Port Shepstone Islamic School

MATHEMATICS

GRADE 12

Term 2 2020

TOPIC: CALCULUS – MOTION IN A STRAIGHT LINE

TEXTBOOKS: Clever Keeping Maths simple GR12 pa 235

EDUCATOR: B GUNDANI

Velocity is $v = \frac{ds}{dt}$

acceleration, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

Displacement $s(t)$

Velocity $v(t) = s'(t) = \frac{ds}{dt}$

Acceleration $a(t) = v'(t) = s''(t) = \frac{dv}{dt}$

EXAMPLE

An object is projected vertically upwards into the air. Its height, s , in metres above the ground after t seconds is given by the formula:

$s = 250t - 5t^2$.

Determine:

1.1 The velocity after 8 seconds. State whether upwards or downwards.
1.2 After how long the velocity upwards will be 75 m/s
1.3 After how long the velocity downwards will be 50 m/s.
1.4 After how long the object will stop moving upwards.
1.5 The maximum height reached by the object.
1.6 The time taken for the object to reach the ground again.
1.7 The height attained during the 5th second.
1.8 The average speed between the 3rd and 7th seconds.

SOLUTIONS

1.1 Velocity $= s'(t) = 250 - 10t$

After 8 seconds,

Velocity $= s'(8) = 250 - 10(8) = 170$

$\therefore$ Velocity is 170 m/s

Since the velocity is positive, it is **upwards.**

1.2 V = 75

$\therefore s'(t) = 75$

$\therefore 250 - 10t = 25$

$10t = 175$

$\therefore t = 17{,}5$ seconds.

1.3 V = – 50 (negative because downwards)

$\therefore s'(t) = -50$

$\therefore 250 - 10t = -50$

$\therefore 10t = 300$

$\therefore t = 30$ seconds

1.4 When the object stops moving, the velocity is zero.

$\therefore s'(t) = 0$

$\therefore 250 - 10t = 0$

$\therefore 10t = 250$

$\therefore t = 25$ seconds

1.5 The max height will be reached at the moment when the object stops moving upwards at t = 25 seconds.

$s = 250t - 5t^2$

$\therefore$ Max height will be given by $f(25)$

$\therefore f(25) = 250(25) - 5(25)^2 = 3\ 125$ metres

1.6 When the object reaches the ground, the height above the ground will be zero.

$s(t) = 0$

$\therefore 250t - 5t^2 = 0$

$t = 0$, n/a or $t = 50$ seconds

1.7 The height attained during the 5th second is given by the height after 5 seconds minus the height after 4 seconds, that is s(5) – s(4).

$s(5) = 250(5) - 5(5)^2 = 1\ 125$ metres

$s(4) = 250(4) - 5(4)^2 = 920$ metres

$\therefore$ the height attained during the 5th second = 1 125 – 920 = 205m

1.8 The **average speed** is given by the **average gradient**.

Average gradient = $\frac{y_1 - y_2}{x_1 - x_2}$

$= \frac{1505 - 705}{7 - 3} = 200$

$\therefore$ Average speed is 200 m/s

ACTIVITY

1. A ball is shot into the air and its height in metres after t seconds is given by

   $f(t) = 80t - 5t^2$. Calculate the :

   (a) Height of the ball after 5 seconds.
   (b) Velocity of the ball after 4 seconds.
   (c) time taken for the ball to reach its maximum height.
   (d) Average velocity between the second and the third seconds.

2. Thabiso is a learner at the Blair Atholl School founded by Gary Player. He is also a rising star in golf. Thabiso hits a ball up into the air. The height, s, metres, above the ground while the ball is in the air, t seconds after being hit, is given by the formula:
   $s = 60t - 5t^2$

   (a) Calculate the velocity with which the ball was launched.
   (b) What is the maximum height that the ball can reach?
   (c) What is the acceleration of the ball throughout its flight?
   (d) How many seconds after the ball is hit will its velocity be 15m/s, and how high will the ball be at this instant?

3. A bullet is fired into air. it reaches a height of s metres t seconds after the trigger is pulled. The formula to calculate the height is :
   $s = -5t^2 + 210t$
   Calculate :
   (a) The velocity of the bullet as it leaves the gun
   (b) How many seconds it takes to reach its maximum height.
   (c) The maximum height
   (d) How long it takes for the bullet to reach the ground.

4. Water hyacinth is a weed that covers part of the Kariba dam. To combat the weed, herbicide was sprayed over the dam. The number of hectares, H, of the dam covered by the water hyacinth is approximately:

H = $-\frac{1}{3}t^3 + 8t^2$ after t months of spraying.

(a) When did the area covered in the section of the dam reach a maximum? How many hectares were covered by the weed at this time?
(b) What time was the average rate of decrease of weed from the 17th to the 20th month?
(c) What was the actual rate of decrease after 20 months?
(d) After how many months was that part of the dam clear of weed?

5. A boy shoots a stone vertically into the air using a catapult. The height of the stone in metres, t seconds after leaving the catapult, is given by the formula: $s = 40t - 5t^2$.
   (a) What was the maximum height that stone reached, and how long did it take to reach this height?
   (b) What was the velocity of the stone when it hit the ground?

6. A water tank is installed at a rural school in Mdantsane. The water tank is available to the school as well as people in the surrounding community. The volume of the water in a tank is given by the equation:

   V = 120 +40t – 5t^2, where V is the volume in litres and t is the time in hours.

   (a) Determine the rate at which the volume is changing after two hours.
   (b) Determine at what time the volume in the tank will be a maximum.
   (c) When does the volume of water in the tank start decreasing?

7. A particle travels in a straight line from a point, P. it reaches a distance of s metres away from P, t seconds after it started moving. The distance is given by :
   $s = 3t^4 - 28t^3 + 60t^2$.
   (a) How far is the particle from P after 1 second?
   (b) When does it first momentarily come to rest again?
   (c) When does the particle reach P again?

8. The water in a dam is pumped out using a new kind of electric pump. Some water still flows into the dam as water is pumped out. The height of the water H in metres, t days after the pump starts operating, is given by:
   H = $\frac{1}{4}\left(110 + 3t - t^3\right)$.
   Calculate:
   (a) When the height of the water in the dam was at a maximum.
   (b) The average rate that the depth was falling from the second to the fourth day.
   (c) The actual rate at which the height was dropping on the third day.
   (d) When the pump had pumped the dam dry.

MEMO

1. (a) 275 m (b) 40m/s (c) 8 sec (d) 55 m/s

2. (a) 60m/s (b) 180m (c) $-10 m/s^2$ (d) 168,75m

3. (a) 210m/s (b) 21sec (c) 2 205m (d) 42 sec

4. (a) 682,67 hec (b) – 47 hec/ m (c) – 80 hec/m (d) after 24 m

5. (a) 80 m (b) 40m/s

6. (a) 20l/hour (b) 4 hours (c) after 4 hours

7. (a) 35 m (b) 2 sec (c) 0 or 6 or $\frac{10}{3}$

8. (a) 1 (b) – 6,25m/day (c) – 6m / day (d) 5 days