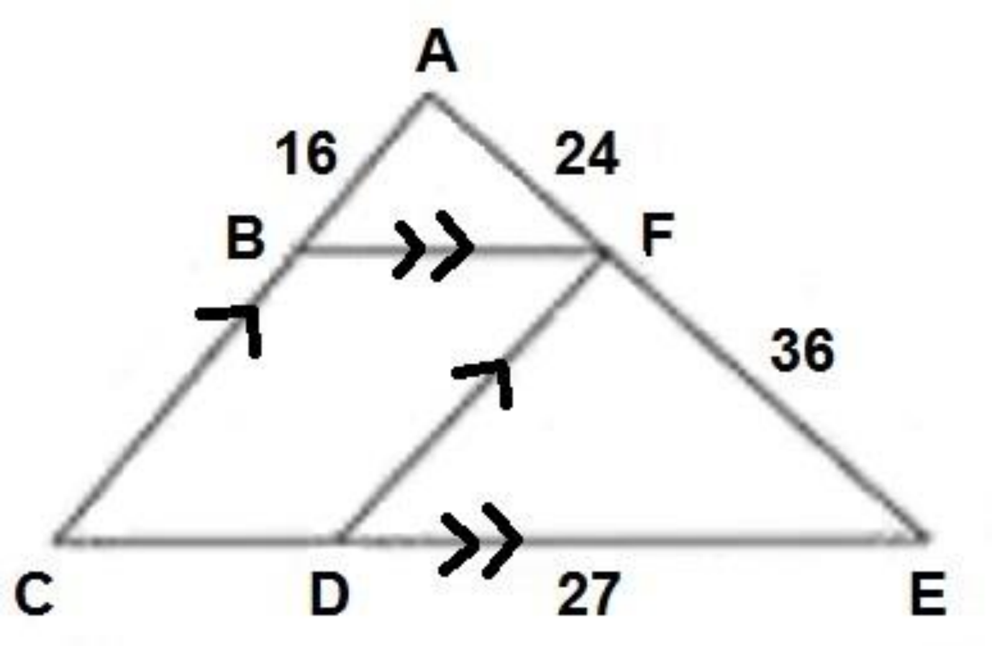
$$\frac{AP}{AB} = \frac{AQ}{AC} \ or$$

$$\frac{AB}{PB} = \frac{AC}{QC}$$



Then: PQ // BC

EXERCISE



Determine the following:

a) CD

b) BC

c) $\angle BCD$ if $\angle BFD = 23°$

MIDPOINT THEOREM: The line joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

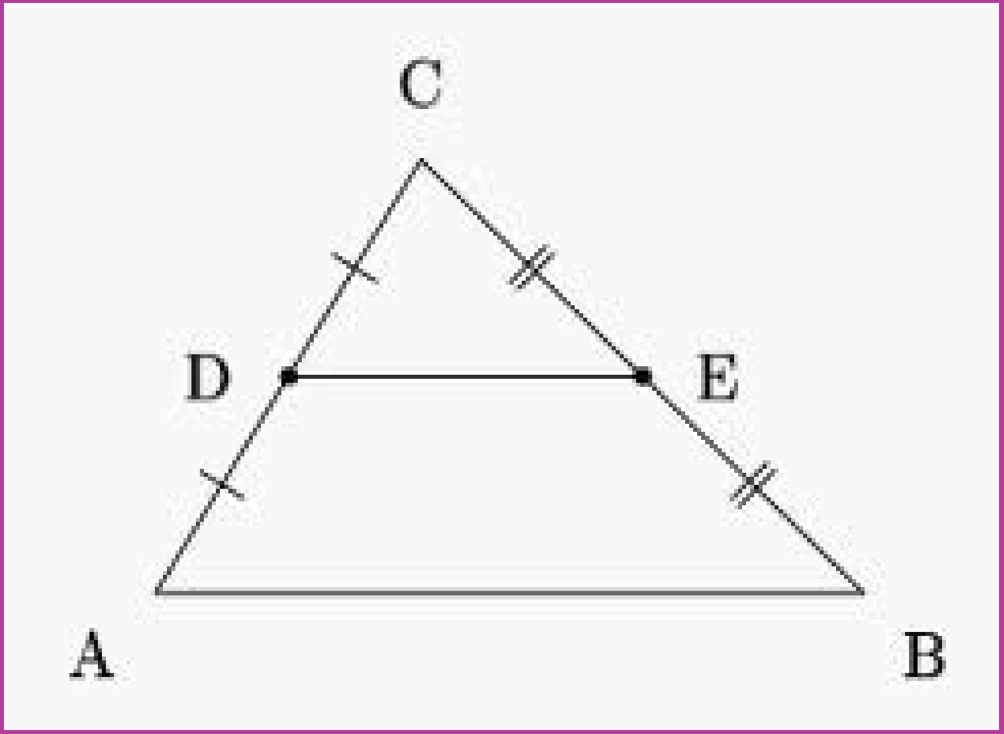
Given:

$$\frac{CD}{DA} = \frac{CE}{EB}$$

Then:

DE // AB and

DB = ½ AB



Example :

a) Solve for x

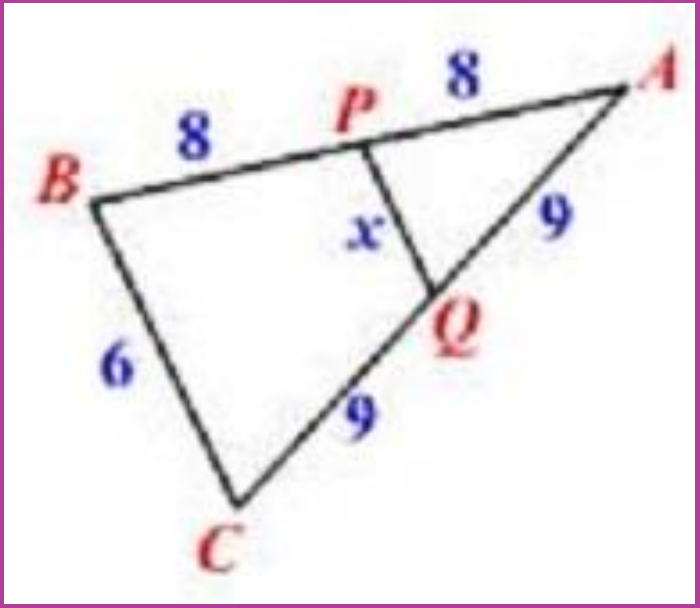
P is the midpoint of AB (given)

Q is the midpoint of AC (given)

$$\therefore PQ = \frac{1}{2}BC$$

$$\therefore x = \frac{1}{2}(6)$$

$$\therefore x = 3$$



Example :

b) If it is given that $\angle CBA = 72°$,

determine the size of $\angle$ QPA.

Using the Midpoint Theorem

P is the midpoint of AB (given)

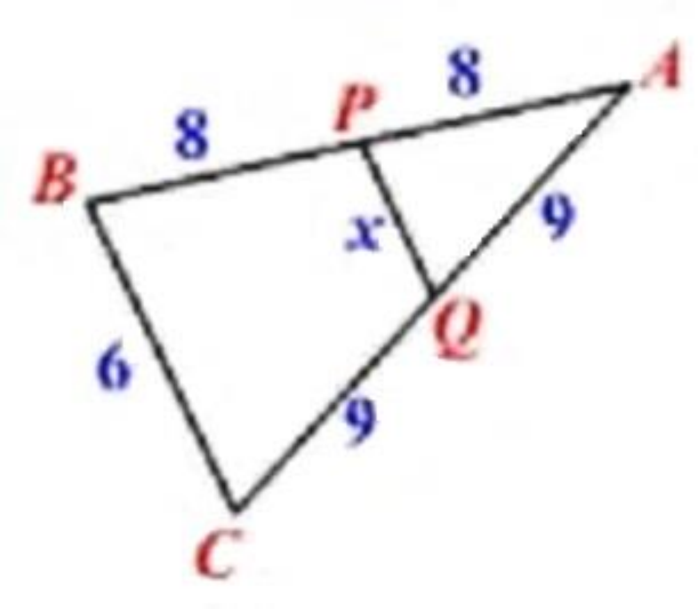
Q is the midpoint of AC (given)

$\therefore PQ$ // BC

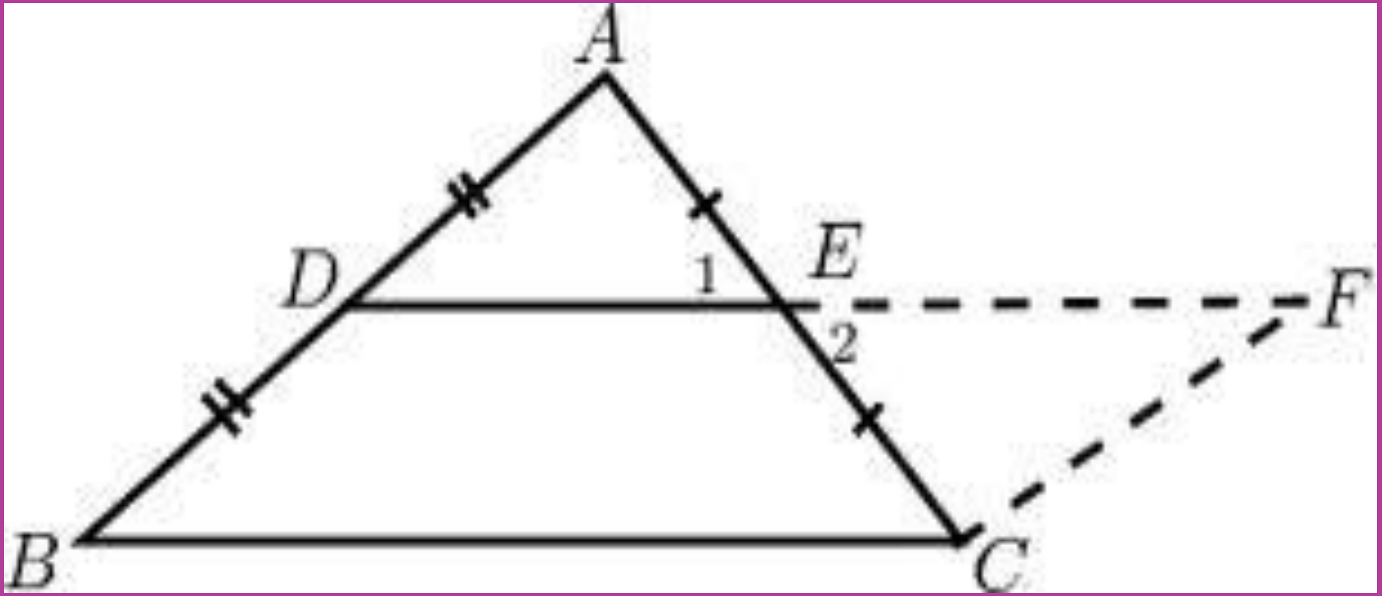
$\angle CBA = 72°$ (given)

$\therefore \angle QPA = 72°$

(corresp. $\angle$'s; PQ // BC)



EXERCISE



Determine the following:

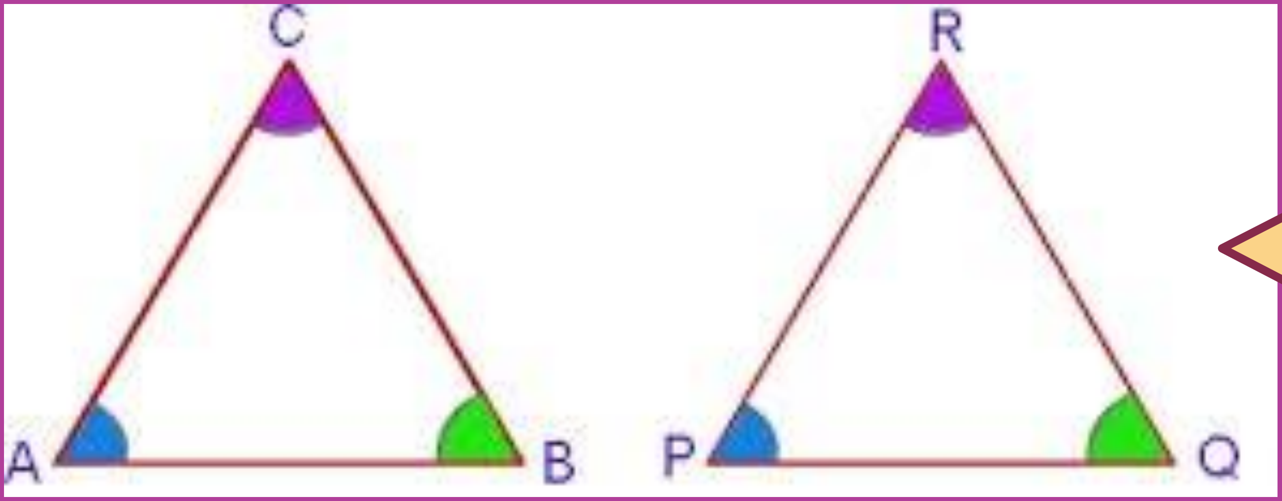
a) Length of BC if DE is 4 units

b) $\angle E_1$ if $\angle BCA = 50°$

c) $\angle CFE$

THEOREM: If two triangles are equiangular, then the corresponding sides are in proportion and hence the triangles are similar.

Given: $\angle A = \angle P$; $\angle B = \angle Q$; and $\angle C = \angle R$



Note! Order of letters is NB!

Then: $\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$ $\therefore \Delta ABC \;|||\; \Delta PQR$

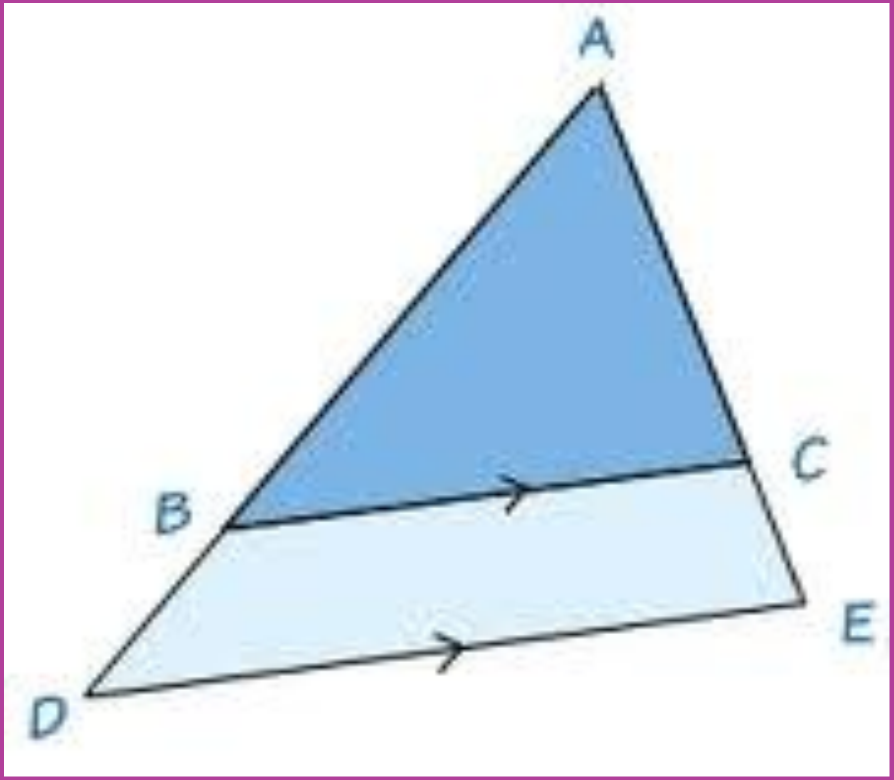
Example 1: Prove that $\frac{AB}{AD} = \frac{AC}{AE}$

$\angle D = \angle B$ (corresp. $\angle$'s; BC // DE)

$\angle E = \angle C$ (corresp. $\angle$'s; BC // DE)

$\therefore\ \Delta ABC\ ||| \ \Delta ADE$

$\therefore \frac{AB}{AD} = \frac{AC}{AE}$



Proving similarity to find proportional sides (0:00 – 4:15)

Example 2: Prove that $\frac{AC}{BF} = \frac{BC}{DF}$

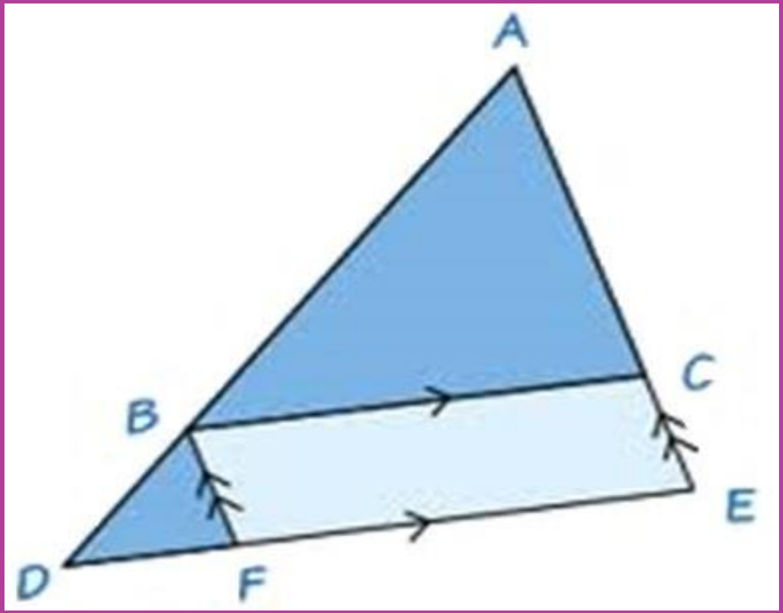
∠ACB = ∠E (corresp. ∠'s; BC // DE)

∠E = ∠BFD (corresp. ∠'s; BF // CE)

∠ABF = ∠D (corresp. ∠'s; BC // DE)

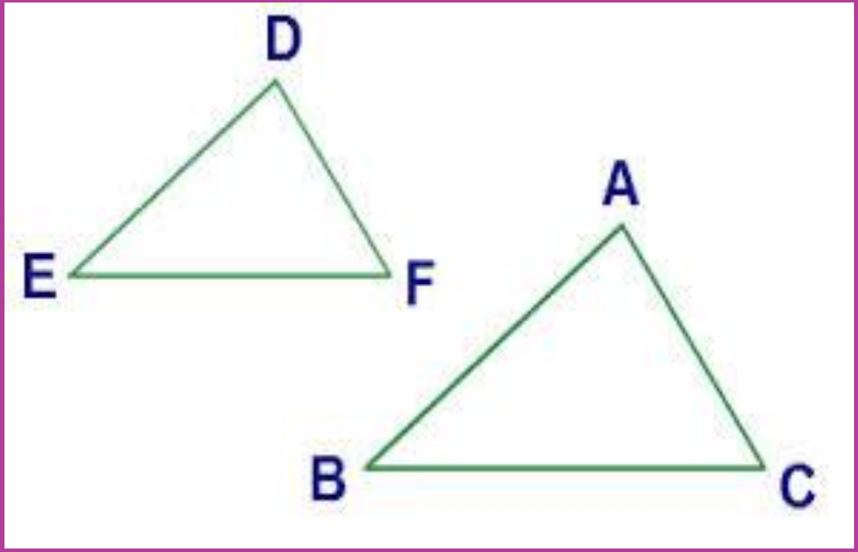
$\therefore\ \Delta ABC\ /\!/\!/\ \Delta BDF$

$\therefore \frac{AC}{BF} = \frac{BC}{DF}$



CONVERSE: If the corresponding sides of two triangles are proportional, then the triangles are equiangular and hence similar.

Given: $\frac{DE}{AB} = \frac{EF}{BC} = \frac{FD}{CA}$



Then:

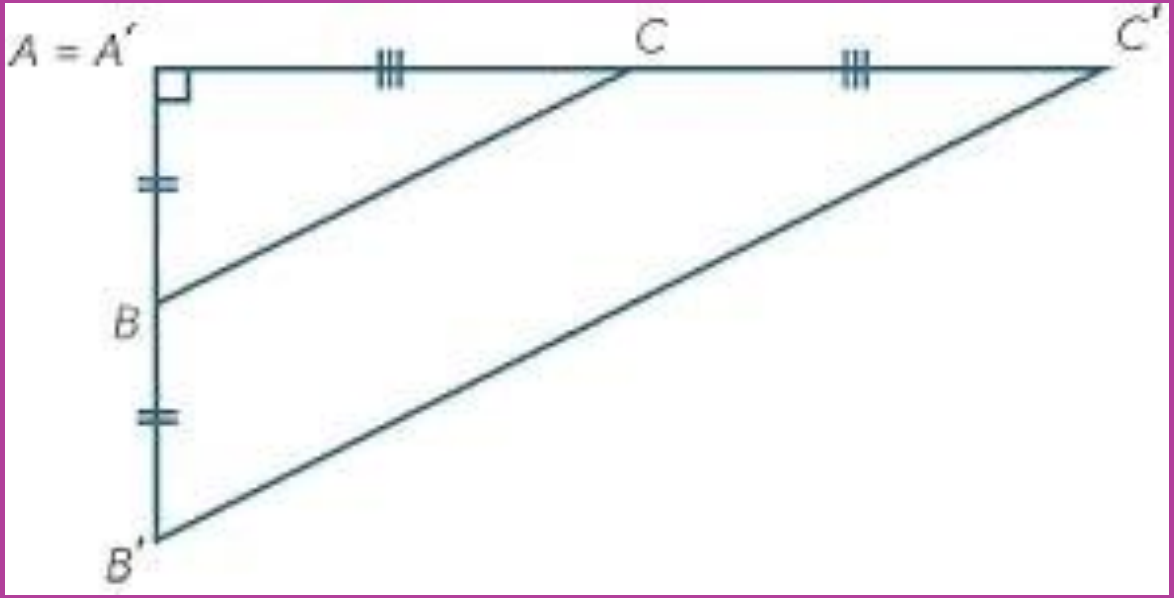
$\angle D = \angle A$;

$\angle E = \angle B$; and

$\angle F = \angle C$

$\therefore \Delta DEF \text{ ||| } \Delta ABC$

EXERCISE



Determine the following:

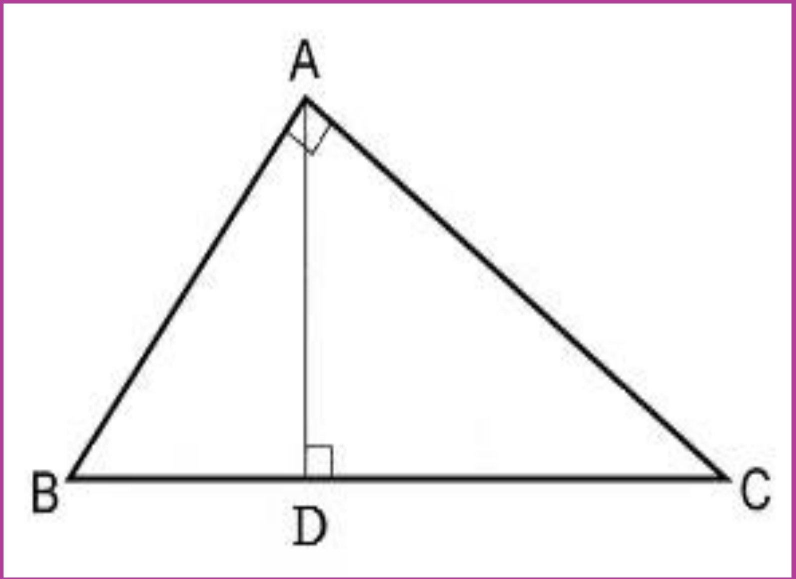
a) Length of BC if B'C' = 12 units

b) Prove that $\frac{A'B'}{BB'} = \frac{BC}{B'C'}$

PYTHAGOREAN THEOREM: In a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on other two sides.

Given:

$\angle BAC = \angle ADC = 90^\circ$



Then:

$\angle BAC = \angle ADC$ (given)

$\angle C$ is common

$\therefore \Delta ABC\ /// \ \Delta DAC$

$(\angle\angle\angle)$

$$\therefore \frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$$

Since ΔABC /// ΔDAC, $\frac{AB}{DA} = \frac{BC}{AC} = \frac{AC}{DC}$

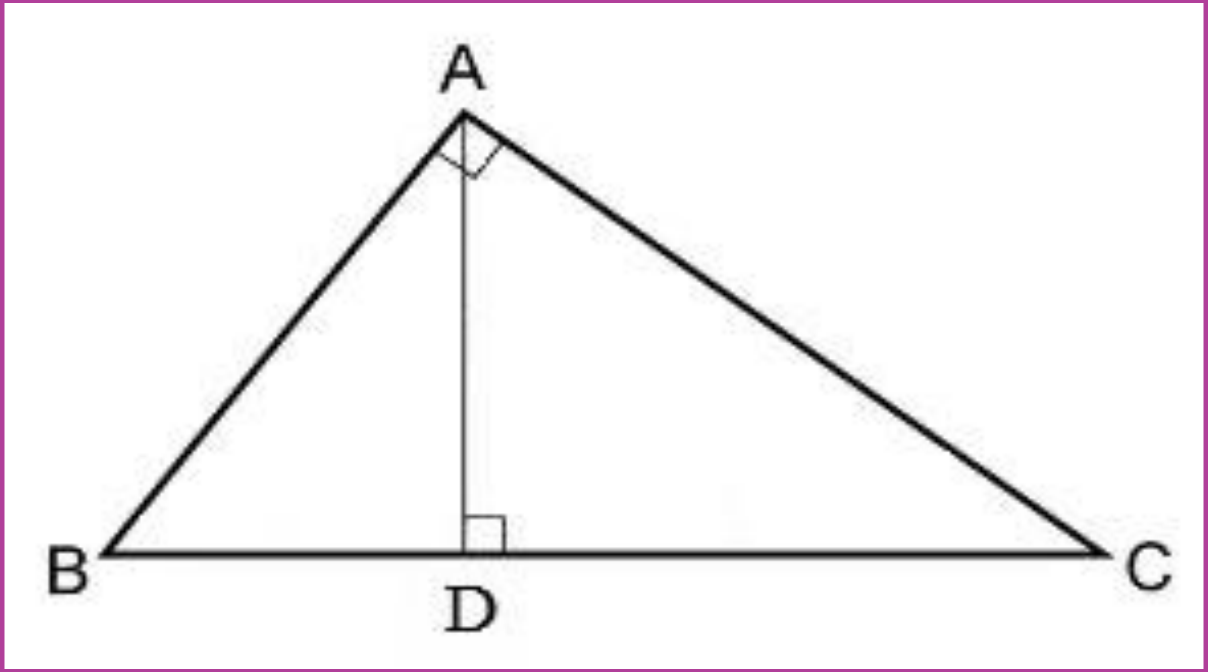
Now manipulate the proportional sides to prove the Pythagorean Theorem:

$$\frac{BC}{AC} = \frac{AC}{DC}$$

Proving Pythagorean Theorem using similarity

$$\therefore AC^2 = BC.DC$$
$$\therefore AB^2 + AC^2 = BD.CB + BC.DC$$
$$\therefore AB^2 + AC^2 = BC(BD + DC)$$
$$\therefore AB^2 + AC^2 = BC.BC$$
$$\therefore AB^2 + AC^2 = BC^2$$

EXERCISE



Prove the following:

a) $AD^2 = CD.DB$

b) $AB^2 + AC^2 = BC^2$

RECAP: SIMILARITY PROBLEMS

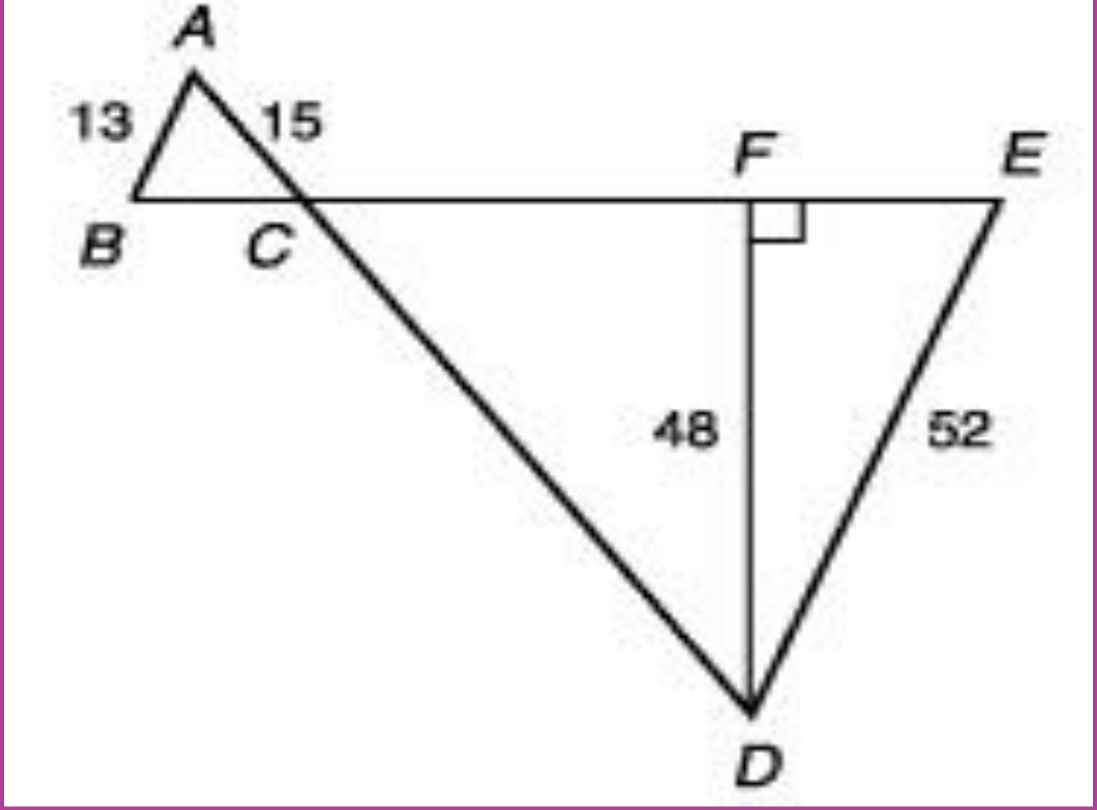
Similarity where one sides plays two different roles

Challenging similarity problem

Extension: Finding area using similarity and congruency

TEST YOUR SKILLS!

1. Determine the perimeter of ΔABC and Δ CDE, given that AB // DE and DF $\perp$ CE.



TEST YOUR SKILLS!

2. Given the following sketch, state the properties of the sides and angles, giving reasons for all your answers.