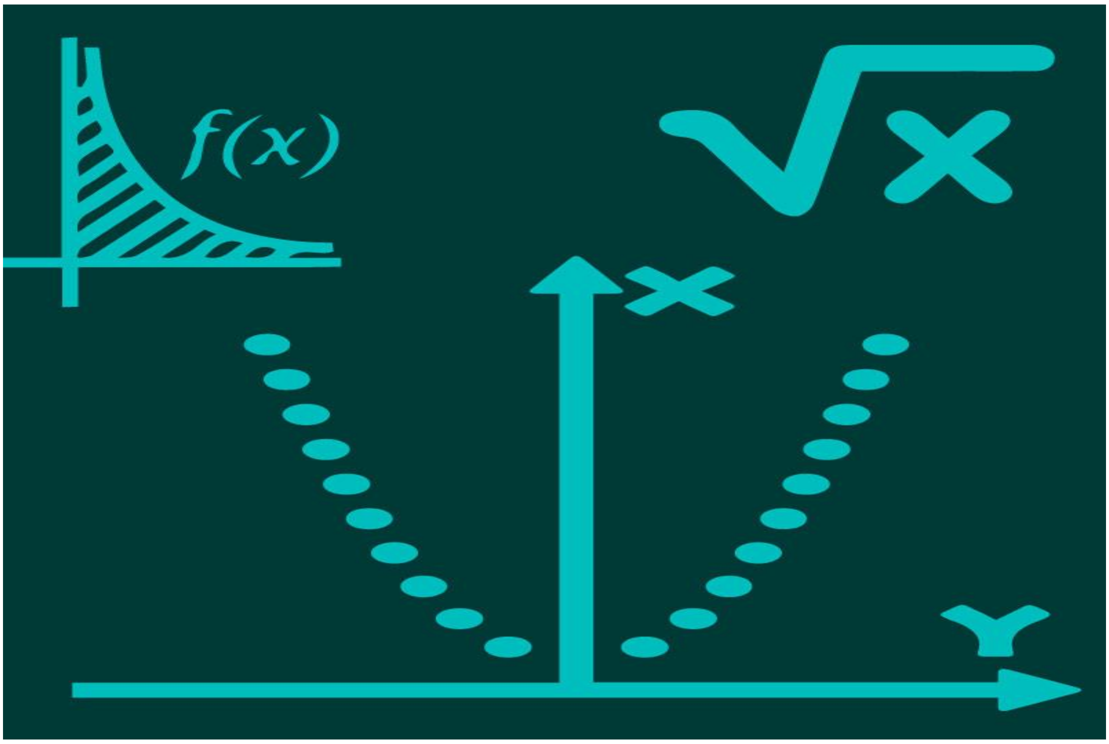



# FUNCTIONS AND GRAPHS

# MATHEMATICS GRADE 12

## REVISION PACK (2019)

## PAST PAPERS

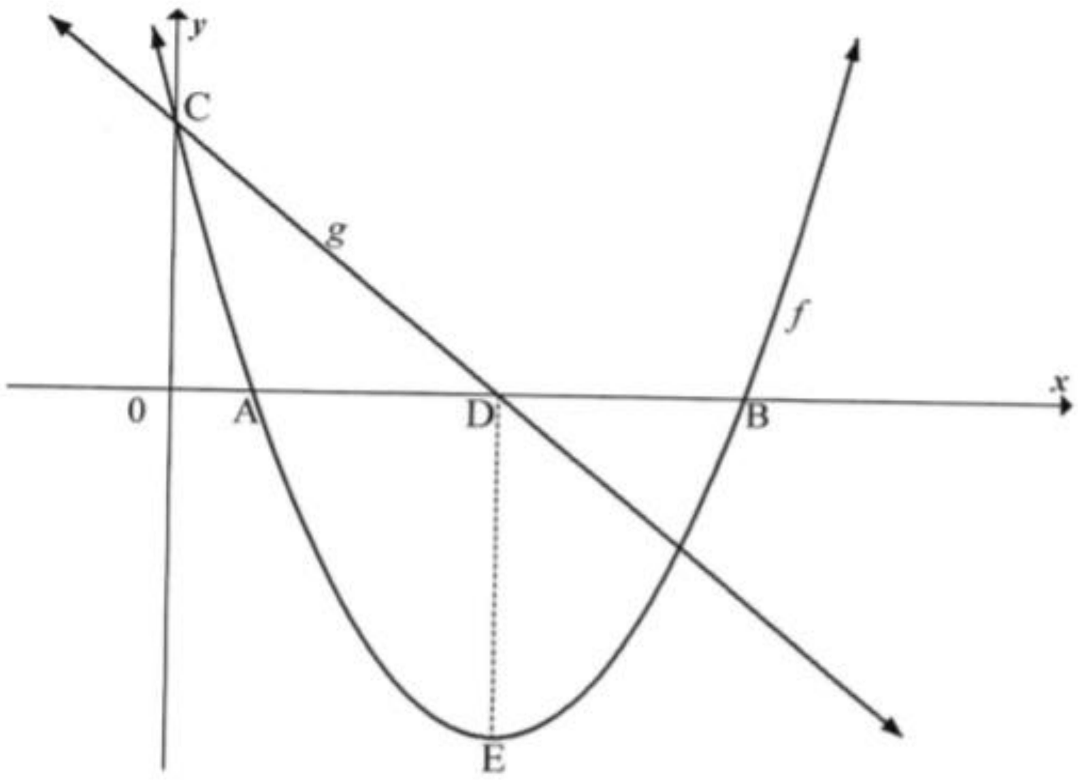
BY AYANDA DLADLA/ 074 994 7970

**FEB 18**

## QUESTION 4

Below are the graphs of $f(x)=(x-4)^2-9$ and a straight line $g$.

- A and B are the $x$-intercepts of $f$ and E is the turning point of $f$.
- C is the $y$-intercept of both $f$ and $g$.
- The $x$-intercept of $g$ is D. DE is parallel to the $y$-axis.




4.1 Write down the coordinates of E. (2)

4.2 Calculate the coordinates of A. (3)

4.3 M is the reflection of C in the axis of symmetry of $f$. Write down the coordinates of M. (3)

4.4 Determine the equation of $g$ in the form $y=mx+c$. (3)

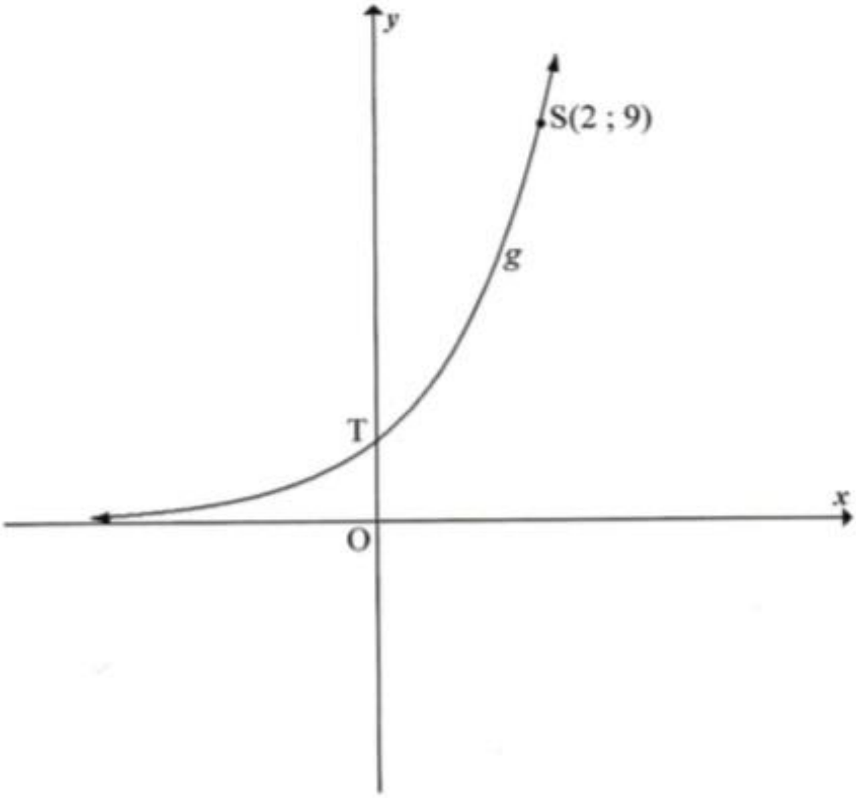
4.5 Write down the equation of $g^{-1}$ in the form $y=\ldots$ (3)

4.6 For which values of $x$ will $x(f(x))\leq 0$? (4)

**[18]**

## QUESTION 5

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2 ; 9) lies on $g$. T is the $y$-intercept of $g$.




5.1 Write down the coordinates of T. (2)

5.2 Calculate the value of $a$. (2)

5.3 The graph $h$ is obtained by reflecting $g$ in the $y$-axis. Write down the equation of $h$. (2)

5.4 Write down the values of $x$ for which $0 < \log_3 x < 1$. (2)

**[8]**

**QUESTION 6**

The function $f$, defined by $f(x)=\dfrac{a}{x+p}+q$ , has the following properties:

- The range of $f$ is $y\in R,\ y\neq 1$.
- The graph $f$ passes through the origin.
- $\mathrm{P}\left(\sqrt{2}+2;\sqrt{2}+1\right)$ lies on the graph $f$.

6.1 Write down the value of $q$. (1)

6.2 Calculate the values of $a$ and $p$. (5)

6.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4)

**[10]**



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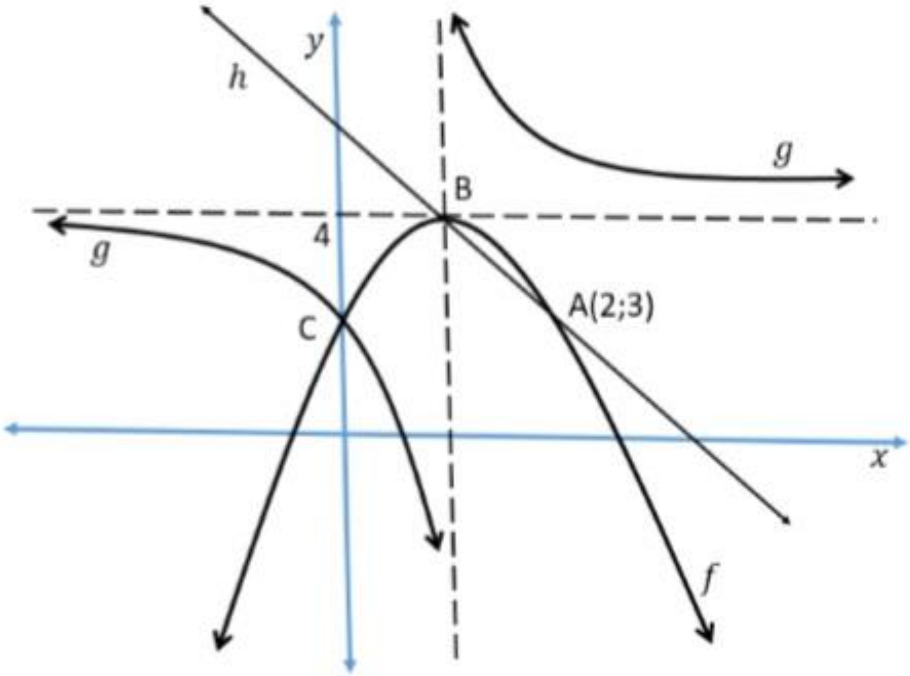
**SEPT 18 MCED**

## QUESTION 4

In the sketch below the graphs of $g(x) = \frac{k}{x+p} + 4$ and $f(x) = ax^2 + bx + c$ are given.
The asymptotes of $g$ intersect at B, the turning point of $f$. The graphs of $f$ and $g$ intersect at C.
The axis of symmetry of $g$ that has a negative gradient, is the line $h(x)$ that intersects the graph of $f$ at A $(2\,;\,3)$ and B.




4.1 Determine the equation of $h$. (2)

4.2 Show that the coordinates of B are $(1; 4)$.
Clearly indicate all calculations. (2)

4.3 Show that the equation of $f(x)$ is given by $f(x) = -x^2 + 2x + 3$.
Clearly indicate all calculations. (4)

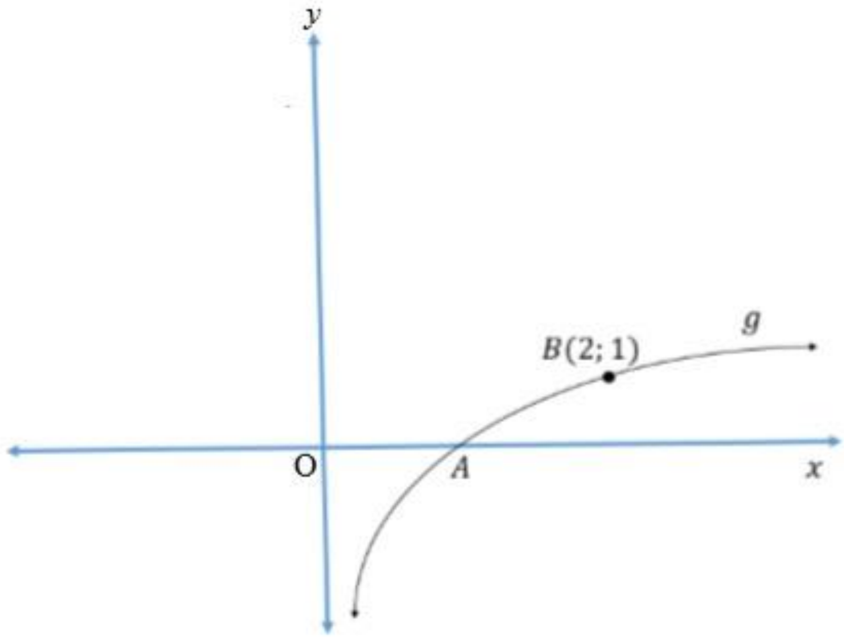
4.4 Determine the equation of $g$. (3)

4.5 Determine the equations of the asymptotes of $g(x+1)$. (2)

4.6 Determine the value(s) of $x$ for which $g'(x).f'(x) \geq 0$. (2)
**[15]**

## QUESTION 5

The graph of $g(x) = log_2 x$ is given below. A is the point of intersection of $g$ with the $x$-axis.




5.1 Write down the coordinates of A. (2)

5.2 Determine the equation of $g^{-1}$ in the form $y =$ ........ (2)

5.3 Sketch the graph of $p(x) = -g(x)$. Clearly indicate all intercept(s) with the axes as well as the coordinates of any other point on the graph. (2)

5.4 Use the graph to determine for which value(s) of $x$ will $log_{½} x \geq 0$ (2)

**[8]**

## QUESTION 6

6.1 Sketch the graph of $f(x) = \frac{k}{x+p} + q$ if:

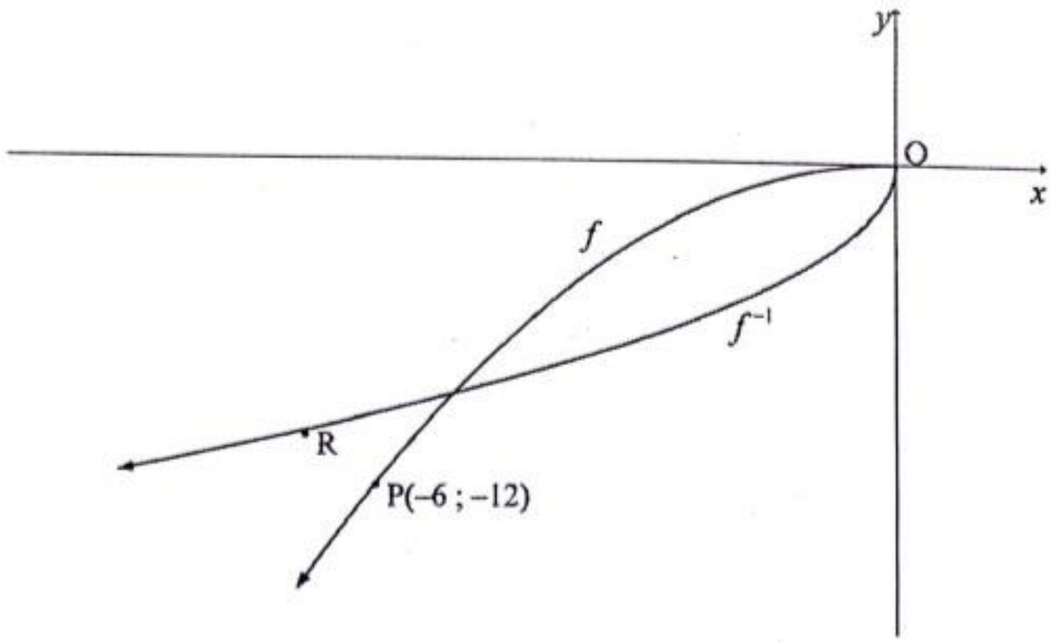
- The domain is given as: $x \in R\,;x \neq -1$.
- The range is given as: $y \in R\,;y \neq 2$
- $k < 0$
- $x - intercept:\ (-\frac{1}{2};0)$
- $f(0) = 1$ (5)

**[5]**

**NOV 18**

## QUESTION 4

In the diagram below, the graph of $f(x) = ax^2$ is drawn in the interval $x \leq 0$.
The graph of $f^{-1}$ is also drawn. P(–6 ; –12) is a point on $f$ and R is a point on $f^{-1}$.




| | | |
|---|---|---|
| 4.1 | Is $f^{-1}$ a function? Motivate your answer. | (2) |
| 4.2 | If R is the reflection of P in the line $y = x$, write down the coordinates of R. | (1) |
| 4.3 | Calculate the value of $a$. | (2) |
| 4.4 | Write down the equation of $f^{-1}$ in the form $y = ...$ | (3) |
| | | **[8]** |

## QUESTION 5

Given: $f(x)=\dfrac{-1}{x-1}$

5.1 Write down the domain of $f$. (1)

5.2 Write down the asymptotes of $f$. (2)

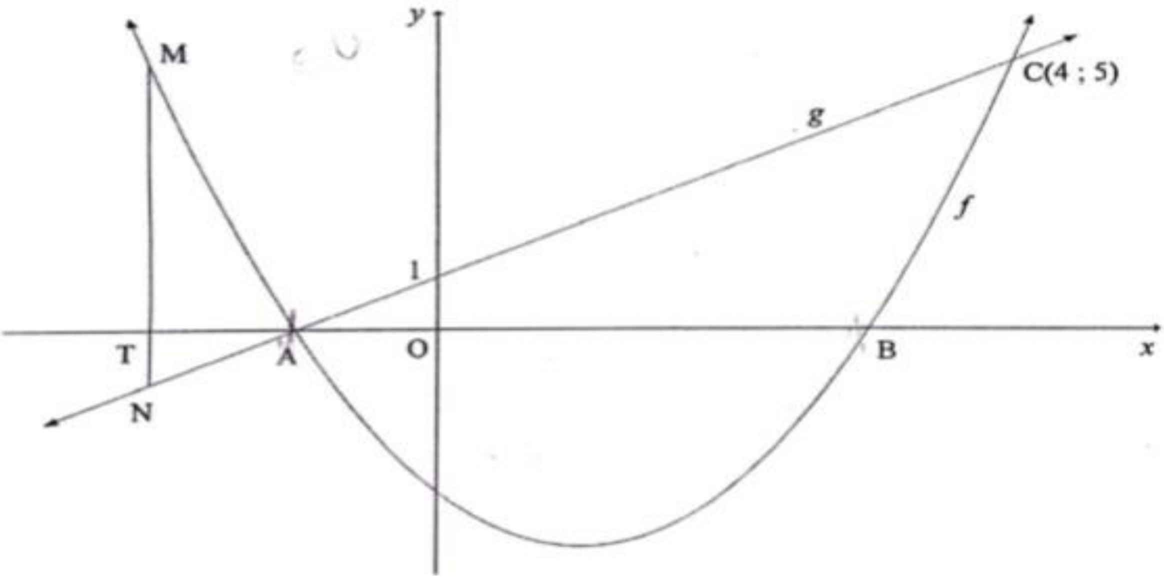
5.3 Sketch the graph of $f$, clearly showing all intercepts with the axes and any asymptotes. (3)

5.4 For which values of $x$ will $x.f'(x)\geq 0$? (2)

**[8]**

## QUESTION 6

In the diagram below, A and B are the $x$-intercepts of the graph of $f(x)=x^2-2x-3$.
A straight line, $g$, through A cuts $f$ at C(4 ; 5) and the $y$-axis at (0 ; 1).
M is a point on $f$ and N is a point on $g$ such that MN is parallel to the $y$-axis.
MN cuts the $x$-axis at T.




6.1 Show that $g(x)=x+1$. (2)

6.2 Calculate the coordinates of A and B. (3)

6.3 Determine the range of $f$. (3)

6.4 If MN = 6:

6.4.1 Determine the length of OT if T lies on the negative $x$-axis. Show ALL your working. (4)

6.4.2 Hence, write down the coordinates of N. (2)

6.5 Determine the equation of the tangent to $f$ drawn parallel to $g$. (5)

6.6 For which value(s) of $k$ will $f(x)=x^2-2x-3$ and $h(x)=x+k$ NOT intersect? (1)

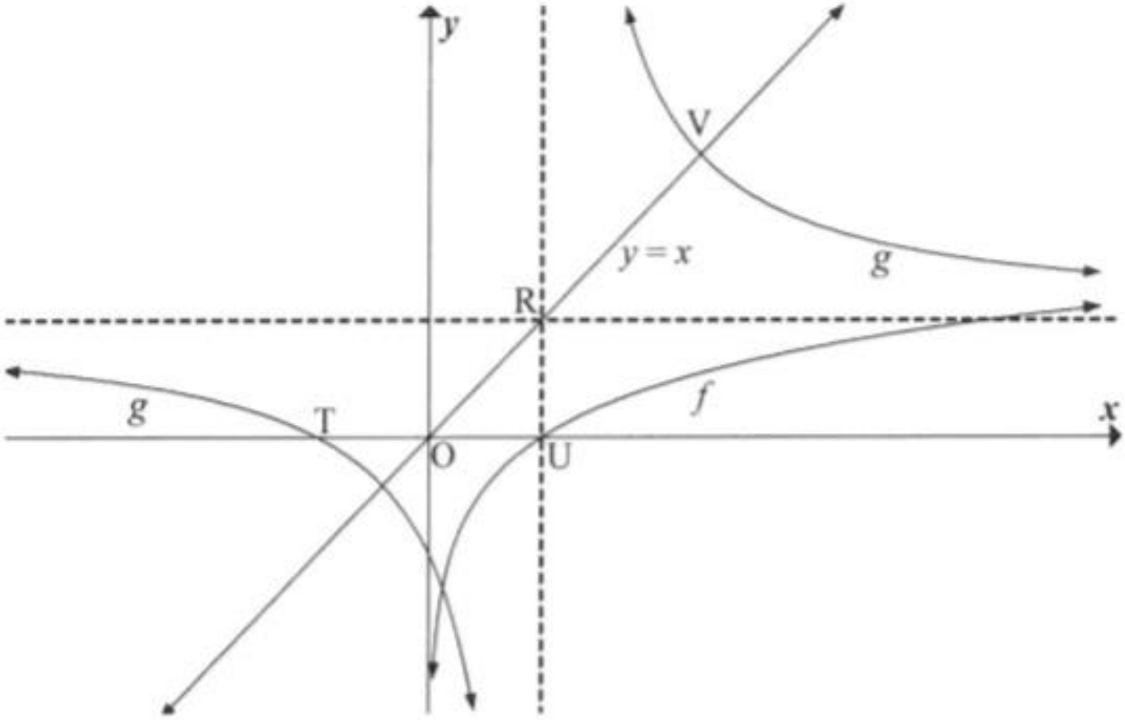
**[20]**

**FEB 17**

## QUESTION 4

The sketch below shows the graphs of $f(x) = \log_5 x$ and $g(x) = \frac{2}{x-1} + 1$.

- T and U are the $x$-intercepts of $g$ and $f$ respectively.
- The line $y = x$ intersects the asymptotes of $g$ at R, and the graph of $g$ at V.



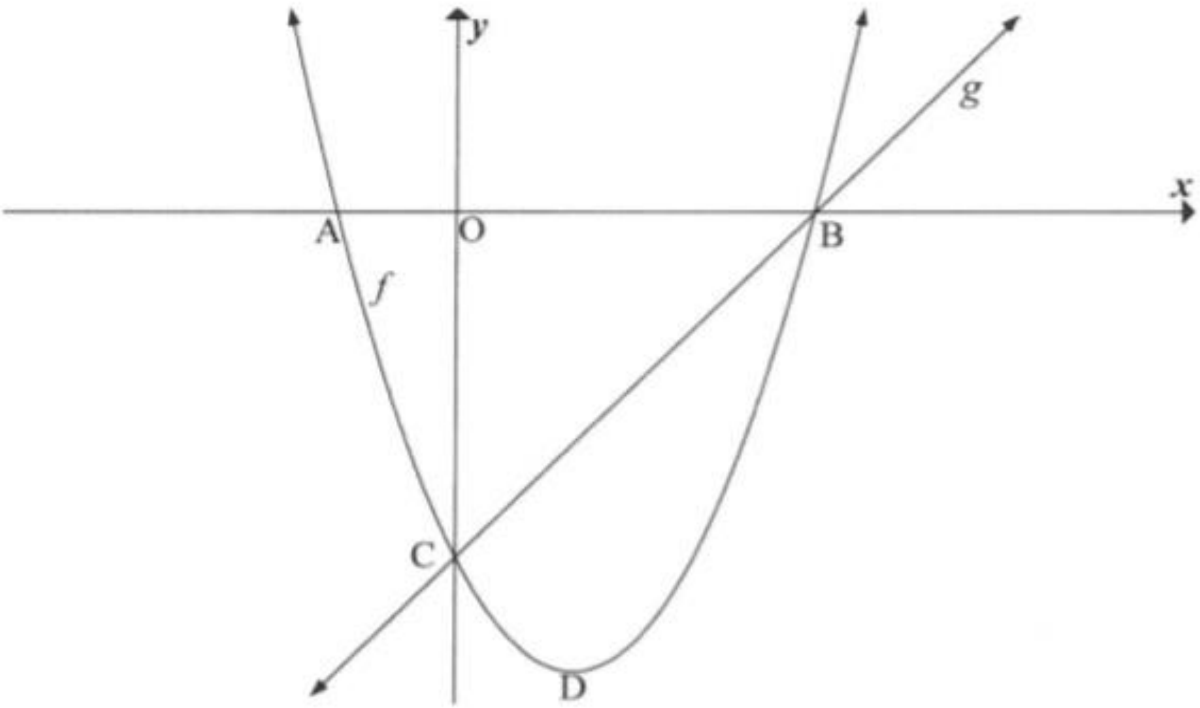

| | | |
|---|---|---|
| 4.1 | Write down the coordinates of U. | (1) |
| 4.2 | Write down the equations of the asymptotes of $g$. | (2) |
| 4.3 | Determine the coordinates of T. | (2) |
| 4.4 | Write down the equation of $h$, the reflection of $f$ in the line $y = x$, in the form $y = \ldots$ | (2) |
| 4.5 | Write down the equation of the asymptote of $h(x-3)$. | (1) |
| 4.6 | Calculate the coordinates of V. | (4) |
| 4.7 | Determine the coordinates of $T'$ the point which is symmetrical to T about the point R. | (2) |
| | | **[14]** |

## QUESTION 5

5.1 The sketch below shows the graphs of $f(x)=x^2-2x-3$ and $g(x)=x-3$.

- A and B are the $x$-intercepts of $f$.
- The graphs of $f$ and $g$ intersect at C and B.

D is the turning point of $f$.




| | | |
|---|---|---|
| 5.1.1 | Determine the coordinates of C. | (1) |
| 5.1.2 | Calculate the length of AB. | (4) |
| 5.1.3 | Determine the coordinates of D. | (2) |
| 5.1.4 | Calculate the average gradient of $f$ between C and D. | (2) |
| 5.1.5 | Calculate the size of $O\hat{C}B$ | (2) |
| 5.1.6 | Determine the values of $k$ for which $f(x)=k$ will have two unequal positive real roots. | (3) |
| 5.1.7 | For which values of $x$ will $f'(x) \cdot f''(x)>0$? | (3) |

5.2 The graph of a parabola $f$ has $x$-intercepts at $x=1$ and $x=5$. $g(x)=4$ is a tangent to $f$ at P, the turning point of $f$. Sketch the graph of $f$, clearly showing the intercepts with the axes and the coordinates of the turning point. (5)

**[22]**

## NOV 17

### QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of $f$ at the point $\left(-1\,;\,\frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the $x$-intercepts of $f$. (3)

4.3 Calculate the coordinates of the turning point of $f$. (3)

4.4 Sketch the graph of $f$. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of $x$ for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as $f$. Clearly indicate ALL intercepts with the axes. (2)

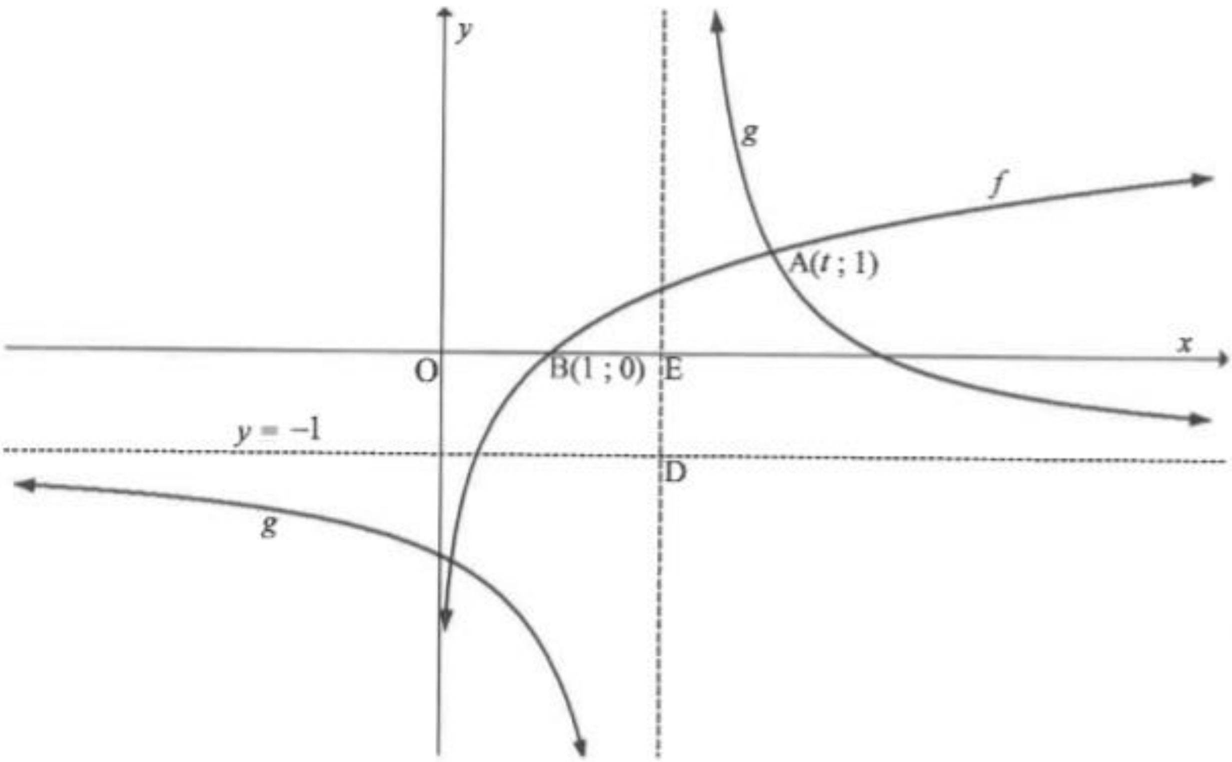
4.7 Write down the values of $x$ for which $f(x).g(x) \leq 0$. (3)

**[23]**

## QUESTION 5

The diagram below shows the graphs of $g(x)=\dfrac{2}{x+p}+q$ and $f(x)=\log_3 x$.

- $y=-1$ is the horizontal asymptote of $g$.
- B(1 ; 0) is the $x$-intercept of $f$.
- A($t$ ; 1) is a point of intersection between $f$ and $g$.
- The vertical asymptote of $g$ intersects the $x$-axis at E and the horizontal asymptote at D.
- OB = BE.




| | | |
|---|---|---|
| 5.1 | Write down the range of $g$. | (2) |
| 5.2 | Determine the equation of $g$. | (2) |
| 5.3 | Calculate the value of $t$. | (3) |
| 5.4 | Write down the equation of $f^{-1}$, the inverse of $f$, in the form $y=\ldots$ | (2) |
| 5.5 | For which values of $x$ will $f^{-1}(x)<3$? | (2) |
| 5.6 | Determine the point of intersection of the graphs of $f$ and the axis of symmetry of $g$ that has a negative gradient. | (3) |

**[14]**

## FEB 16

### QUESTION 4

Given: $f(x) = 2^{-x} + 1$

4.1 Determine the coordinates of the $y$-intercept of $f$. (1)

4.2 Sketch the graph of $f$, clearly indicating ALL intercepts with the axes as well as any asymptotes. (3)

4.3 Calculate the average gradient of $f$ between the points on the graph where $x = -2$ and $x = 1$. (3)

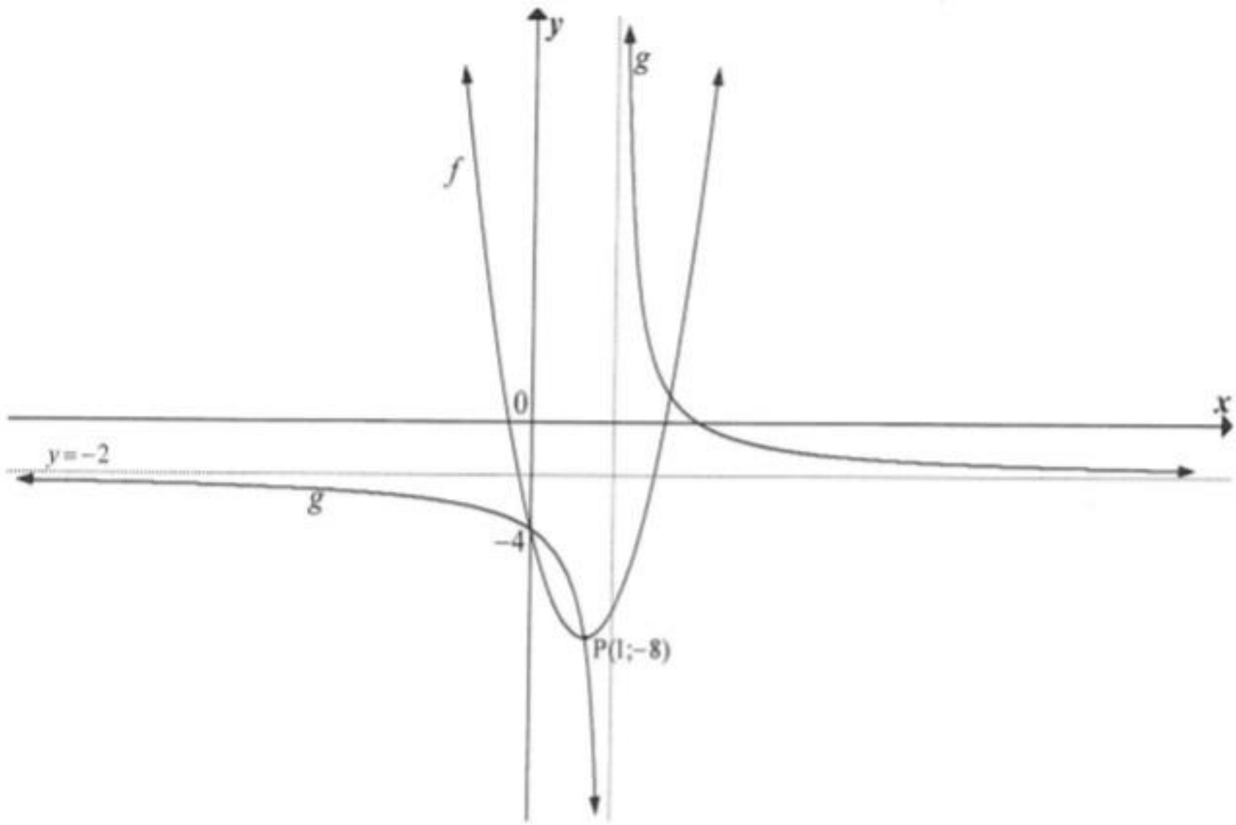
4.4 If $h(x) = 3f(x)$, write down an equation of the asymptote of $h$. (1)

**[8]**

## QUESTION 5

The graphs of the functions $f(x)=a(x+p)^2+q$ and $g(x)=\dfrac{k}{x+r}+d$ are sketched below.

Both graphs cut the $y$-axis at $-4$. One of the points of intersection of the graphs is P(1 ; –8), which is also the turning point of $f$. The horizontal asymptote of $g$ is $y=-2$.




| | | |
|---|---|---|
| 5.1 | Calculate the values of $a$, $p$ and $q$. | (4) |
| 5.2 | Calculate the values of $k$, $r$ and $d$. | (6) |
| 5.3 | Determine the value(s) of $x$ in the interval $x \leq 1$ for which $g(x) \geq f(x)$. | (2) |
| 5.4 | Determine the value(s) of $k$ for which $f(x)=k$ has two, unequal positive roots. | (2) |
| 5.5 | Write down an equation for the axis of symmetry of $g$ that has a negative gradient. | (3) |
| 5.6 | The point P is reflected in the line determined in QUESTION 5.5 to give the point Q. Write down the coordinates of Q. | (2) |
| | | **[19]** |

## QUESTION 6

Given: $f(x)=\frac{1}{4}x^2,\ x\leq 0$

6.1 Determine the equation of $f^{-1}$ in the form $f^{-1}(x)=\ldots$ (3)

6.2 On the same system of axes, sketch the graphs of $f$ and $f^{-1}$. Indicate clearly the intercepts with the axes, as well as another point on the graph of each of $f$ and $f^{-1}$. (3)

6.3 Is $f^{-1}$ a function? Give a reason for your answer. (2)

**[8]**

**SEPT 16 MP**

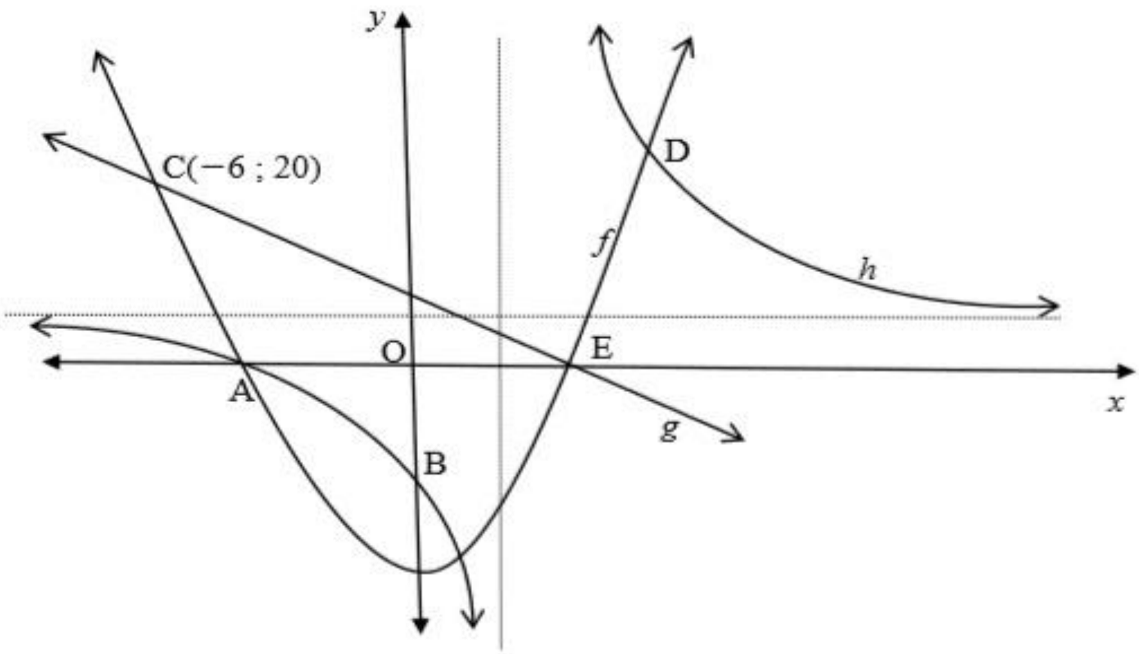
**QUESTION 3**

Sketched below are the graphs of: $g(x) = -2x + 8$

$f(x) = x^2 + c$

and $h(x) = \dfrac{6}{x-2} + 1$

A and B are the $x$- and $y$ - intercepts of $h$ respectively, C $(-6\ ;\ 20)$ and E are the points of intersection of $f$ and $g$.




3.1 Calculate the coordinates of A, B and E. (4)

3.2 Show that the value of $c = -16$ (2)

3.3 Write down the values of $x$ for which $g(x) - f(x) \geq 0$ (2)

3.4 Determine the equation of the symmetry axis of $h$ if the gradient is negative. (2)

3.5 Show that the length of BE $= 2\sqrt{5}$ (2)

3.6 Write down the range of $s$, if $s(x) = h(x) + 2$. (2)

3.7 If $t$ is a tangent to $f$ and parallel to $g$, determine the equation of the tangent, $t$ in the form $y = mx + c$. (5)

**[19]**

## QUESTION 4

Given: $g(x) = \log_{\frac{1}{2}} x$ and $f(x) = 2x^2 - 4x - 6$

4.1 Determine the equation of $g^{-1}$, the inverse of $g$, in the form $g^{-1}(x) = \ldots$ (2)

4.2 Write $f$ in the form $f(x) = a(x+p)^2 + q$ (4)

4.3 The graph of $f$ is shifted 2 unit to the left to form $p$. Write down the equation of the new graph in the form $p(x) = \ldots\ldots$ (2)

4.4 Write down the values of $x$ for which $\log_2 x \leq 0$ (2)

4.5 If $h$ is the reflection of $g^{-1}$ about the $x$-axis, for which value of $x$ will $h(x) - f'(x) = 3$ (3)

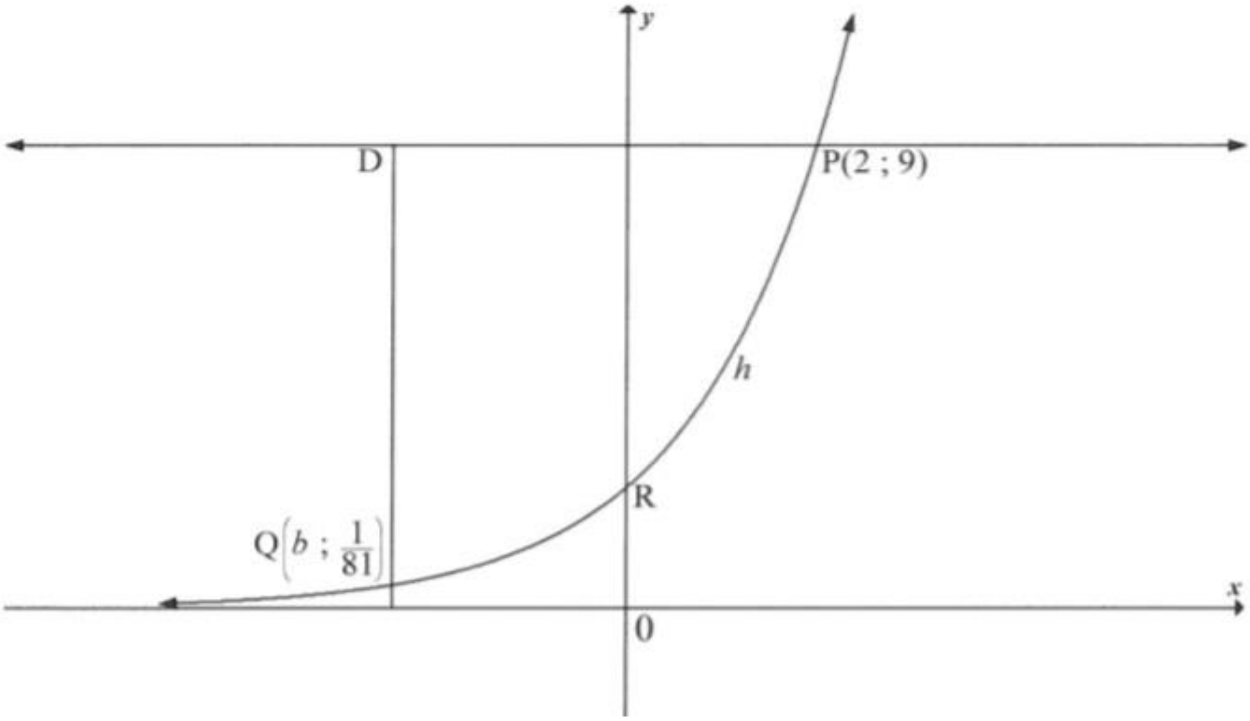
**[13]**

## NOV 16

### QUESTION 4

Sketched below is the graph of $h(x) = a^x$, $a > 0$. R is the $y$-intercept of $h$.

The points P(2 ; 9) and $Q\left(b ; \frac{1}{81}\right)$ lie on $h$.

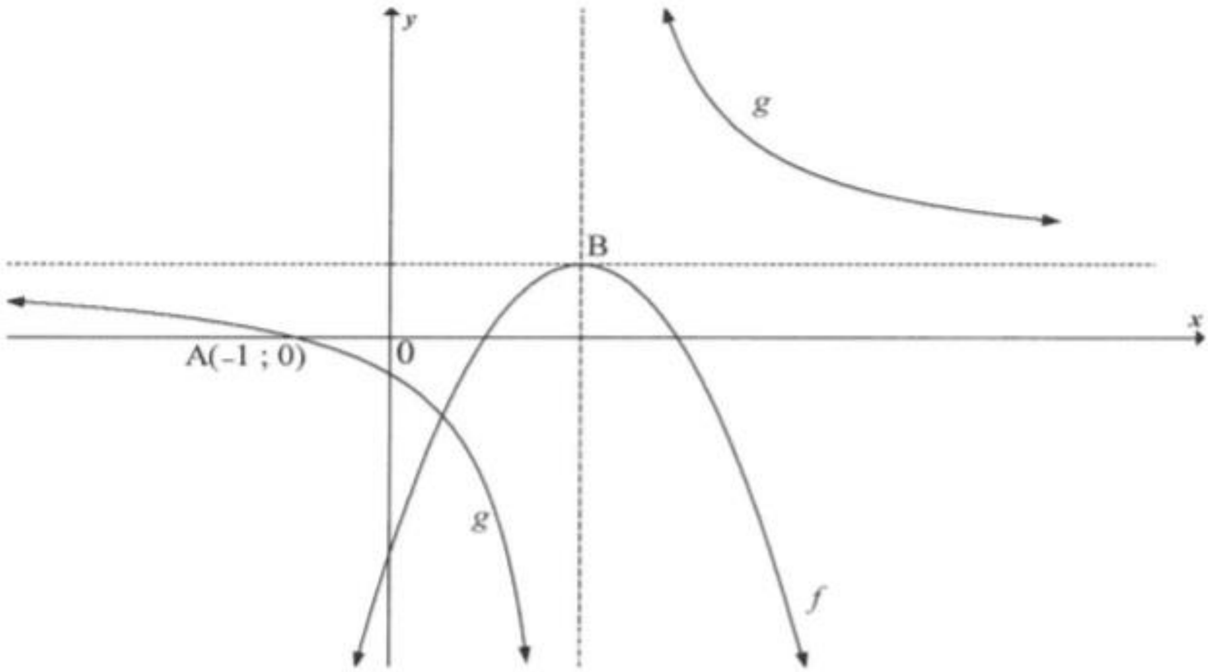



| | | |
|---|---|---|
| 4.1 | Write down the equation of the asymptote of $h$. | (1) |
| 4.2 | Determine the coordinates of R. | (1) |
| 4.3 | Calculate the value of $a$. | (2) |
| 4.4 | D is a point such that DQ ∥ $y$-axis and DP ∥ $x$-axis. Calculate the length of DP. | (4) |
| 4.5 | Determine the values of $k$ for which the equation $h(x+2)+k=0$ will have a root that is less than $-6$. | (3) |
| | | **[11]** |

## QUESTION 5

Sketched below is the parabola $f$, with equation $f(x) = -x^2 + 4x - 3$ and a hyperbola $g$, with equation $(x-p)(y+t) = 3$.

- B, the turning point of $f$, lies at the point of intersection of the asymptotes of $g$.
- $A(-1\,;0)$ is the $x$-intercept of $g$.




5.1 Show that the coordinates of B are $(2\,;1)$ (2)

5.2 Write down the range of $f$. (1)

5.3 For which value(s) of $x$ will $g(x) \geq 0$? (2)

5.4 Determine the equation of the vertical asymptote of the graph of $h$ if $h(x) = g(x+4)$ (1)

5.5 Determine the values of $p$ and $t$. (4)

5.6 Write down the values of $x$ for which $f(x).g'(x) \geq 0$ (4)

**[14]**

## QUESTION 6

Given: $f(x) = -x + 3$ and $g(x) = \log_2 x$

6.1 On the same set of axes, sketch the graphs of $f$ and $g$, clearly showing ALL intercepts with the axes. (4)

6.2 Write down the equation of $g^{-1}(x)$, the inverse of $g$, in the form $y = \ldots$ (2)

6.3 Explain how you will use QUESTION 6.1 and/or QUESTION 6.2 to solve the equation $\log_2(3 - x) = x$. (3)

6.4 Write down the solution to $\log_2(3 - x) = x$. (1)

**[10]**

# FEB 15

## QUESTION 4

Given: $g(x)=\dfrac{6}{x+2}-1$

4.1 Write down the equations of the asymptotes of $g$. (2)

4.2 Calculate:

4.2.1 The $y$-intercept of $g$ (1)

4.2.2 The $x$-intercept of $g$ (2)

4.3 Draw the graph of $g$, showing clearly the asymptotes and the intercepts with the axes. (3)

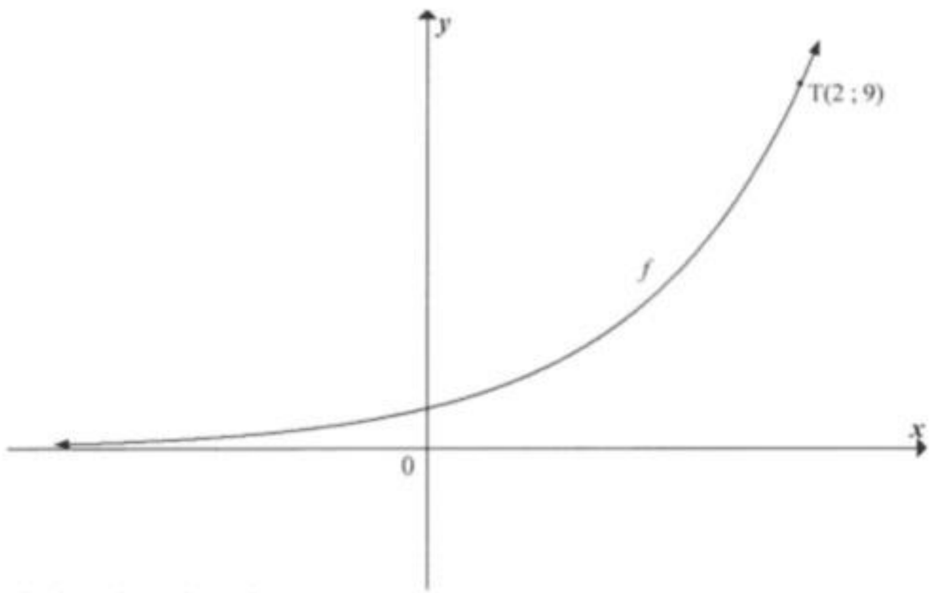
4.4 Determine the equation of the line of symmetry that has a negative gradient, in the form $y = ....$ (3)

4.5 Determine the value(s) of $x$ for which $\dfrac{6}{x+2}-1\geq -x-3$. (2)

**[13]**

## QUESTION 5

The graph of $f(x)=a^x, a>1$ is shown below. T(2 ; 9) lies on $f$.




5.1 Calculate the value of $a$. (2)

5.2 Determine the equation of $g(x)$ if $g(x)=f(-x)$. (1)

5.3 Determine the value(s) of $x$ for which $f^{-1}(x)\geq 2$. (2)

5.4 Is the inverse of $f$ a function? Explain your answer. (2)

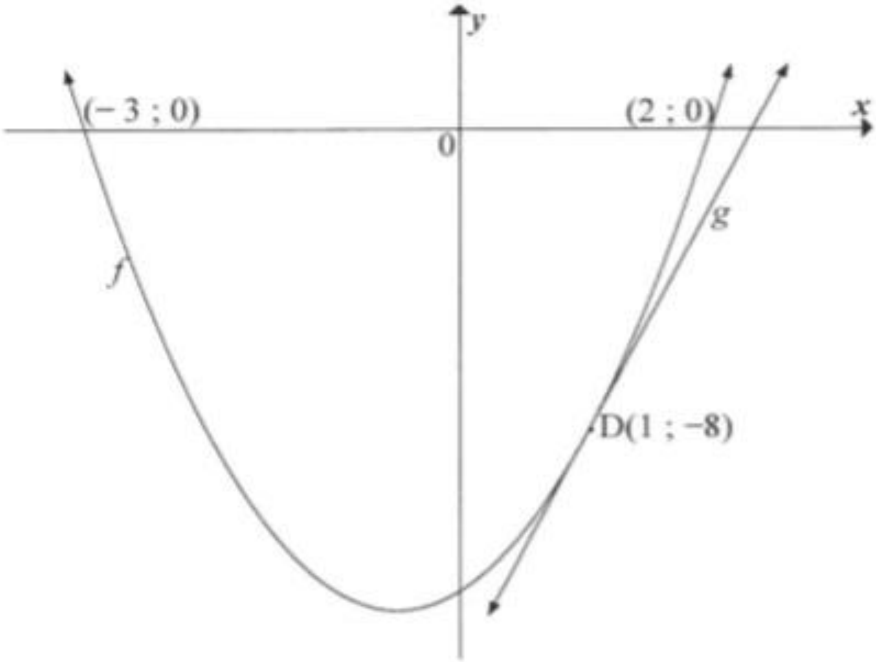
**[7]**

## QUESTION 6

The graphs of $f(x) = ax^2 + bx + c$ ; $a \neq 0$ and $g(x) = mx + k$ are drawn below.

D(1 ; – 8) is a common point on $f$ and $g$.

- $f$ intersects the $x$-axis at $(-3 ; 0)$ and $(2 ; 0)$.
- $g$ is the tangent to $f$ at D.




| | | |
|---|---|---|
| 6.1 | For which value(s) of $x$ is $f(x) \leq 0$? | (2) |
| 6.2 | Determine the values of $a$, $b$ and $c$. | (5) |
| 6.3 | Determine the coordinates of the turning point of $f$. | (3) |
| 6.4 | Write down the equation of the axis of symmetry of $h$ if $h(x) = f(x-7) + 2$. | (2) |
| 6.5 | Calculate the gradient of $g$. | (3) |
| | | **[15]** |

## NOV 15

### QUESTION 4

Given: $f(x) = 2^{x+1} - 8$

4.1 Write down the equation of the asymptote of $f$. (1)

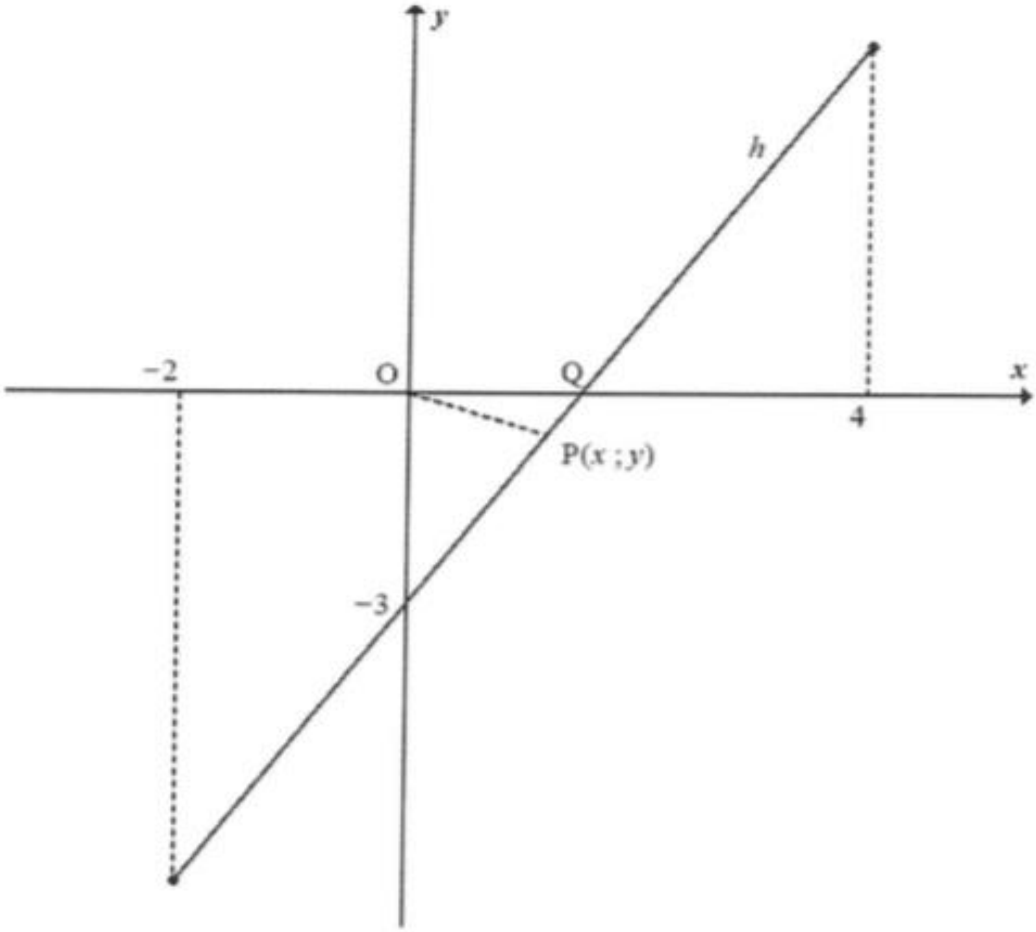
4.2 Sketch the graph of $f$. Clearly indicate ALL intercepts with the axes as well as the asymptote. (4)

4.3 The graph of $g$ is obtained by reflecting the graph of $f$ in the $y$-axis. Write down the equation of $g$. (1)

**[6]**

## QUESTION 5

Given: $h(x)=2x-3$ for $-2\le x\le 4$. The $x$-intercept of $h$ is Q.

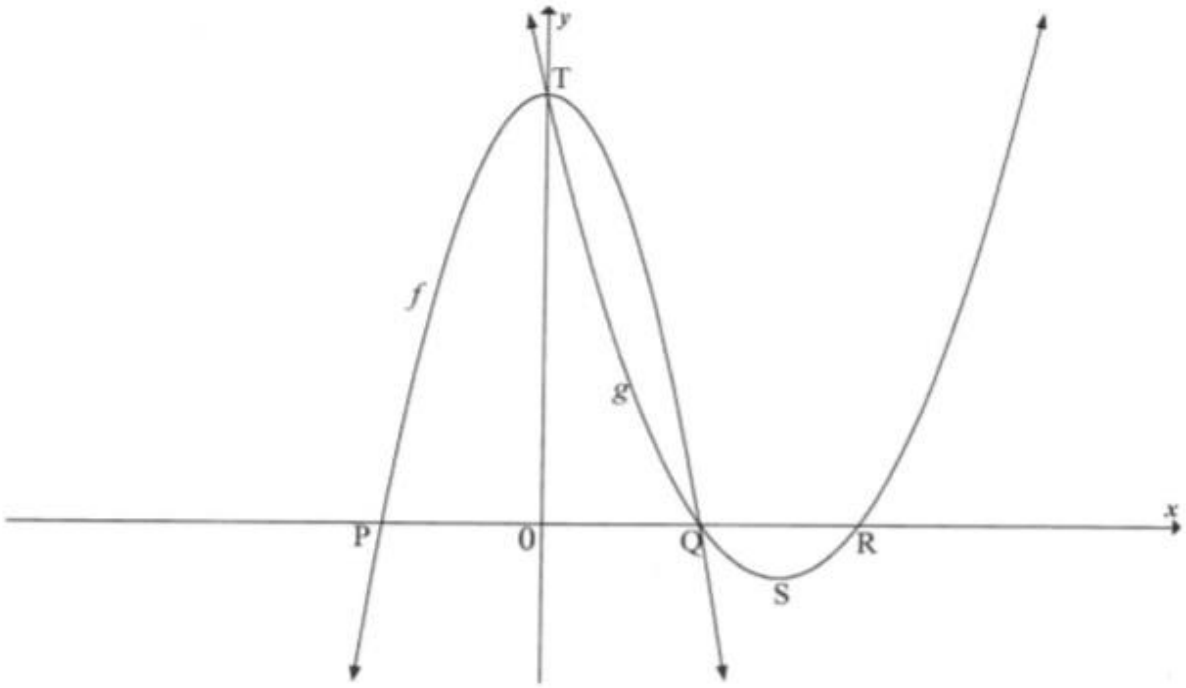



| | | |
|---|---|---|
| 5.1 | Determine the coordinates of Q. | (2) |
| 5.2 | Write down the domain of $h^{-1}$. | (3) |
| 5.3 | Sketch the graph of $h^{-1}$ in your ANSWER BOOK, clearly indicating the $y$-intercept and the end points. | (3) |
| 5.4 | For which value(s) of $x$ will $h(x)=h^{-1}(x)$? | (3) |
| 5.5 | P$(x\,;\,y)$ is the point on the graph of $h$ that is closest to the origin. Calculate the distance OP. | (5) |
| 5.6 | Given: $h(x)=f'(x)$ where $f$ is a function defined for $-2\le x\le 4$. | |
| 5.6.1 | Explain why $f$ has a local minimum. | (2) |
| 5.6.2 | Write down the value of the maximum gradient of the tangent to the graph of $f$. | (1) |
| | | **[19]** |

## QUESTION 6

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.

Points P and Q are the $x$-intercepts of $f$. Points Q and R are the $x$-intercepts of $g$. S is the turning point of $g$. T is the $y$-intercept of both $f$ and $g$.




6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that $x = 4{,}5$ at S, determine the coordinates of R. (2)

6.1.4 Determine the value(s) of $x$ for which $g''(x) > 0$. (2)

6.2 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

- The domain is $x \in R,\ x \neq -2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R,\ x \neq -2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

**[12]**

**FEB 14**

## QUESTION 5

5.1 Draw the graphs of $x^2 + y^2 = 16$ and $x + y = 4$ on the same set of axes in your ANSWER BOOK. (4)

5.2 Write down the coordinates of the points of intersection of the two graphs. (2)

**[6]**

## QUESTION 6

Consider: $f(x) = \dfrac{6}{x-2} + 3$

6.1 Write down the equations of the asymptotes of the graph of $f$. (2)

6.2 Write down the domain of $f$. (1)

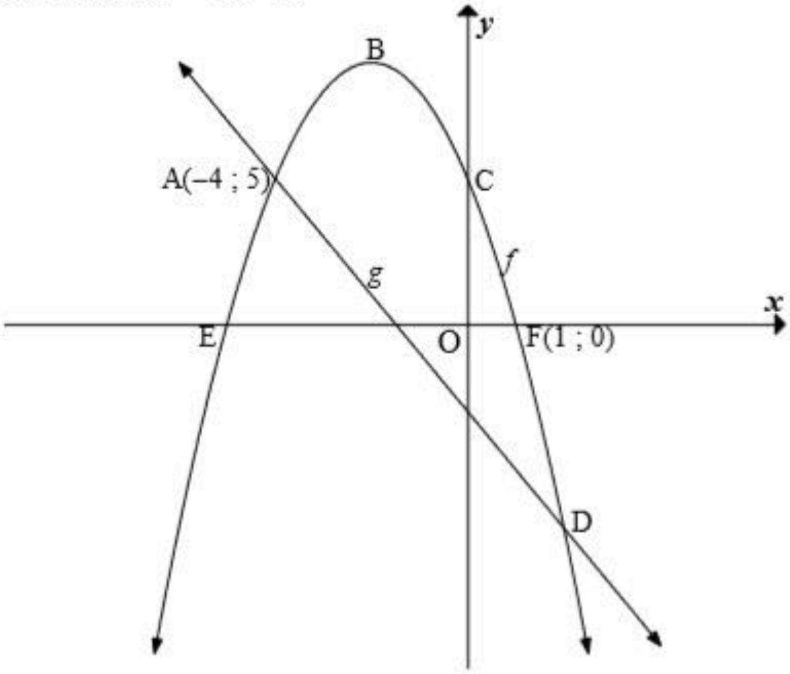
6.3 Draw a sketch graph of $f$ in your ANSWER BOOK, indicating the intercept(s) with the axes and the asymptotes. (4)

6.4 The graph of $f$ is translated to $g$. Describe the transformation in the form $(x\,;y) \rightarrow \ldots$ if the axes of symmetry of $g$ are $y = x + 3$ and $y = -x + 1$. (4)

**[11]**

## QUESTION 7

The graph of $f(x)=a(x-p)^2+q$ where $a$, $p$ and $q$ are constants, is given below.
Points E, F(1 ; 0) and C are its intercepts with the coordinates axes. A(− 4 ; 5) is the reflection of C across the axis of symmetry of $f$. D is a point on the graph such that the straight line through A and D has equation $g(x)=-2x-3$.

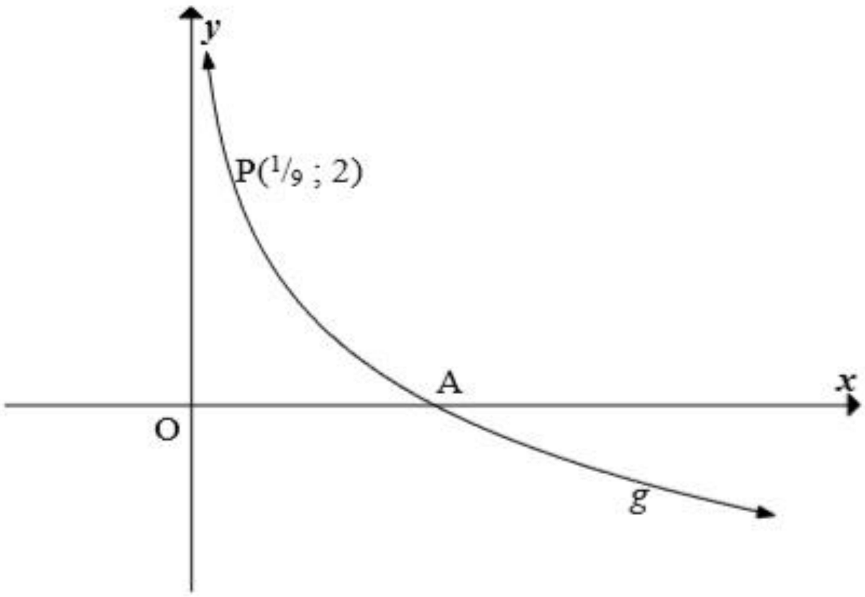



| | | |
|---|---|---|
| 7.1 | Write down the coordinates of C. | (1) |
| 7.2 | Write down the equation of the axis of symmetry of $f$. | (1) |
| 7.3 | Calculate the values of $a$, $p$ and $q$. | (6) |
| 7.4 | If $f(x)=-x^2-4x+5$, calculate the $x$-coordinate of D. | (4) |
| 7.5 | The graph of $f$ is reflected about the $x$-axis. | |
| | Write down the coordinates of the turning point of the new parabola. | (2) |
| | | **[14]** |

## QUESTION 8

Given the graph of $g(x)=\log_{\frac{1}{3}} x$.

- A is the $x$-intercept of $g$.
- P $\left(\frac{1}{9};2\right)$ is a point on $g$.




8.1 Write down the coordinates of A. (1)

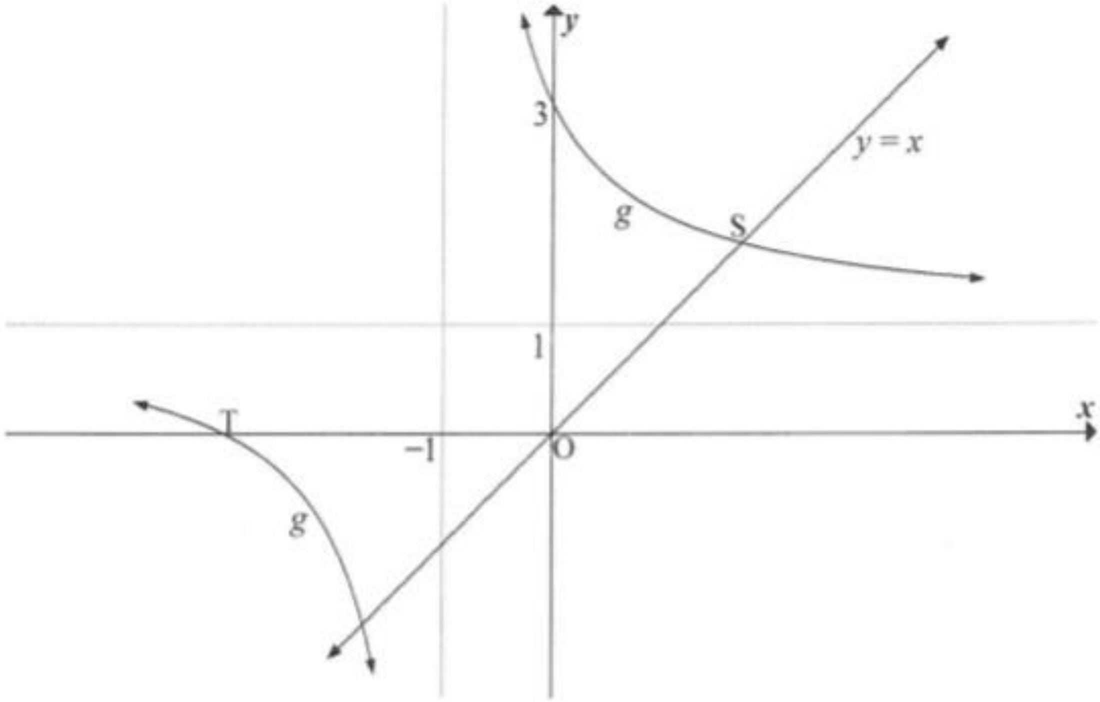
8.2 Sketch the graph of $g^{-1}$ indicating an intercept with the axes and ONE other point on the graph. (3)

8.3 Write down the domain of $g^{-1}$. (1)
**[5]**

**NOV 14**

**QUESTION 4**

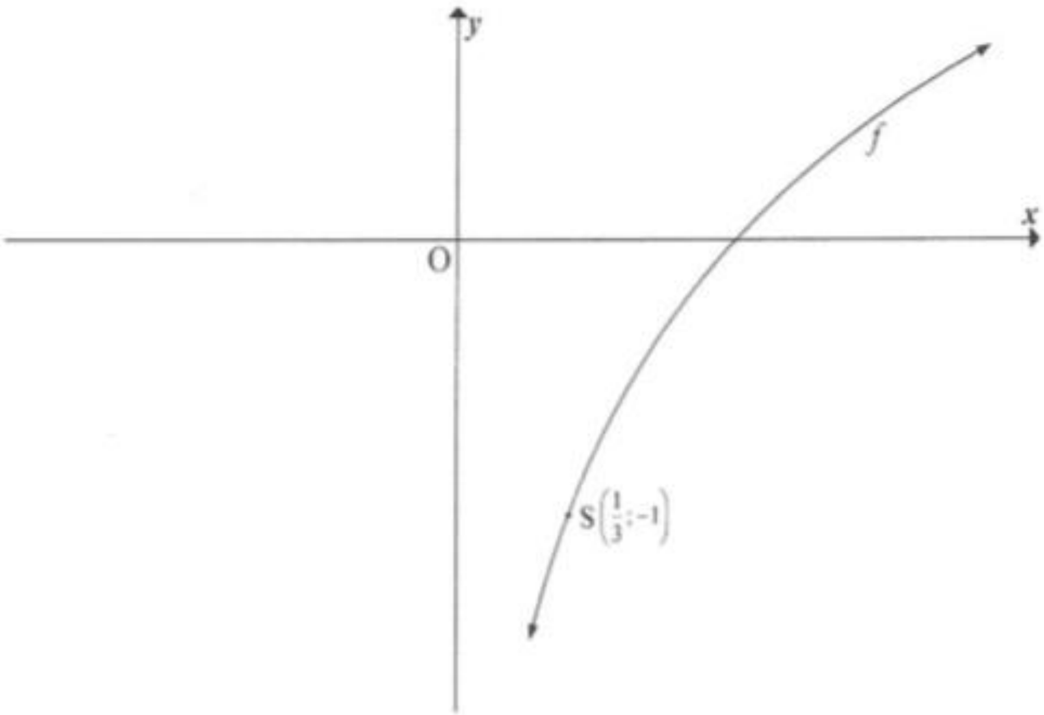
The diagram below shows the hyperbola $g$ defined by $g(x) = \frac{2}{x+p} + q$ with asymptotes $y = 1$ and $x = -1$. The graph of $g$ intersects the $x$-axis at T and the $y$-axis at $(0\ ;\ 3)$. The line $y = x$ intersects the hyperbola in the first quadrant at S.




| | | |
|---|---|---|
| 4.1 | Write down the values of $p$ and $q$. | (2) |
| 4.2 | Calculate the $x$-coordinate of T. | (2) |
| 4.3 | Write down the equation of the vertical asymptote of the graph of $h$, if $h(x) = g(x+5)$ | (1) |
| 4.4 | Calculate the length of OS. | (5) |
| 4.5 | For which values of $k$ will the equation $g(x) = x + k$ have two real roots that are of opposite signs? | (1) |
| | | **[11]** |

## QUESTION 5

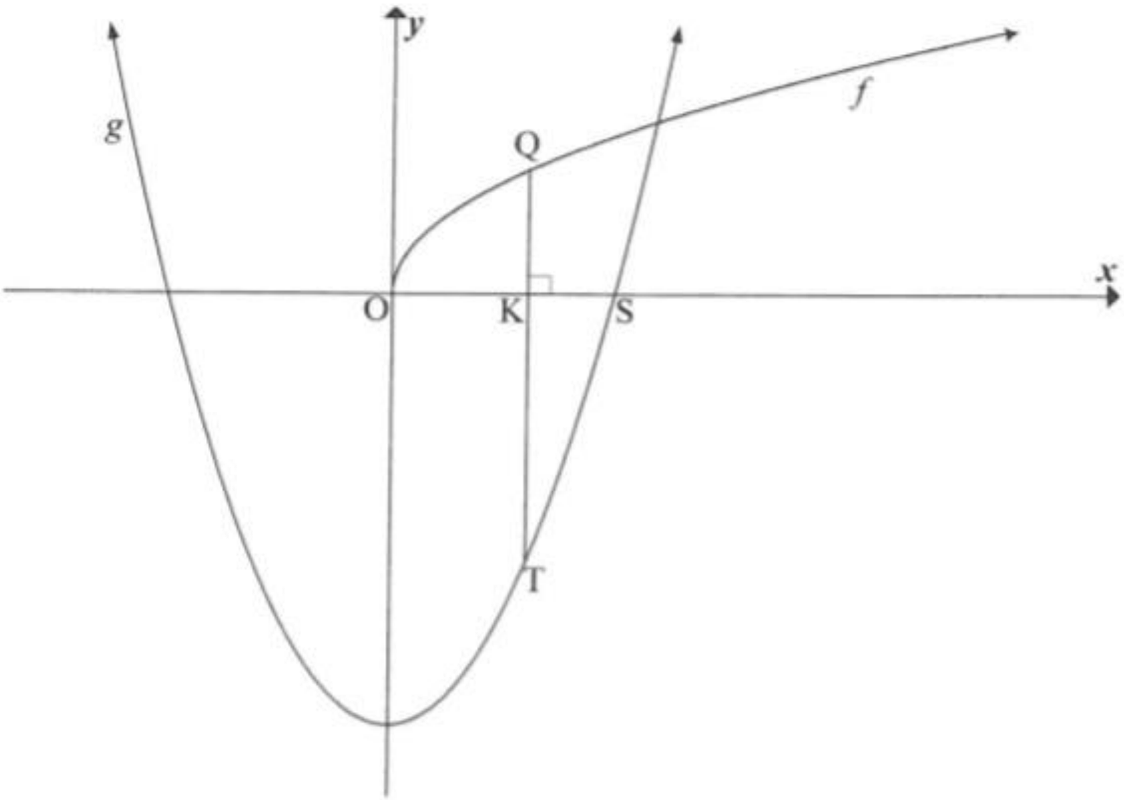
Given: $f(x) = \log_a x$ where $a > 0$. $S\left(\frac{1}{3};-1\right)$ is a point on the graph of $f$.




| | | |
|---|---|---|
| 5.1 | Prove that $a = 3$. | (2) |
| 5.2 | Write down the equation of $h$, the inverse of $f$, in the form $y = ...$ | (2) |
| 5.3 | If $g(x) = -f(x)$, determine the equation of $g$. | (1) |
| 5.4 | Write down the domain of $g$. | (1) |
| 5.5 | Determine the values of $x$ for which $f(x) \geq -3$. | (3) |
| | | **[9]** |

## QUESTION 6

Given: $g(x)=4x^2-6$ and $f(x)=2\sqrt{x}$. The graphs of $g$ and $f$ are sketched below. S is an $x$-intercept of $g$ and K is a point between O and S. The straight line QKT with Q on the graph of $f$ and T on the graph of $g$, is parallel to the $y$-axis.




| | | | |
|---|---|---|---|
| 6.1 | Determine the $x$-coordinate of S, correct to TWO decimal places. | | (2) |
| 6.2 | Write down the coordinates of the turning point of $g$. | | (2) |
| 6.3 | 6.3.1 | Write down the length of QKT in terms of $x$, where $x$ is the $x$-coordinate of K. | (3) |
| | 6.3.2 | Calculate the maximum length of QT. | (6) |
| | | | **[13]** |

## EXEMPL 14

### QUESTION 4

4.1 Given: $f(x)=\dfrac{2}{x+1}-3$

4.1.1 Calculate the coordinates of the $y$-intercept of $f$. (2)

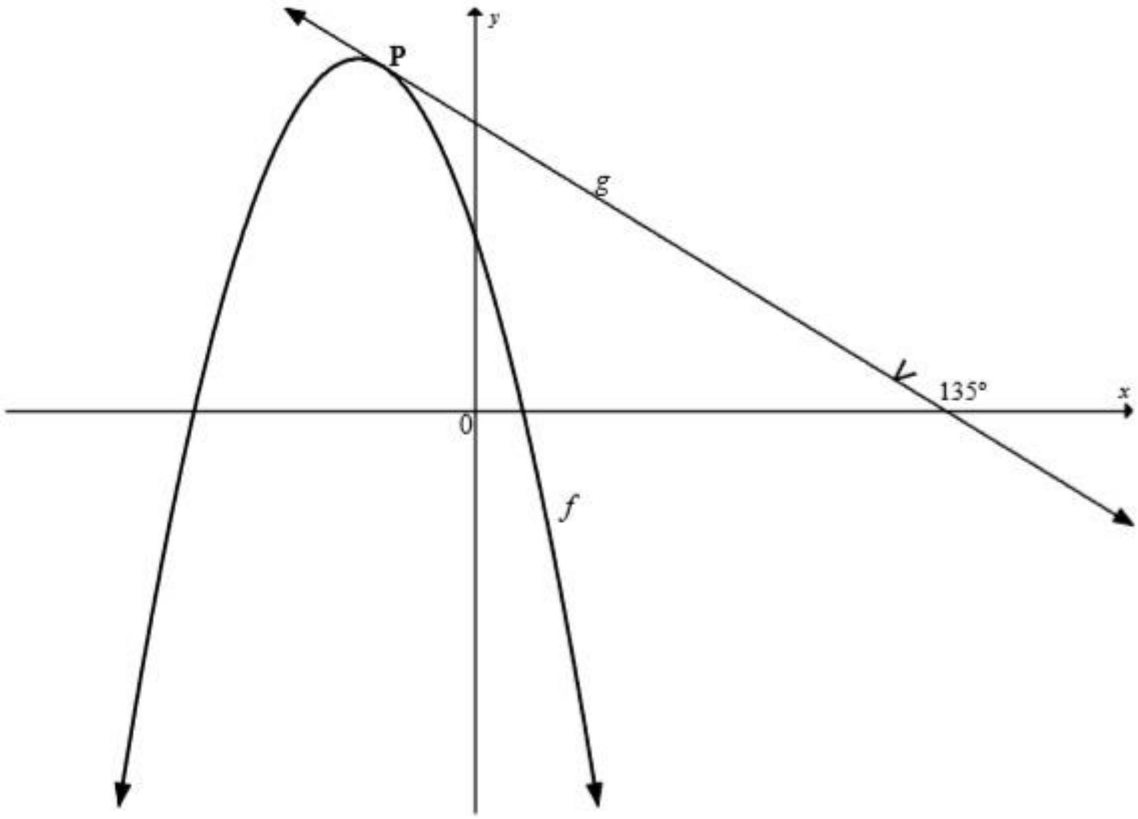
4.1.2 Calculate the coordinates of the $x$-intercept of $f$. (2)

4.1.3 Sketch the graph of $f$ in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (3)

4.1.4 One of the axes of symmetry of $f$ is a decreasing function. Write down the equation of this axis of symmetry. (2)

## QUESTION 5

The sketch below shows the graphs of $f(x) = -2x^2 - 5x + 3$ and $g(x) = ax + q$. The angle of inclination of graph $g$ is $135°$ in the direction of the positive $x$-axis. P is the point of intersection of $f$ and $g$ such that $g$ is a tangent to the graph of $f$ at P.




| | | |
|---|---|---|
| 5.1 | Calculate the coordinates of the turning point of the graph of $f$. | (3) |
| 5.2 | Calculate the coordinates of P, the point of contact between $f$ and $g$. | (4) |
| 5.3 | Hence or otherwise, determine the equation of $g$. | (2) |
| 5.4 | Determine the values of $d$ for which the line $k(x) = -x + d$ will not intersect the graph of $f$. | (1) |

**[10]**

## QUESTION 6

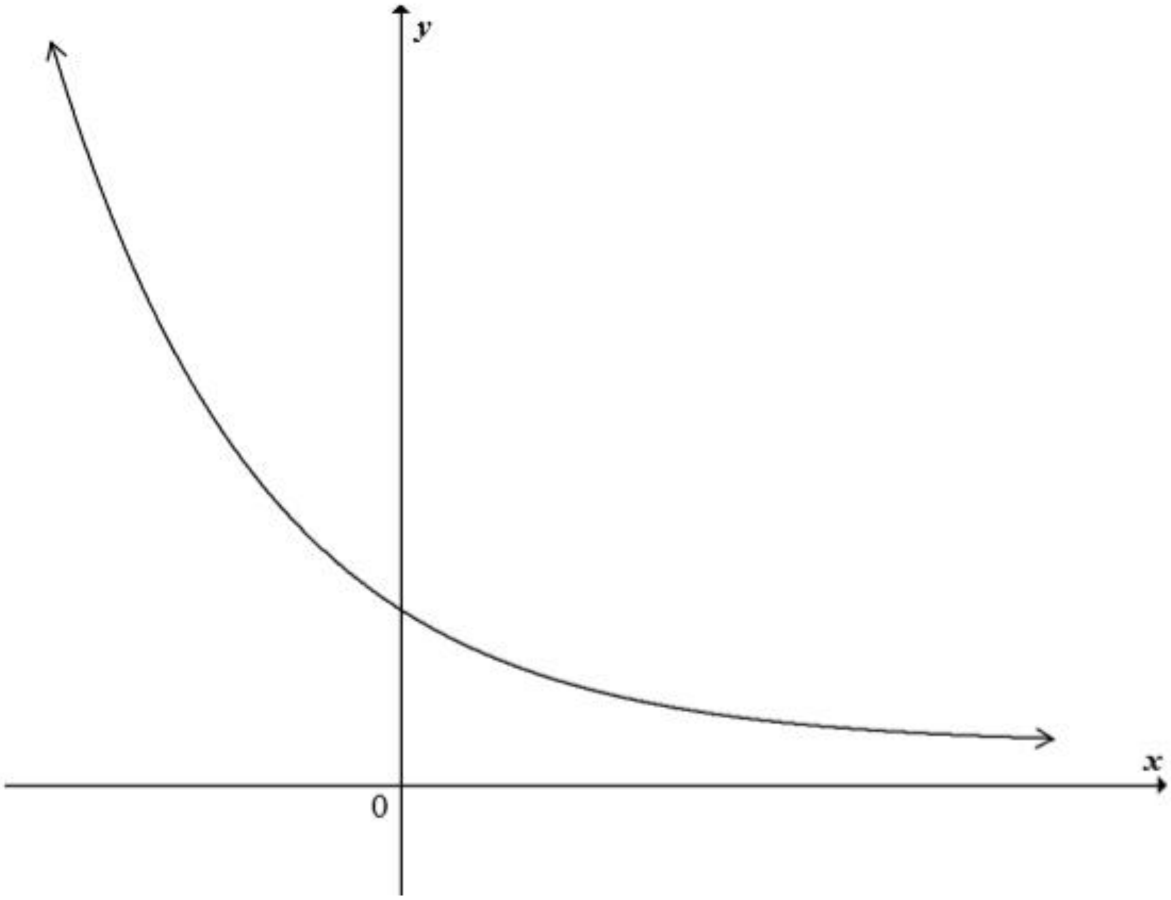
The graph of $g$ is defined by the equation $g(x)=\sqrt{ax}$. The point (8 ; 4) lies on $g$.

| | | |
|---|---|---|
| 6.1 | Calculate the value of $a$. | (2) |
| 6.2 | If $g(x)>0$, for what values of $x$ will $g$ be defined? | (1) |
| 6.3 | Determine the range of $g$. | (1) |
| 6.4 | Write down the equation of $g^{-1}$, the inverse of $g$, in the form $y=\ldots$ | (2) |
| 6.5 | If $h(x)=x-4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of $h$ and $g$. | (4) |
| 6.6 | Hence, or otherwise, determine the values of $x$ for which $g(x)>h(x)$. | (2) |
| | | **[12]** |

**FEB 13**

## QUESTION 4

The graph of $f(x)=\left(\frac{1}{3}\right)^x$ is sketched below.




4.1 Write down the domain of $f$. (1)

4.2 Write down the equation of the asymptote of $f$. (1)

4.3 Write down the equation of $f^{-1}$ in the form $y = \ldots$ (2)

4.4 Sketch the graph of $f^{-1}$ in your ANSWER BOOK. Indicate the $x$-intercept and ONE other point. (3)

4.5 Write down the equation of the asymptote of $f^{-1}(x+2)$. (2)

4.6 Prove that: $[f(x)]^2 - [f(-x)]^2 = f(2x) - f(-2x)$ for all values of $x$. (3)

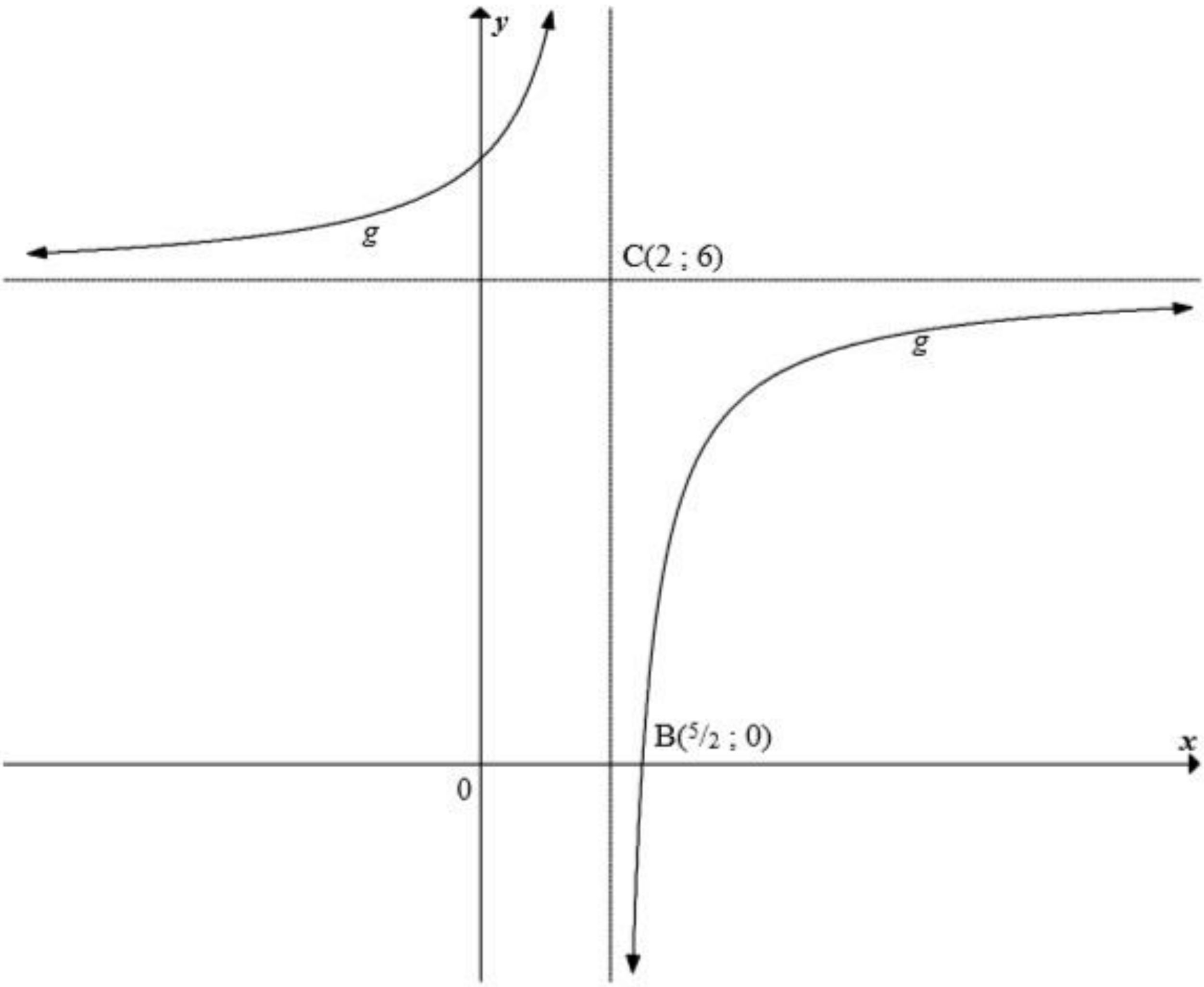
**[12]**

## QUESTION 5

Sketched below is the graph of $g(x) = \dfrac{a}{x-p} + q$ .

C(2 ; 6) is the point of intersection of the asymptotes of $g$.

B$\left(\dfrac{5}{2}\,;\,0\right)$ is the $x$-intercept of $g$.



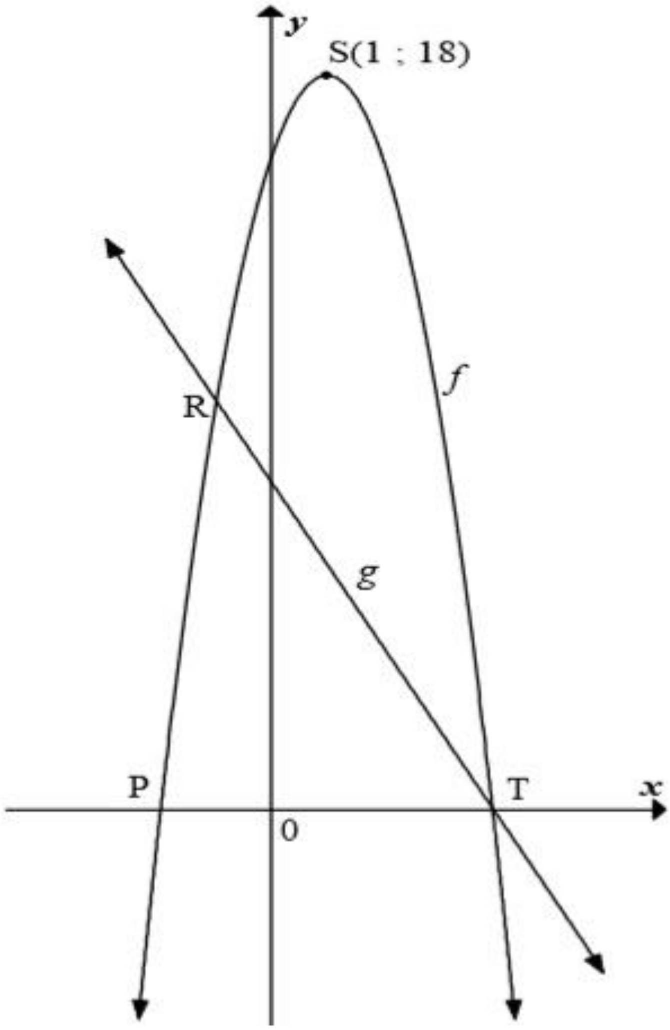

5.1 Determine the equation for $g$ in the form $g(x) = \dfrac{a}{x-p} + q$ (4)

5.2 F is the reflection of B across C. Determine the coordinates of F. (2)

**[6]**

## QUESTION 6

S(1 ; 18) is the turning point of the graph of $f(x) = ax^2 + bx + c$. P and T are $x$-intercepts of $f$. The graph of $g(x) = -2x + 8$ has an $x$-intercept at T. R is a point of intersection of $f$ and $g$.




6.1 Calculate the coordinates of T. (2)

6.2 Determine the equation for $f$ in the form $f(x) = ax^2 + bx + c$. Show ALL your working. (4)

6.3 If $f(x) = -2x^2 + 4x + 16$, calculate the coordinates of R. (4)

6.4 Use your graphs to solve for $x$ where:

6.4.1 $f(x) \geq g(x)$ (2)

6.4.2 $-2x^2 + 4x - 2 < 0$ (4)

**[16]**

**NOV 13**

**QUESTION 4**

Given: $f(x) = -2x^2 - 5x + 3$

4.1 Write down the coordinates of the $y$-intercept of $f$. (1)

4.2 Determine the coordinates of the $x$-intercepts of $f$. (3)

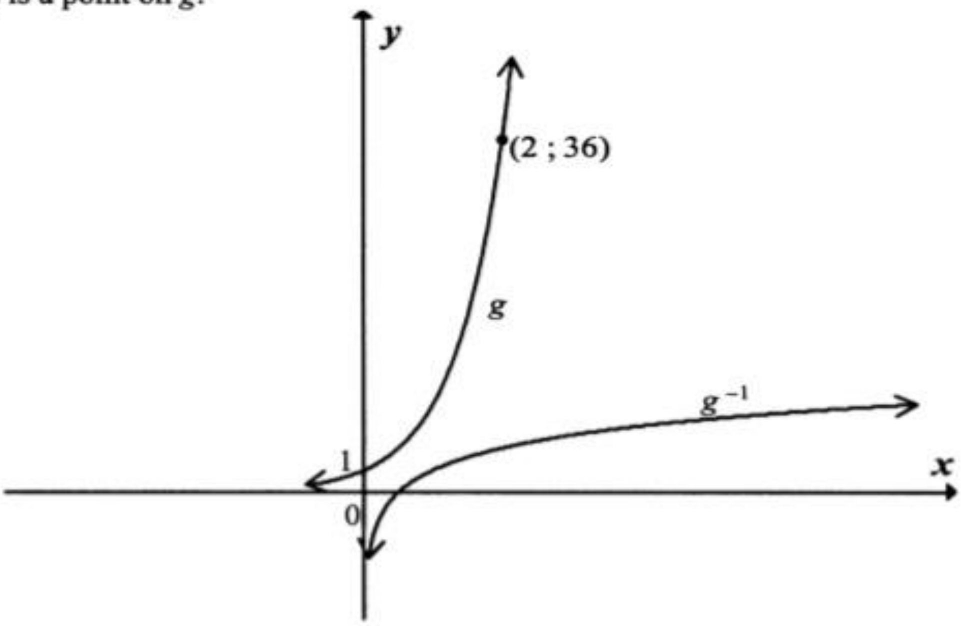
4.3 Determine the coordinates of the turning point of $f$. (3)

4.4 Sketch the graph of $y = f(x)$, clearly showing the coordinates of the turning points and the three intercepts with the axes. (3)

**[10]**

**QUESTION 5**

5.1 Sketched below are the graphs of $g(x) = k^x$, where $k > 0$ and $y = g^{-1}(x)$. (2 ; 36) is a point on $g$.




5.1.1 Determine the value of $k$. (2)

5.1.2 Give the equation of $g^{-1}$ in the form $y = \ldots$ (2)

5.1.3 For which value(s) of $x$ is $g^{-1}(x) \leq 0$? (2)

5.1.4 Write down the domain of $h$ if $h(x) = g^{-1}(x-3)$. (1)

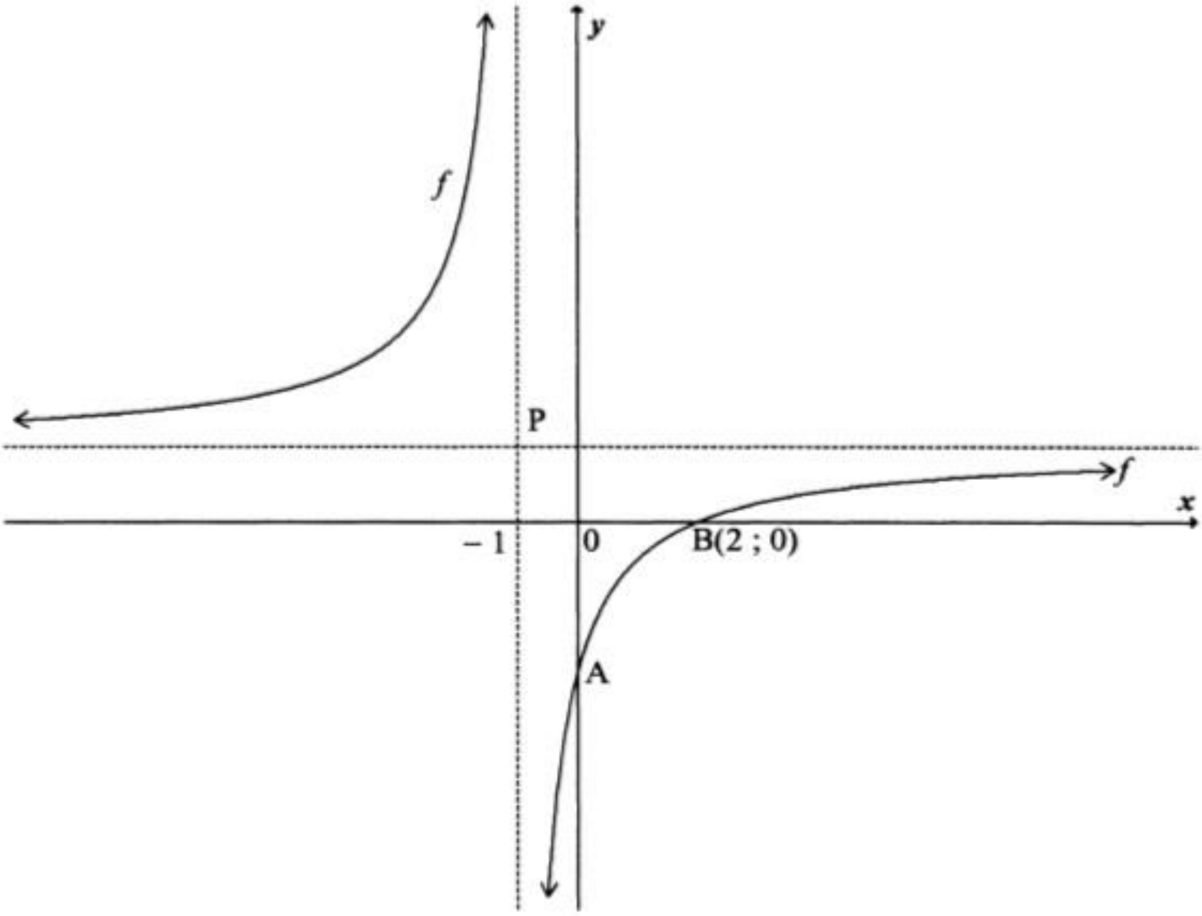
5.2 5.2.1 Sketch the graph of the inverse of $y = 1$. (2)

5.2.2 Is the inverse of $y = 1$ a function? Motivate your answer. (2)

**[11]**

**QUESTION 6**

A sketch of the hyperbola $f(x)=\frac{x-d}{x-p}$, where $d$ and $p$ are constants, is given below. The dotted lines are the asymptotes. The asymptotes intersect at P and B(2 ; 0) is a point on $f$.




6.1.1 Determine the values of $d$ and $p$. (2)

6.1.2 Show that the equation of $f$ can be written as $y=\frac{-3}{x+1}+1$. (2)

6.1.3 Write down the coordinates of P. (2)

6.1.4 Write down the coordinates of the image of $B(2\,;0)$ if B is reflected about the axis of symmetry $y=x+2$. (2)

6.2 The exponential function, $g(x)=p.2^{x}+q$ has a horizontal asymptote at $y=1$ and passes through $(0\,;-2)$. Determine the values of $p$ and $q$. (3)

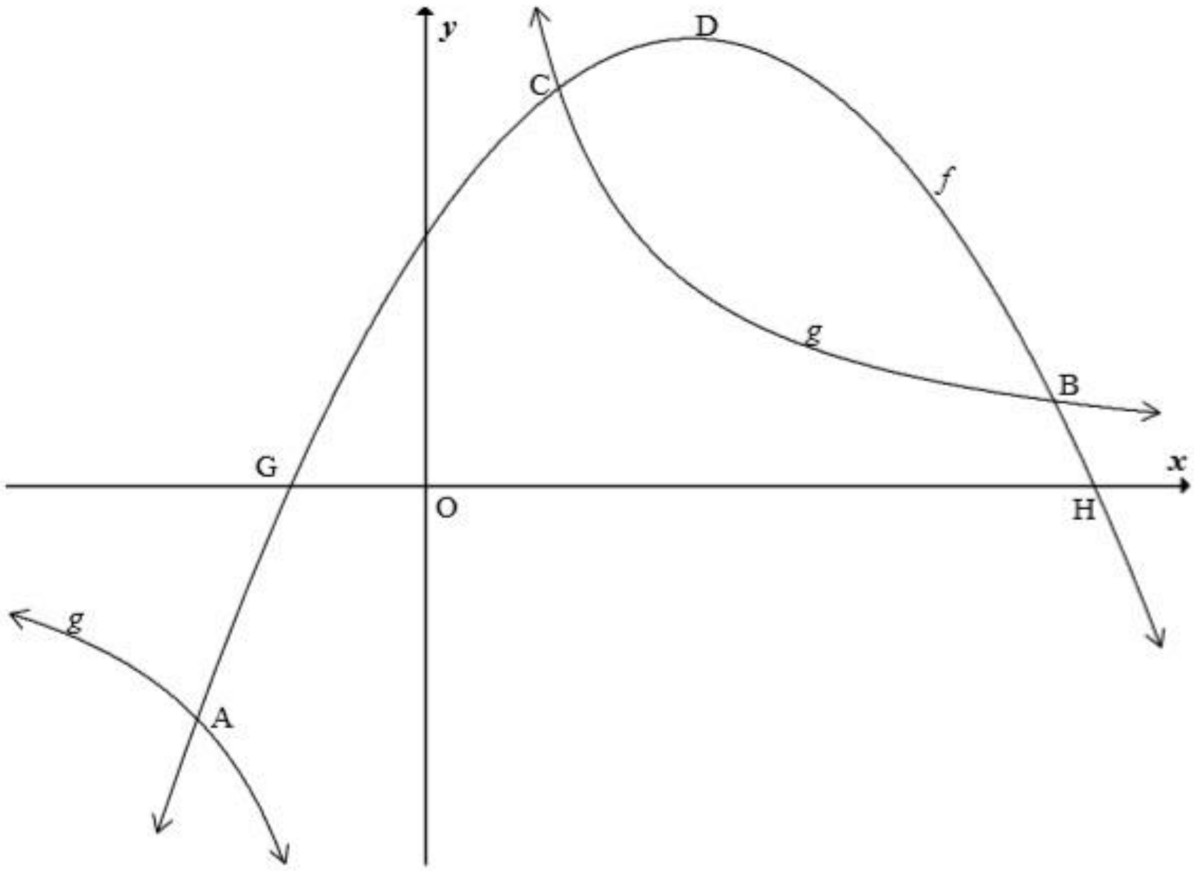
**[11]**

**FEB 12**

## QUESTION 5

The graphs of the functions $f(x) = -2x^2 + 8x + 10$ and $g(x) = \dfrac{16}{x}$ are sketched below.

G and H are the $x$-intercepts of $f$.
D is the turning point of $f$.
Points A, B and C are the points of intersection of $f$ and $g$.




| | | |
|---|---|---|
| 5.1 | Write down the equations of the asymptotes of the graph of $g$. | (2) |
| 5.2 | Determine the coordinates of H. | (4) |
| 5.3 | Determine the range of $f$. | (4) |
| 5.4 | Verify that C is the point (1 ; 16). | (2) |
| 5.5 | Determine the coordinates of the turning point of $p$ if $p(x) = f(3x)$. | (3) |
| | | **[15]** |

## QUESTION 6

Given: $f(x)=3^x$

6.1 Determine an equation for $f^{-1}$ in the form $f^{-1}(x)=...$ (1)

6.2 Sketch, in your ANSWER BOOK, the graphs of $f$ and $f^{-1}$, showing clearly ALL intercepts with the axes. (4)

6.3 Write down the domain of $f^{-1}$. (2)

6.4 For which values of $x$ will $f(x).f^{-1}(x)\leq 0$ ? (2)

6.5 Write down the range of $h(x)=3^{-x}-4$ (2)

6.6 Write down an equation for $g$ if the graph of $g$ is the image of the graph of $f$ after $f$ has been translated two units to the right and reflected about the $x$-axis. (2)

**[13]**

## NOV 12

### QUESTION 4

4.1 Consider the function $f(x) = 3.2^x - 6$.

4.1.1 Calculate the coordinates of the $y$-intercept of the graph of $f$. (1)

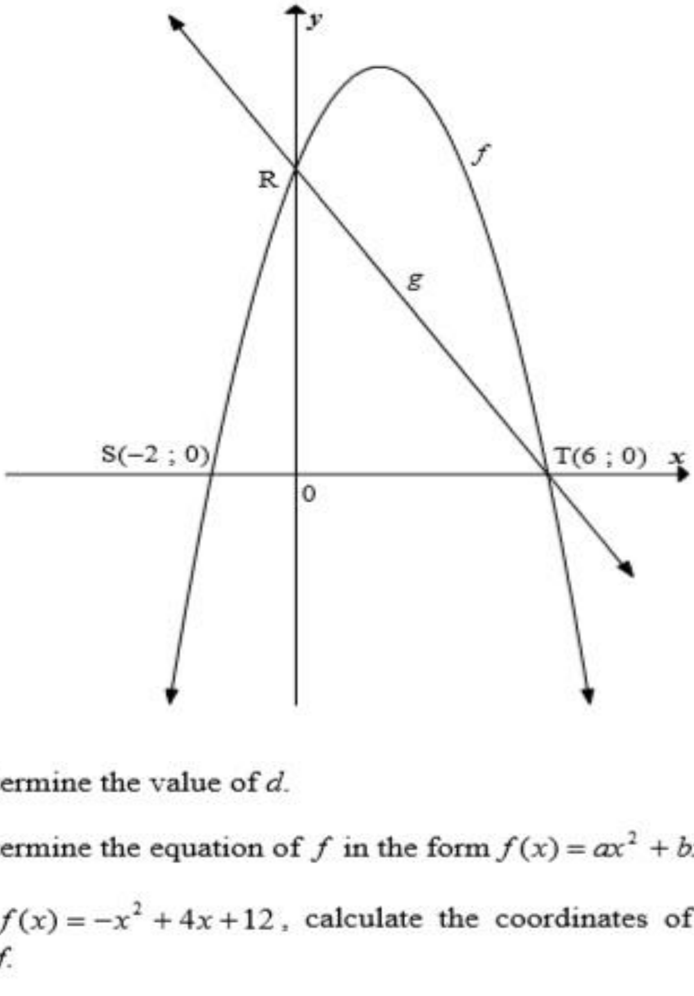
4.1.2 Calculate the coordinates of the $x$-intercept of the graph of $f$. (2)

4.1.3 Sketch the graph of $f$ in your ANSWER BOOK.

Clearly show ALL asymptotes and intercepts with the axes. (3)

4.1.4 Write down the range of $f$. (1)

4.2 S(–2 ; 0) and T(6 ; 0) are the $x$-intercepts of the graph of $f(x) = ax^2 + bx + c$ and R is the $y$-intercept. The straight line through R and T represents the graph of $g(x) = -2x + d$.




4.2.1 Determine the value of $d$. (2)

4.2.2 Determine the equation of $f$ in the form $f(x) = ax^2 + bx + c$. (4)

4.2.3 If $f(x) = -x^2 + 4x + 12$, calculate the coordinates of the turning point of $f$. (2)

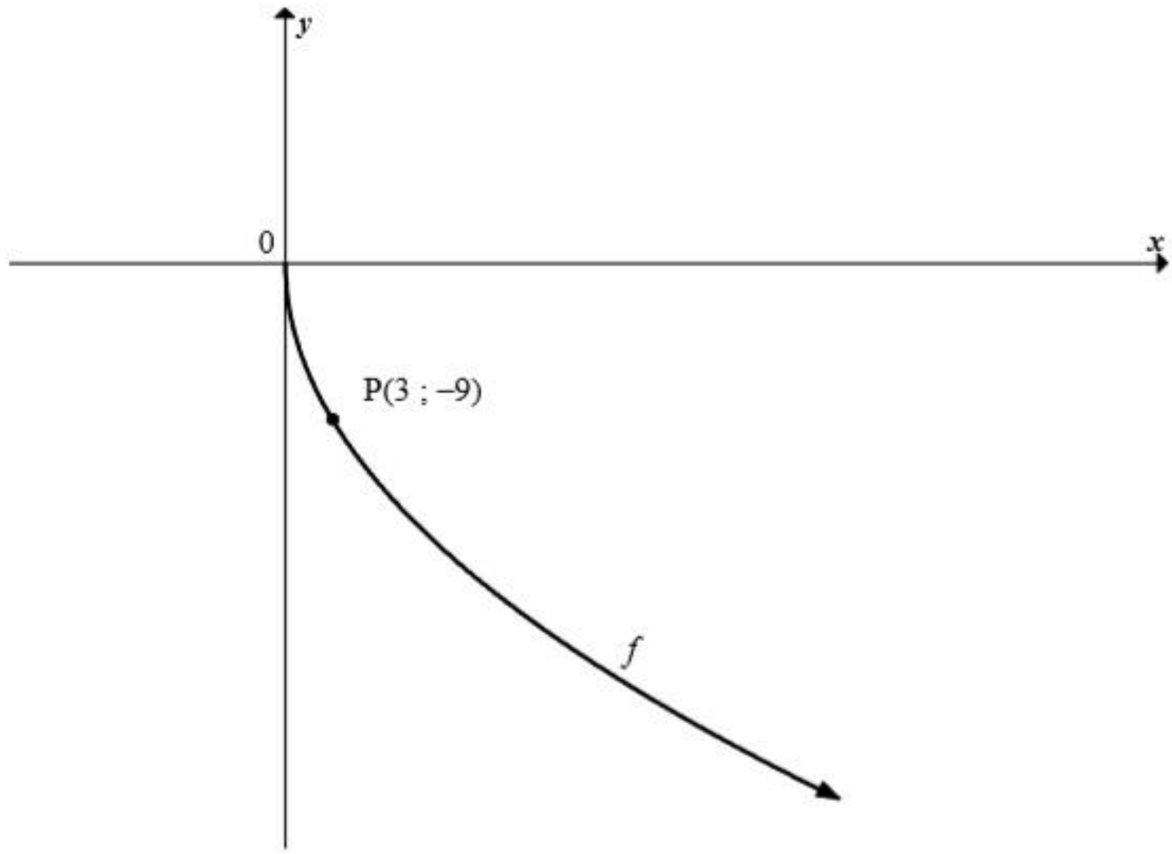
4.2.4 For which values of $k$ will $f(x) = k$ have two distinct roots? (2)

4.2.5 Determine the maximum value of $h(x) = 3^{f(x)-12}$. (3)

**[20]**

## QUESTION 5

The graph of $f(x) = -\sqrt{27x}$ for $x \geq 0$ is sketched below.
The point P(3 ; –9) lies on the graph of $f$.




5.1 Use your graph to determine the values of $x$ for which $f(x) \geq -9$. (2)

5.2 Write down the equation of $f^{-1}$ in the form $y = ...$ Include ALL restrictions. (3)

5.3 Sketch $f^{-1}$, the inverse of $f$, in your ANSWER BOOK.
Indicate the intercept(s) with the axes and the coordinates of ONE other point. (3)

5.4 Describe the transformation from $f$ to $g$ if $g(x) = \sqrt{27x}$, where $x \geq 0$. (1)
**[9]**

## QUESTION 6

The graph of a hyperbola with equation $y = f(x)$ has the following properties:

- Domain: $x \in \mathbf{R},\ x \neq 5$
- Range: $y \in \mathbf{R},\ y \neq 1$
- Passes through the point (2 ; 0)

Determine $f(x)$. **[4]**

**FEB 11**

**QUESTION 5**

Consider the function $f(x)=\dfrac{3}{x-1}-2$.

5.1 Write down the equations of the asymptotes of $f$. (2)

5.2 Calculate the intercepts of the graph of $f$ with the axes. (3)

5.3 Sketch the graph of $f$ on DIAGRAM SHEET 1. (3)

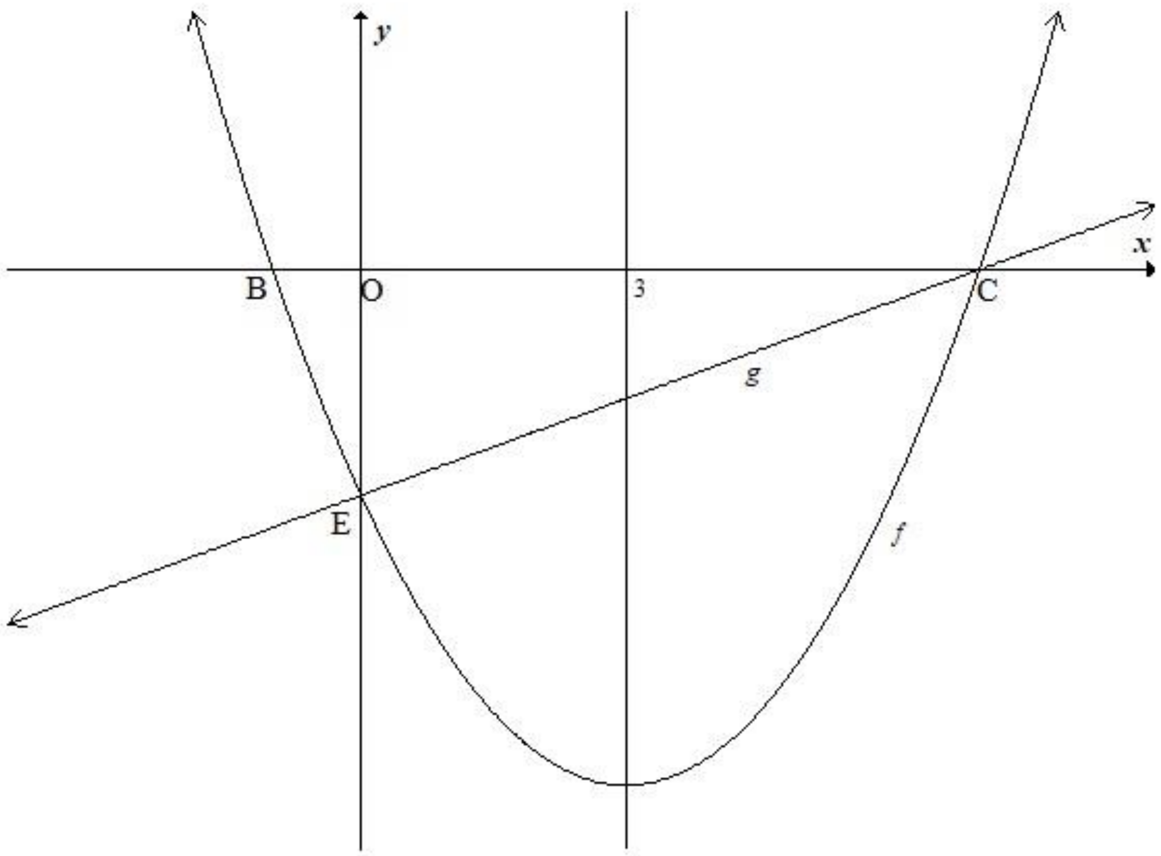
5.4 Write down the range of $y = -f(x)$. (1)

5.5 Describe, in words, the transformation of $f$ to $g$ if $g(x)=\dfrac{-3}{x+1}-2$. (2)

**[11]**

## QUESTION 6

A parabola $f$ intersects the $x$-axis at B and C and the $y$-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.




| | | |
|---|---|---|
| 6.1 | Show that the coordinates of C are (7; 0). | (1) |
| 6.2 | Calculate the $x$-coordinate of B. | (1) |
| 6.3 | Determine the equation of $f$ in the form $y = a(x-p)^2 + q$. | (6) |
| 6.4 | Write down the equation of the graph of $h$, the reflection of $f$ in the $x$-axis. | (1) |
| 6.5 | Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$. | (2) |
| 6.6 | Solve for $x$ if $f(x^2 - 2) = 0$. | (4) |
| | | **[15]** |

**QUESTION 7**

Consider the function $f(x)=\left(\frac{1}{3}\right)^x$.

7.1 Is $f$ an increasing or decreasing function? Give a reason for your answer. (2)

7.2 Determine $f^{-1}(x)$ in the form $y = \ldots$ (2)

7.3 Write down the equation of the asymptote of $f(x)-5$. (1)

7.4 Describe the transformation from $f$ to $g$ if $g(x)=\log_3 x$. (2)
**[7]**

**NOV 11**

**QUESTION 5**

5.1 Consider the function: $f(x)=\dfrac{-6}{x-3}-1$

5.1.1 Calculate the coordinates of the $y$-intercept of $f$. (2)

5.1.2 Calculate the coordinates of the $x$-intercept of $f$. (3)

5.1.3 Sketch the graph of $f$ in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)

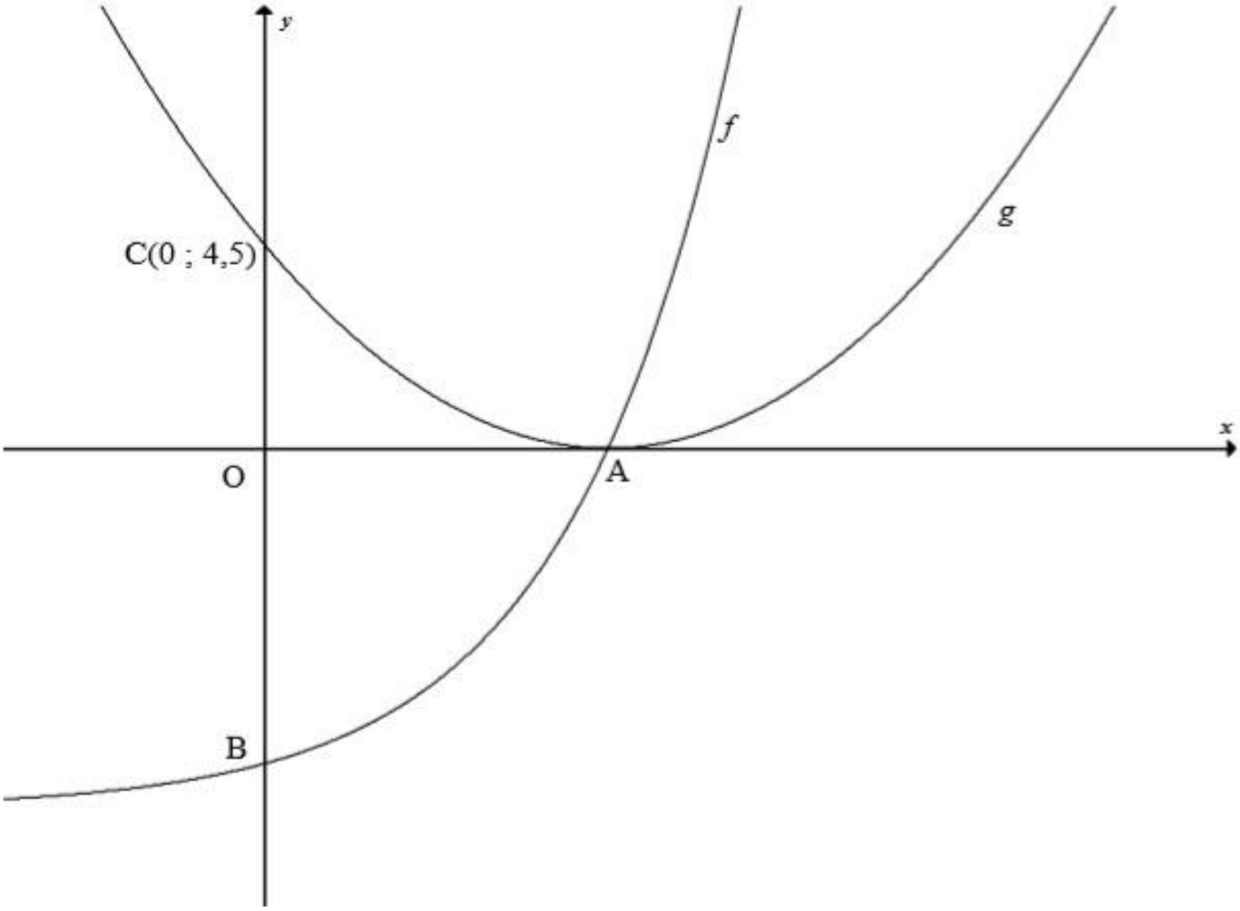
5.1.4 For which values of $x$ is $f(x)>0$? (2)

5.1.5 Calculate the average gradient of $f$ between $x=-2$ and $x=0$. (4)

5.2 Draw a sketch graph of $y=ax^2+bx+c$, where $a<0$, $b<0$, $c<0$ and $ax^2+bx+c=0$ has only ONE solution. (4)
**[19]**

## QUESTION 6

The graphs of $f(x) = 2^x - 8$ and $g(x) = ax^2 + bx + c$ are sketched below.
B and C(0 ; 4,5) are the $y$-intercepts of the graphs of $f$ and $g$ respectively.
The two graphs intersect at A, which is the turning point of the graph of $g$ and the $x$-intercept of the graphs of $f$ and $g$.




6.1 Determine the coordinates of A and B. (4)

6.2 Write down an equation of the asymptote of the graph of $f$. (1)

6.3 Determine an equation of $h$ if $h(x) = f(2x) + 8$. (2)

6.4 Determine an equation of $h^{-1}$ in the form $y = ...$ (2)

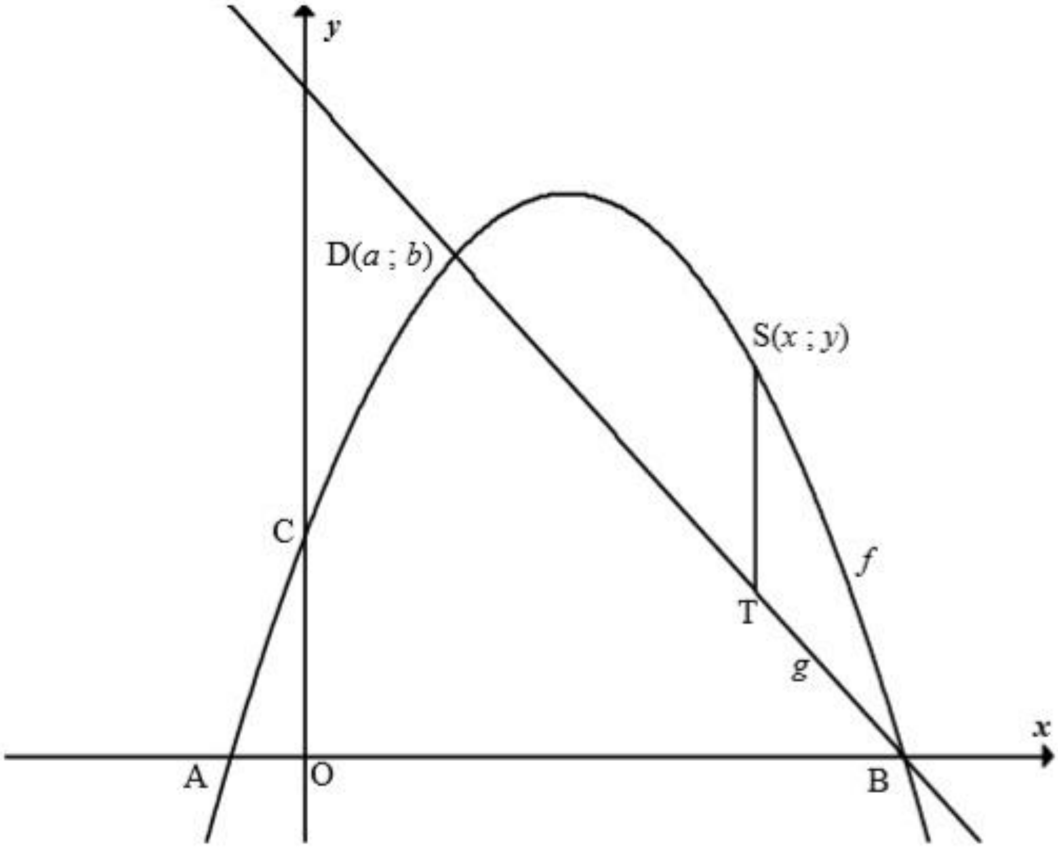
6.5 Write down an equation of $p$, if $p$ is the reflection of $h^{-1}$ about the $x$-axis. (1)

6.6 Calculate $\sum_{k=0}^{3} g(k) - \sum_{k=4}^{5} g(k)$. Show ALL your working. (4)
**[14]**

**FEB 10**

## QUESTION 6

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$ are sketched below.
$f$ and $g$ intersect in D and B. A and B are the $x$-intercepts of $f$.




6.1 Determine the coordinates of A and B. (4)

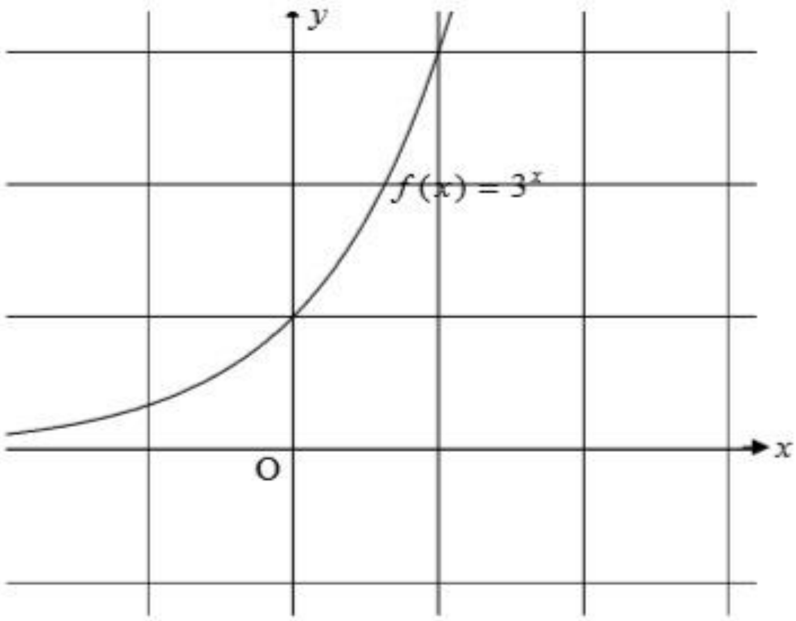
6.2 Calculate $a$, the $x$-coordinate of D. (4)

6.3 S($x$ ; $y$) is a point on the graph of $f$, where $a \leq x \leq 8$. ST is drawn parallel to the $y$-axis with T on the graph of $g$. Determine ST in terms of $x$. (2)

6.4 Calculate the maximum length of ST. (2)
**[12]**

## QUESTION 7

The graph of $f(x) = 3^x$ is drawn below.




7.1 Write $f^{-1}$ in the form $y = \ldots$ (1)

7.2 Sketch the graphs of $y = f^{-1}(x)$ and $y = f^{-1}(x-2)$ on the grid provided in DIAGRAM SHEET 2. (4)

7.3 Use your graphs to solve for $x$ if $\log_3(x-2) < 1$. (2)

**[7]**

## QUESTION 8

Given: $f(x) = \tan(x - 30°)$

8.1 Sketch the graph of $y = f(x)$ for $-90° \leq x \leq 90°$ on the grid provided in DIAGRAM SHEET 2. (3)

8.2 Write down the equation of an asymptote of *f*. (1)

8.3 Describe in words the transformation of *f* to *g* if $g(x) = \tan(30° - x)$. (2)

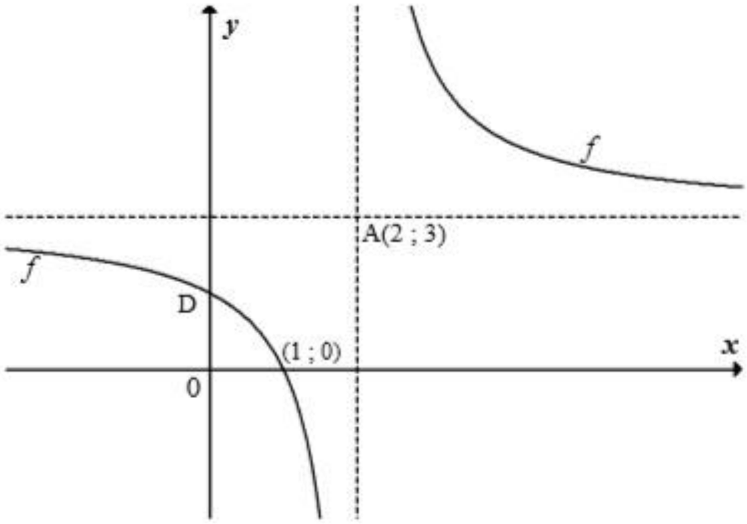
**[6]**

## NOV 10

### QUESTION 4

Given $f(x)=\dfrac{a}{x-p}+q$. The point A(2 ; 3) is the point of intersection of the asymptotes of $f$.

The graph of $f$ intersects the $x$-axis at (1 ; 0).
D is the $y$-intercept of $f$.




| | | |
|---|---|---|
| 4.1 | Write down the equations of the asymptotes of $f$. | (2) |
| 4.2 | Determine an equation of $f$. | (3) |
| 4.3 | Write down the coordinates of D. | (2) |
| 4.4 | Write down an equation of $g$ if $g$ is the straight line joining A and D. | (3) |
| 4.5 | Write down the coordinates of the other point of intersection of $f$ and $g$. | (4) |
| | | **[14]** |

## QUESTION 5

Consider the function $f(x) = 4^{-x} - 2$.

5.1 Calculate the coordinates of the intercepts of $f$ with the axes. (4)

5.2 Write down the equation of the asymptote of $f$. (1)

5.3 Sketch the graph of $f$ on DIAGRAM SHEET 1. (3)

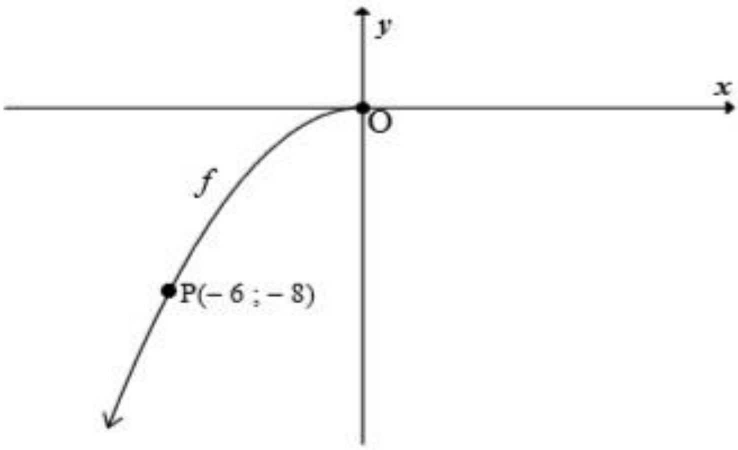
5.4 Write down the equation of $g$ if $g$ is the graph of $f$ shifted 2 units upwards. (1)

5.5 Solve for $x$ if $f(x) = 3$. (You need not simplify your answer.) (3)

**[12]**

## QUESTION 6

The graph of $f(x) = ax^2$, $x \leq 0$ is sketched below.
The point P($-6$ ; $-8$) lies on the graph of $f$.




6.1 Calculate the value of $a$. (2)

6.2 Determine the equation of $f^{-1}$, in the form $y = \ldots$ (3)

6.3 Write down the range of $f^{-1}$. (1)

6.4 Draw the graph of $f^{-1}$ on DIAGRAM SHEET 1. Indicate the coordinates of a point on the graph different from (0 ; 0). (2)

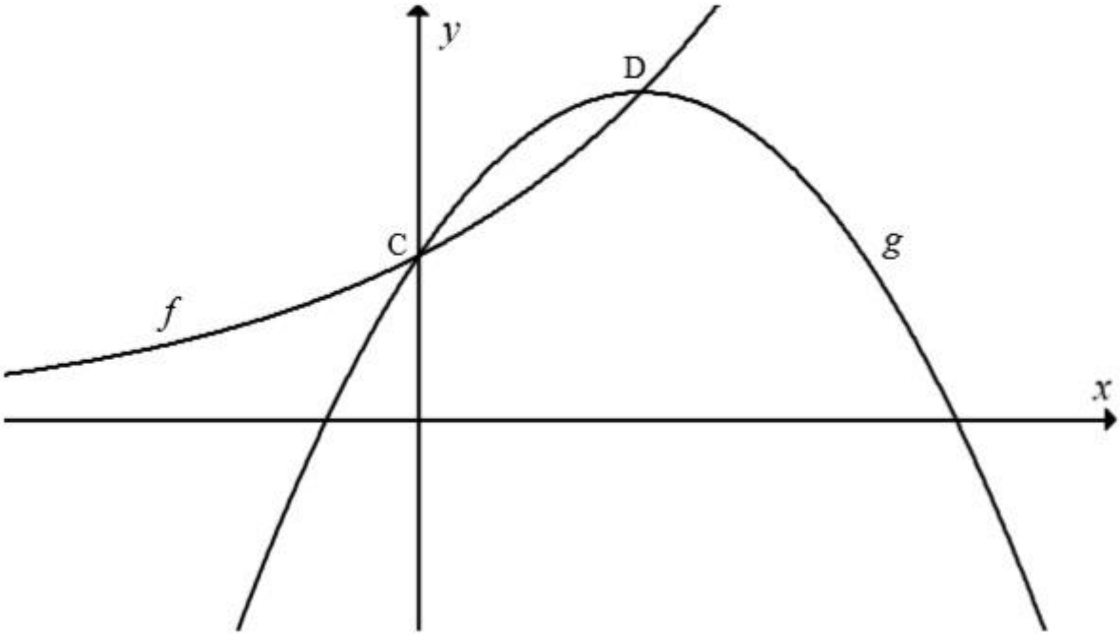
6.5 The graph of $f$ is reflected across the line $y = x$ and thereafter it is reflected across the $x$-axis. Determine the equation of the new function in the form $y = \ldots$ (3)

**[11]**

**NOV 09**

**QUESTION 5**

Sketched below are the graphs of $f(x) = 2^x$ and $g(x) = -(x-1)^2 + b$, where $b$ is a constant. The graphs of $f$ and $g$ intersect the $y$-axis at C. D is the turning point of $g$.

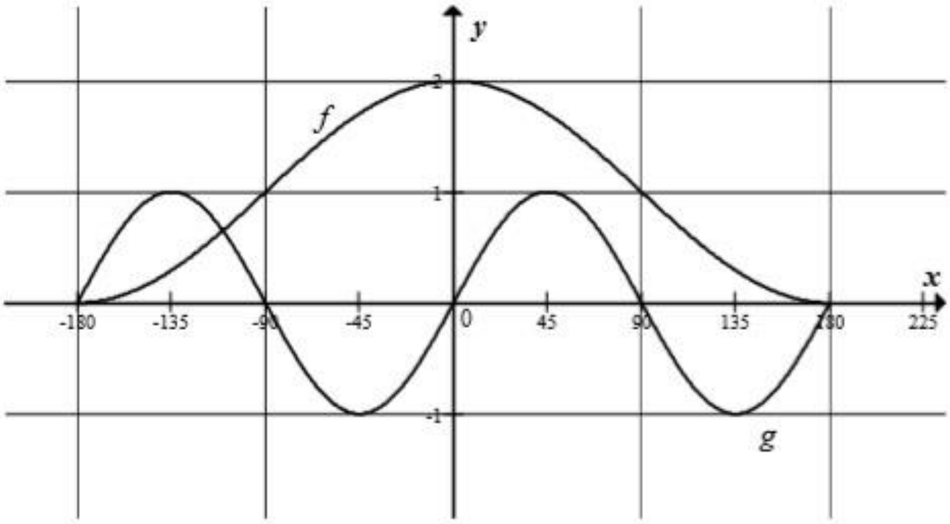



| | | |
|---|---|---|
| 5.1 | Show that $b = 2$. | (2) |
| 5.2 | Write down the coordinates of the turning point of $g$. | (2) |
| 5.3 | Write down the equation of $f^{-1}(x)$ in the form $y = ...$ | (1) |
| 5.4 | Sketch the graph of $f^{-1}$ on the system of axes on DIAGRAM SHEET 1 (attached). Indicate the $x$-intercept and the coordinates of one other point on your graph. | (3) |
| 5.5 | Write down the equation of $h$ if $h(x) = g(x+1) - 2$. | (2) |
| 5.6 | How can the domain of $h$ be restricted so that $h^{-1}$ will be a function? | (1) |
| 5.7 | Determine the maximum value of $2^{2-(x-1)^2}$. | (2) |

**[13]**

## QUESTION 6

The sketch below shows the graphs of $f(x) = q + \cos x$ and $g(x) = \sin px$ for $x \in [-180° ; 180°]$.




6.1 Write down the values of $p$ and $q$. (2)

6.2 Write down the range of $f$. (1)

6.3 Use the graphs to answer the following:

6.3.1 Explain how you would solve the equation $(2\sin x - 1)\cos x = 1$. (2)

6.3.2 Give ONE solution to the equation in QUESTION 6.3.1. (1)

**[6]**

## QUESTION 7

Given: $f(x) = \dfrac{x+3}{x+1}$

7.1 Calculate the $x$- and $y$-intercepts of $f$. (4)

7.2 Show that $f(x) = \dfrac{2}{x+1} + 1$ (2)

7.3 Write down the equations of the vertical and horizontal asymptotes of $f$. (2)

7.4 Draw a sketch graph of $f(x)$ showing clearly the intercepts and asymptotes on the axes provided on DIAGRAM SHEET 1 (attached). (4)

7.5 Use your graph to solve: $\dfrac{2}{x+1} \geq -1$ (3)

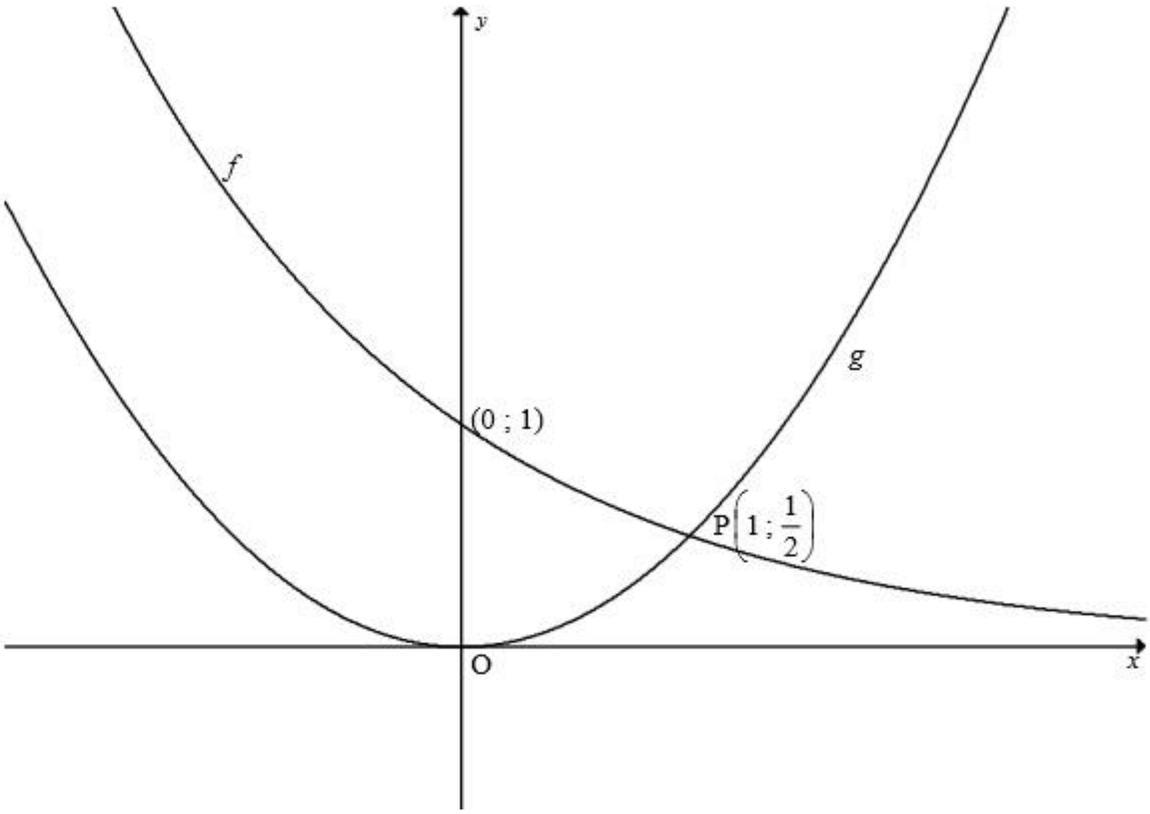
**[15]**

# EXEMPL08

## QUESTION 6

The diagram below shows the graphs of $f(x) = a^x$ and $g(x) = bx^2$

The point $P\left(1\,;\frac{1}{2}\right)$ is the point of intersection of $f$ and $g$.




| | | |
|---|---|---|
| 6.1 | Calculate the values of $a$ and $b$. | (2) |
| 6.2 | Write down the equation of $f^{-1}$ in the form $y = ...$ | (3) |
| 6.3 | Explain why the inverse of $g$ is not a function. | (2) |
| 6.4 | Write down TWO ways in which the domain of $g$ could be restricted in order that $g^{-1}$ is a function. | (2) |
| 6.5 | Determine the $x$-values for which: | |
| 6.5.1 | $f^{-1}(x) > 0$ | (2) |
| 6.5.2 | $f(x) - 1 = g(x)$ | (2) |

**[13]**

## QUESTION 7

Given: $g(x) = \dfrac{2}{x-3} - 1$

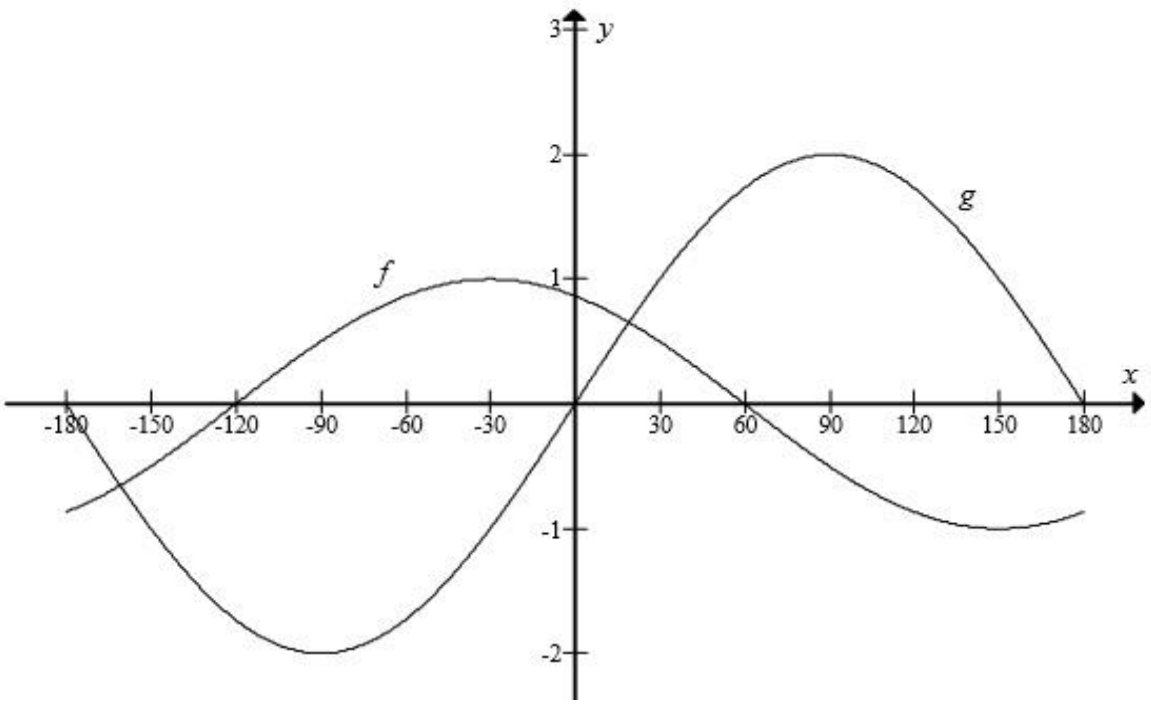
7.1 Write down the equations of the asymptotes of $g$. (2)

7.2 Calculate the intercepts of $g$ with the axes. (3)

7.3 Use DIAGRAM SHEET 1 to draw the asymptotes and make a neat sketch of $g$. (3)
**[8]**

## QUESTION 8

Sketched below are the functions $f(x) = \cos(x + 30°)$ and $g(x) = 2\sin x$ for $x \in [-180°\ ;\ 180°]$




8.1 Write down the period of $f$. (1)

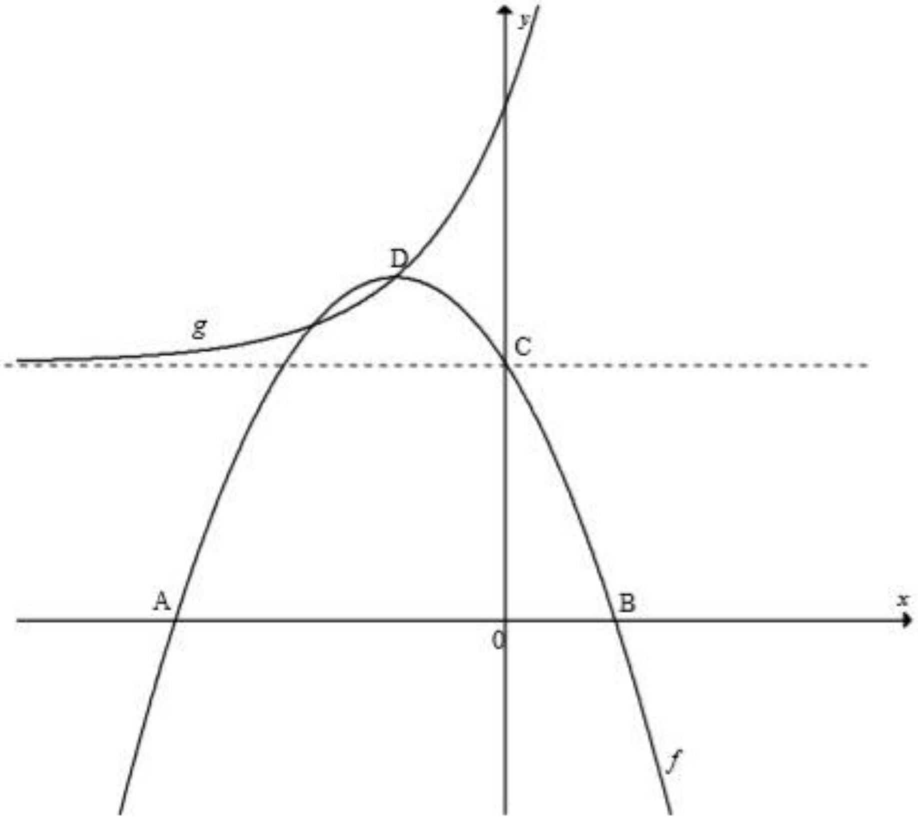
8.2 Give the new range of $g$ if $g$ undergoes a positive vertical shift of 1 unit. (2)

8.3 Write down the new equation of $f$ if it is shifted 30° horizontally to the right. (1)
**[4]**

**ADD EXEMPL 08**

## QUESTION 4

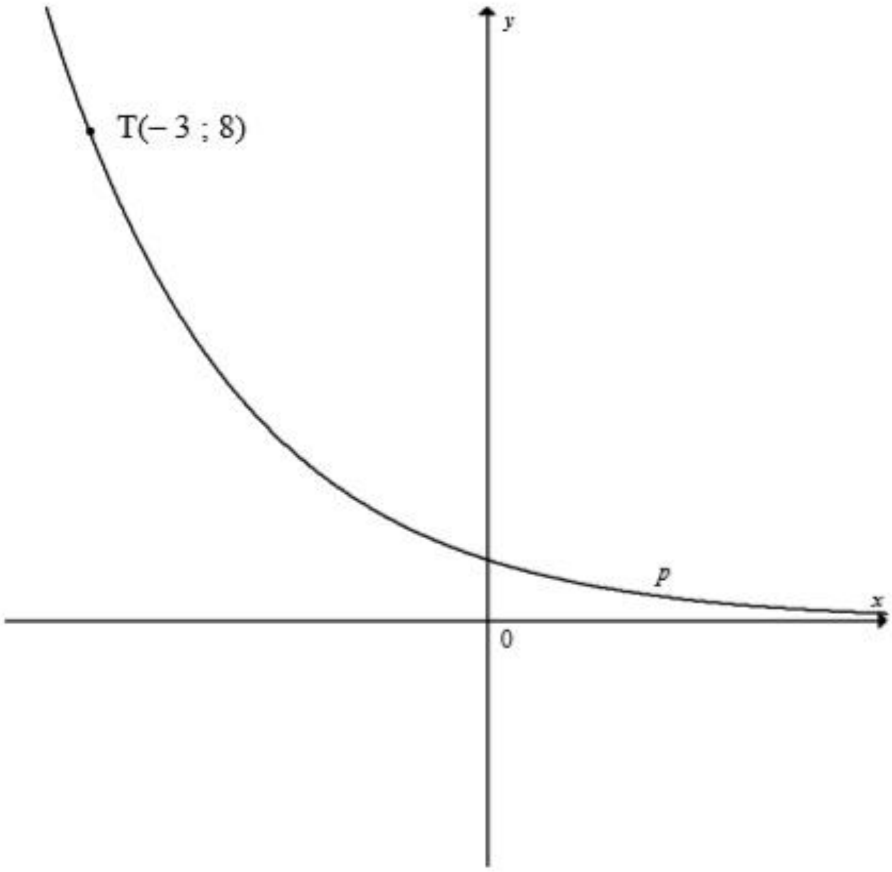
The graphs of $f(x) = -(x+1)^2 + 4$ and $g(x) = a.3^x + q$ are sketched below.
A and B are the $x$-intercepts of $f$.
C is the $y$-intercept of $f$ and lies on the asymptote of $g$. The two graphs intersect in D, the turning point of $f$.




| | | |
|---|---|---|
| 4.1 | Determine the coordinates of A and B. | (4) |
| 4.2 | Determine the equation of $g$. | (3) |
| 4.3 | Calculate the coordinates of the point on $f$ for which the tangent to $f$ will have a gradient of 1. | (4) |
| 4.4 | Write down the values of $k$ for which $f(x) - k$ will always be a negative value. | (2) |
| | | **[13]** |

## QUESTION 5

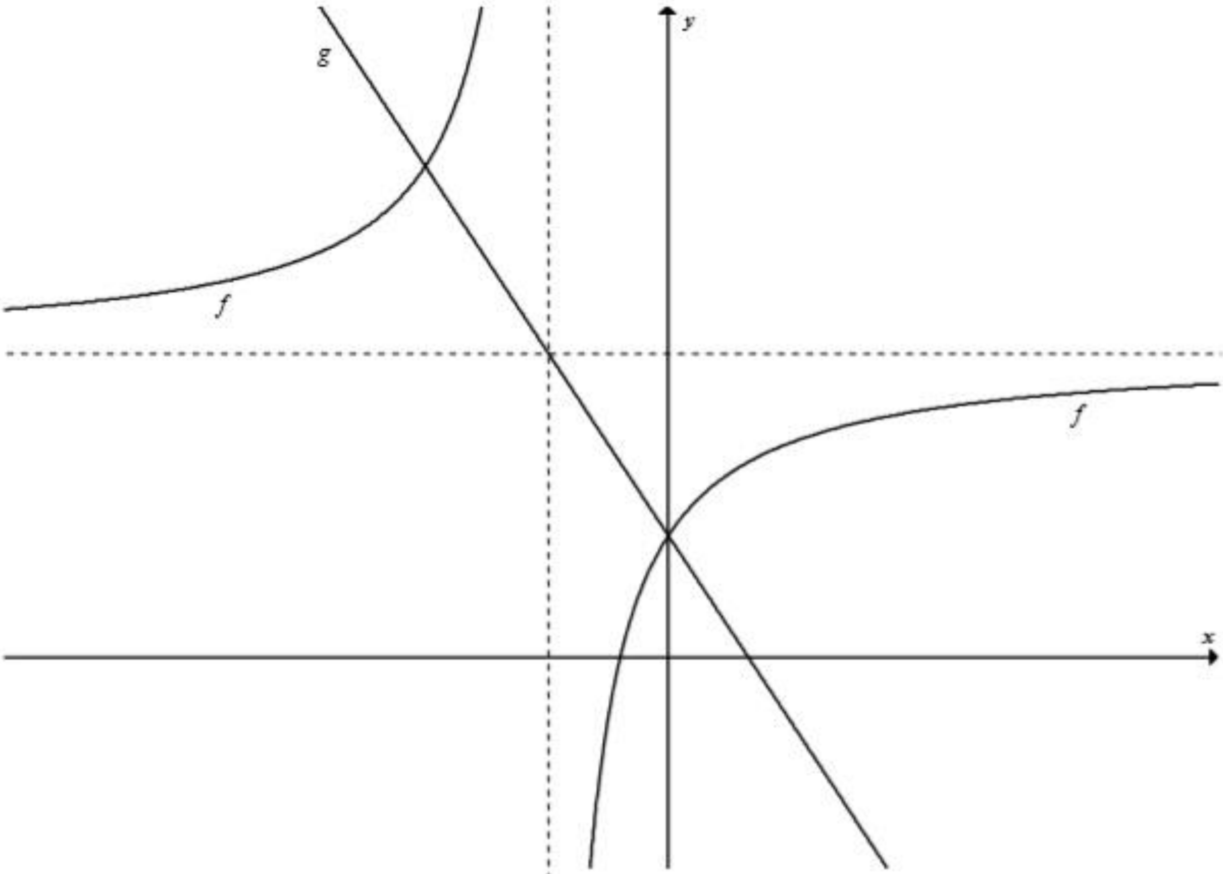
The graph of $p(x) = a^x$ is sketched below. The point T$(-3\ ;\ 8)$ lies on the graph of $p$.




| | | |
|---|---|---|
| 5.1 | Calculate the value of $a$. | (3) |
| 5.2 | Write down the equation of $p^{-1}(x)$ in the form $y = \ldots$ | (2) |
| 5.3 | For which values of $x$ will $p^{-1}(x) > -3$ ? | (2) |
| 5.4 | Write down the equation of $q$ if $q$ is the result of $p$ shifted 3 units to the right. | (2) |
| | | **[9]** |

## QUESTION 6

The graphs of $f(x) = \dfrac{-3}{x+1} + 5$ and $g(x) = -3x + 2$ are sketched below.




6.1 Write down the range of $f(x)$. (2)

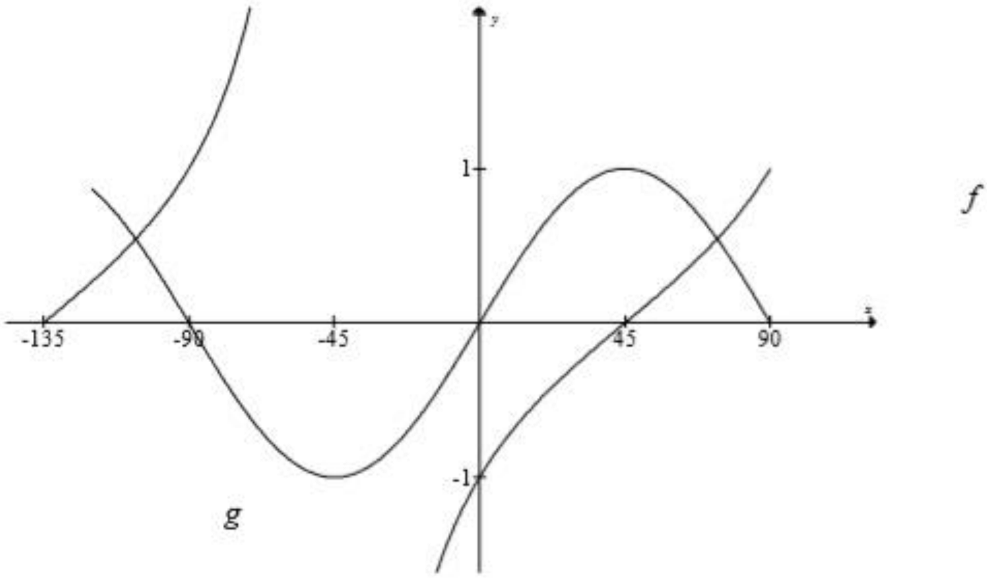
6.2 Determine the coordinates of the points of intersection of $f$ and $g$. (4)

6.3 Describe the transformation of $f$ to $h$ if $h(x) = \dfrac{3}{x+1} + 5$ (2)

**[8]**

## QUESTION 7

The graphs of the functions $f(x) = \tan(x - 45°)$ and $g(x) = \sin 2x$ for $x \in [-135°\,;\ 90°]$ are sketched below.




7.1 Write down the period of $g$. (1)

7.2 For which value(s) of $x$ will $f$ have an asymptote if $x \in [-135°\,;\ 90°]$? (2)

7.3 Write down the equation of $k$ if $k$ is the reflection of $g$ in the $x$-axis. (2)

**[5]**

## QUESTION 8

Karabo does not have a calculator and has to find the value of the following expression:

$2008^2 + 2009 \times 2007 - 2006 \times 2010 - 2016 \times 2000$

Show Karabo how she can calculate the answer without using a calculator. **[4]**