3.

FINANCE, GROWTH & DECAY

FINANCE RECAP!

Simple Interest:

$$A = P\,(1 + ni)$$

Compound Interest:

$$A = P(1 + i)^n$$

A = total amount
P = principle amount
n = no. of years
i = interest rate

Note! Principle is a ONCE-OFF event!

SOLVING FOR *n*: Example 1

If Alison invests R6 000 each year, how long will it take her to reach her goal of R24 000, if the bank offers her 12% p.a. simple interest?

$$A = P(1 + ni)$$
$$24\,000 = 6\,000(1 + n \times 0,12)$$
$$4 = 1 + 0,12n$$
$$3 = 0,12n$$
$$n = 25\ years$$

SOLVING FOR *n*: Example 2

How many months will it take Justin to save for his motorbike, if he is able to save R800 per month, at an interest rate of 14% p.a. compounded monthly; and the motorbike costs R42 000?

$$A = P(1+i)^n$$

$$42\,000 = 800\left(1+\frac{0{,}14}{12}\right)^n$$

$$52{,}5 = \left(1+\frac{0{,}14}{12}\right)^n$$

$$52{,}5 = \left(\frac{607}{600}\right)^n$$

Now what? How do you solve for *n*?

Since we can't make the bases the same, we use LOGS!

$$52,5 = \left(\frac{607}{600}\right)^{n}$$

$log\ 52,5 = log\left(\frac{607}{600}\right)^{n}$ … log both sides

$log\ 52,5 = n\ log\left(\frac{607}{600}\right)$ … *n* moves to front

$\frac{log\ 52,5}{log\ (\frac{607}{600})} = n$ … solve for *n*

$n = 341,47\ \ldots$

$n = 342\ months$

Working with Logs

ANNUITIES

- Characterized by regular monthly payments

- Two types of annuities:
  1)Future Value Annuities
    - used for investments, savings and sinking funds
  2) Present Value Annuities
    - used for bonds and loans

FUTURE VALUE ANNUITIES

$$F_v = \frac{x\,[\,(1+i)^n - 1\,]}{i}$$

F_v = Future Value of annuity
x = Monthly regular payments
i = Interest rate
n = Time period

Calculate your Savings

FUTURE VALUE: Example 1

Josephine invests R500 per month into a long-term savings account, at an interest rate of 8% p.a. compounded monthly for ten years, in order to make provision for her grandchildren's education.
Determine the value of Josephine's education savings fund after 10 years.

$F_v = ?$

$x = R500$

$\therefore i = \frac{0{,}08}{12}$ (8% p.a. compounded monthly)

$\therefore n = 10 \times 12 = 120$ (10 yrs compounded monthly)

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$

$$= \frac{500\left[\left(1 + \frac{0{,}08}{12}\right)^{120} - 1\right]}{\frac{0{,}08}{12}}$$

$$= R91\ 473{,}02$$

FUTURE VALUE: Example 2

Pedro wants to save R600 000 by the end of 10 years. If the bank offers an interest rate of 10% p.a. compounded monthly, determine how much Pedro must invest every month.

$F_v = 600\,000$

$x = ?$

$\therefore i = \frac{0{,}10}{12}$ (10% p.a. compounded monthly)

$\therefore n = 10 \times 12 = 120$ (10 yrs compounded monthly)

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$

$$600000 = \frac{x\left[\left(1 + \frac{0{,}10}{12}\right)^{120} - 1\right]}{\frac{0{,}10}{12}}$$

$$5000 = x\left[\left(1 + \frac{0{,}10}{12}\right)^{120} - 1\right]$$

$$x = R2929{,}04$$

FUTURE VALUE: Example 3

How long will it take Judy to save R550 000, if she invests R1 500 each month at an interest rate of 9,7% p.a. compounded monthly?

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$

$$550\,000 = \frac{1\,500[\left(1+\frac{0{,}097}{12}\right)^n - 1]}{\frac{0{,}097}{12}}$$

$$550\,000 = \frac{1\,500\left[\left(1+\frac{0{,}097}{12}\right)^{n}-1\right]}{\frac{0{,}097}{12}}$$

$$4\,445{,}83\ldots = 1\,500\left[\left(1+\frac{0{,}097}{12}\right)^{n}-1\right]$$

$$\frac{1067}{360} = \left(1+\frac{0{,}097}{12}\right)^{n}-1$$

$$\frac{1427}{360} = \left(1+\frac{0{,}097}{12}\right)^{n}$$

$$\frac{1427}{360} = \left(1 + \frac{0{,}097}{12}\right)^{n}$$

$$log\left(\frac{1427}{360}\right) = log\left(1 + \frac{0{,}097}{12}\right)^{n}$$

$$log\left(\frac{1427}{360}\right) = n.\log\left(\frac{12097}{12000}\right)$$

$$n = \frac{\log\left(\frac{1427}{360}\right)}{\log\left(\frac{12097}{12000}\right)}$$

$$n = 172\ months$$

FUTURE VALUE: Example 4

A bus company would like to set up a sinking fund in order to make provision for replacing buses. A bus currently costs R890 000 with inflation at 7% p.a. However, depreciation is at 6,5% p.a. Determine the bus company's monthly investments into a sinking fund, if it accrues interest at 12% p.a. compounded monthly.

Inflation:

$A = ?$

$P = 890\,000$

$i = 0,07$

$n = 8$

$$A = 890000(1 + 0,07)^8$$
$$= R1\,529\,185,70$$

∴ The value of the bus in 8 years' time

Depreciation:

$A = ?$

$P = 890\,000$

$i = 0,065$

$n = 8$

$$A = 890000(1 - 0,065)^8$$
$$= R519\,855,80$$

∴ The trade-in value of the bus in 8 years' time

Sinking Fund = Inflation – Depreciation value

$$\therefore F_v = R1\,529\,185,70 - R519\,855,80$$
$$= R1\,009\,329,90$$

$$F_v = \frac{x[(1+i)^n - 1]}{i}$$

$$1009329,90 = \frac{x\left[\left(1+\frac{0,12}{12}\right)^{8\times12} - 1\right]}{\frac{0,12}{12}}$$

$$10\,093,299 = x\left[\left(1+\frac{0,12}{12}\right)^{96} - 1\right]$$

$$x = R6\,311,18$$

PRESENT VALUE ANNUITIES

$$P_v = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$$

P_v = Present Value of annuity
x = Monthly regular payments
i = Interest rate
n = Time period

PRESENT VALUE: Example 1

Callan would like to buy a new car. He is able to afford monthly payments of R1 200 per month. Determine the value of the car loan if the bank will charge 11% p.a. compounded monthly for a loan that will be paid back at the end of 5 years.

$P_v = ?$

$x = 1\,200$

$\therefore\ i = \frac{0{,}11}{12}$ (11% p.a. compounded monthly)

$\therefore\ n = 5 \times 12 = 60$ (5 yrs compounded monthly)

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$= \frac{1200\left[1-\left(1+\frac{0{,}11}{12}\right)^{-60}\right]}{\frac{0{,}11}{12}}$$

$$= R55\ 191{,}64$$

PRESENT VALUE: Example 2

Anna would like to borrow R120 000 for an overseas holiday. Determine her monthly repayments, if the bank charges 9,7% p.a. compounded monthly and she pays off her loan at the end of 2 years.

$P_v = 120\,000$

$x = ?$

$\therefore i = \frac{0{,}097}{12}$ (9,7% p.a. compounded monthly)

$\therefore n = 2 \times 12 = 24$ (2 yrs compounded monthly)

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$120\,000 = \frac{x\left[1-\left(1+\frac{0{,}097}{12}\right)^{-24}\right]}{\frac{0{,}097}{12}}$$

$$970 = x\left[1-\left(1+\frac{0{,}097}{12}\right)^{-24}\right]$$

$$x = R5520{,}79$$

Calculate Bond Repayments

PRESENT VALUE: Example 3

Determine how long it would take Candice to pay back a loan of R50 000, if she is able to repay R2 500 per month; at an interest rate of 10% p.a. compounded monthly.

$P_v = 50\,000$

$x = 2\,500$

$\therefore\ i = \frac{0{,}10}{12}$ (10% p.a. compounded monthly)

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$50\,000 = \frac{2\,500\left[1-\left(1+\frac{0{,}10}{12}\right)^{-n}\right]}{\frac{0{,}10}{12}}$$

$$\frac{1250}{3} = 2\,500\left[1-\left(1+\frac{0{,}1}{12}\right)^{-n}\right]$$

$$\frac{1250}{3} = 2\,500[1 - \left(1 + \frac{0{,}1}{12}\right)^{-n}]$$

$$\frac{1}{6} = 1 - \left(1 + \frac{0{,}1}{12}\right)^{-n}$$

$$-\frac{5}{6} = -\left(1 + \frac{0{,}1}{12}\right)^{-n}$$

$$\frac{5}{6} = \left(1 + \frac{0{,}1}{12}\right)^{-n}$$

$$\log\frac{5}{6} = log\left(1 + \frac{0{,}1}{12}\right)^{-n}$$

$$\log\frac{5}{6} = log\left(1+\frac{0{,}1}{12}\right)^{-n}$$

$$\log\frac{5}{6} = -n.\,log\left(1+\frac{0{,}1}{12}\right)$$

$$-n = \frac{\log\frac{5}{6}}{log\left(1+\frac{0{,}1}{12}\right)}$$

$$-n = -21{,}969\ldots$$

$$n = 22\ months$$

PRESENT VALUE: Example 4

Byron takes out a loan of R300 000 to buy a piece of land, which he intends to pay back over 20 years, at an interest rate of 10,5% p.a. compounded monthly. His monthly instalments are R2995,14. If Byron inherits money and would like to settle the outstanding balance on the property after 5 years, determine how much he will have to still pay.

$P_v = ?$

$x = 2995,14$

$\therefore i = \frac{0,105}{12}$ (10,5% p.a. compounded monthly)

$\therefore n = 15 \times 12 = 180$ (15 years remaining on loan)

$$P_v = \frac{x[1-(1+i)^{-n}]}{i}$$

$$= \frac{2995,14\left[1-\left(1+\frac{0,105}{12}\right)^{-180}\right]}{\frac{0,105}{12}}$$

$$= R270\ 955,57$$