12

Mathematics

X-kit

Achieve!

EXAM PRACTICE BOOK

PEARSON

Pearson Education South Africa (Pty) Ltd
Forest Drive, Pinelands, Cape Town
www.pearsoned.co.za

First published 2014
ISBN: 9781775953999

Editor: Margaret Roussouw
Proofreader: Daniel Krupanandan
Reviewer: Daniel Krupanandan
Book design by:
Cover design by:
Artwork: Marco Casati
Typesetting and layout by: Stronghold Publishing CC
Printer:

CONTENTS

STUDY AND EXAMINATION TIPS FOR MATHEMATICS CANDIDATES IN GRADE 12

A. BEFORE THE EXAMINATION PERIOD STARTS

- You must prepare yourself well ahead of time.
- Remember that you do not "read" mathematics; instead, you "do". Therefore you should solve as many mathematical problems as possible. Exposure to a large variety of problems also helps to eliminate possible fear and thus prevents panic during examination time.
- You should have a private study time table to do revision in a more orderly fashion.
- Get into the habit of doing mathematics casually. For example, during any five to ten minutes of spare time you could solve one short mathematical problem.
- When solving mathematical problems, do not lose patience when you fail to find the solution upon the first attempt. Remember that failure is an opportunity for success and experience. It helps you to discover your weak areas so that you can work on them well ahead of time.
- Devote your time to working on your weak areas rather than being anxious about coming examinations.
- Avoid friends who do not encourage you to work hard.
- Ensure that you get enough sleep (at least seven hours) each day. This helps your body to reorganise itself so that it can function optimally on the next day.

B. DURING EXAMINATIONS

- Be in a relaxed mood.
- Do not panic. Assure yourself that you know all the work.
- A stressful situation tends to deprive the brain of oxygen, which has a negative impact on your thought processes. In such a case, take a deep breath to relax your whole body.
- Read the instructions very carefully.
- Decide how much time you should spend on each question by dividing the total time given for answering the paper by the total number of marks. For example, a mathematics paper is usually allocated three hours (180 minutes). Thus: $180 \div 150 = 1{,}2$ minutes per mark. Therefore, if a question carries 15 marks, you should spend a maximum of $15 \times 1{,}2 = 18$ minutes on it. This will help you manage your time as you write the examination.
- Remember that you are competing with only yourself. Therefore, do not start panicking when you see a learner who appears to have finished the examination when you have not yet finished.
- Start with the questions that you know best and do the most difficult ones last.
- Read each question very carefully. Look for key words and underline them, as shown in the example below.

EXAMPLE

QUESTION 5

The <u>first two terms</u> of an <u>infinite geometric</u> sequence are 8 and $\frac{8}{\sqrt{2}}$. Prove, without using a calculator, that the <u>sum of the series to infinity</u> is $16 + 8\sqrt{2}$. (4)

PAPER 1 A

A maximum of 6 marks for bookwork

Description	Weighting of marks	
Algebra, equations and inequalities	25 ± 3	20
Patterns and sequences	25 ± 3	27
Finance, growth and decay	15 ± 3	15
Functions and graphs	35 ± 3	34
Differential calculus	35 ± 3	40
Probability	15 ± 3	14
TOTAL	**150**	**150**

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams and graphs which you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the exam practice book.
9. Number the answers according to the numbering system used in this question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(3x + 1)(x - 5) = 0$ (2)

1.1.2 $\sqrt{2x - 1} + x = 2$ (3)

1.2 Solve simultaneously for x and y if $\frac{8^x}{4^y} = 32$ and $x - y = 3$. (5)

1.3 Solve for x if $x^2 < 15 + 2x$. (3)

1.4 Without solving the equation, discuss the nature of the roots of $x^2 - 3x + 4 = 0$. (2)

1.5 If $x = -2 + \frac{\sqrt{12 - r}}{3}$ is the solution to a quadratic equation, then calculate:

1.5.1 the value of r for which the equation has equal roots (3)

1.5.2 the largest integral value of r for which the equation has rational roots. (2)

[20]

QUESTION 2

2.1 Consider the following arithmetic series: $3 + 10 + 17 + 24 + 31 + 38 + \ldots + 332$

2.1.1 How many terms are there in the series? (3)

2.1.2 Write the series in sigma notation. (2)

2.1.3 Calculate the sum of the first 30 terms in the series. (2)

2.2 Consider the following geometric sequence: $3x$; $x + 2$; $x - 1$

2.2.1 Calculate the value(s) of x. (5)

2.2.1 Explain why the sequence is only convergent when $x = 4$. (3)

2.2.3 If $x = 4$, calculate $S_\infty - S_6$. (4)

2.3 The sum of the first n terms of a sequence is given by the formula $S_n = 4n(4 - n)$.

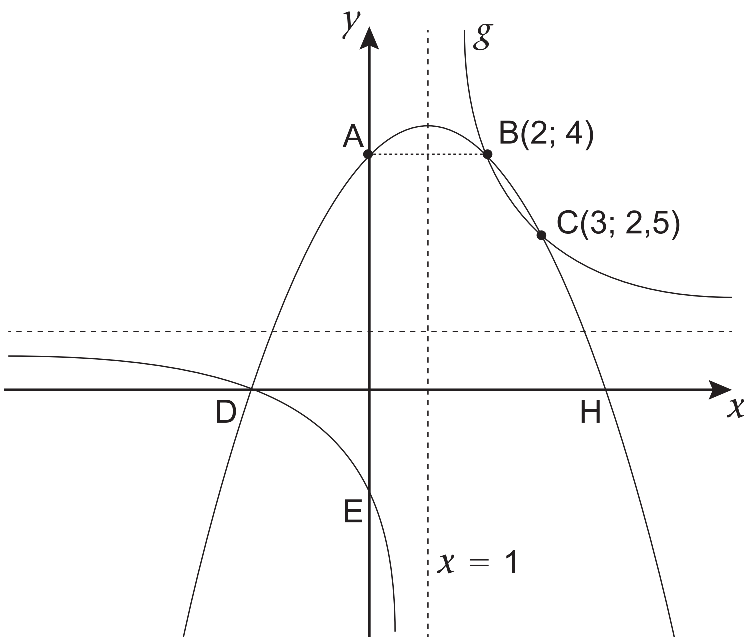
2.3.1 Determine the first three terms in the sequence. (3)

2.3.2 Is thesequence arithmetic or geometric? Give a reason (1)

2.4 Determine p if 4; p; 14; 25 are the first four terms of a quadratic sequence. (4)

[27]

QUESTION 3



$f(x) = ax^2 + bx + c$ and $g(x) = \dfrac{k}{x - p} + q$ are sketched above.

Points A and B are symmetrical to each other about the line $x = 1$.

f and g intersect each other at B(2; 4), C(3; 2,5) and D.

3.1 Write down the value of p. (1)

3.2 Determine the values of k and q. (4)

3.3 Write down the coordinates of A. (1)

3.4 Determine the coordinates of D. (2)

3.5 Determine the values of a and b. (5)

3.6 For which values of x is:

3.6.1 $f(x) \geq 0$ (2)

3.6.2 $f(x) < g(x)$? (3)

3.6.3 $\dfrac{f(x)}{g(x)} = 1$ (1)

3.7 If $h(x)$ is a transformation of $f(x)$ so that $f^{-1}(x)$ is a function with restricted domain, determine the equation of $h(x)$. (2)

[21]

QUESTION 4

$h(x) = \log_3 x$

4.1 Write down the domain and range of h. (2)

4.2 Determine the equation of $h^{-1}(x)$. (1)

4.3 Write down the equation of the line in which h and h^{-1} are reflections of each other. (1)

4.4 Write down the equation of $p(x)$ if p is the reflection of h in the y-axis. (2)

4.5 Write down the equation of q if q is the reflection of h in the x-axis. (1)

4.6 Sketch h, h^{-1} and p on the same set of axes. Label the graphs. (6)

[13]

QUESTION 5

Lubelo plans to buy a new tractor in 8 years' time. He plans to set up a sinking fund to buy the tractor.

- The tractor he plans to buy is currently priced at R2 400 000. He has been advised to work on an inflation rate of 7,5% per annum in order to estimate the price of the new tractor in 8 years' time.
- Lubelo currently owns a tractor which is valued at R1 800 000. This tractor is expected to depreciate at a rate of 8,7% on a straight-line basis. He plans to sell the tractor and use the money from this sale towards buying the new tractor.
- The first payment into the sinking fund will be made at the end of the 8-year period and the last payment at the end of 8 years.
- He will make monthly payments into the fund, which will earn interest at a rate of 9,5% per annum, compounded monthly.

5.1 Calculate the book value of his current tractor after 8 years. (2)

5.2 Calculate the cash price of the new tractor in 8 years' time. (3)

5.3 How much money does he need to save in his sinking fund so that he can buy the new tractor in 8 years' time without needing any additional funding? Assume that he uses the book value that he receives for his current tractor towards the cost of the new tractor. (1)

5.4 Based on his calculations, Lubelo decides that he should have R3 735 000 in the sinking fund at the end of the 8-year period. Calculate the size of Lubelo's monthly payments into the sinking fund. (5)

5.5 Lubelo would like to make a once-off withdrawal of R60 000 from the sinking fund at the end of the first 5 years. Calculate the resulting shortfall between the amount that he will have in his sinking fund and the estimated amount that he will need at the end of the 8-year period as determined in 5.3. Assume that the fund would have been worth R3 735 000 if the withdrawal had not been made. (4)

[15]

QUESTION 6

6.1 Differentiate $f(x)$ from first principles if

6.1.1 $f(x) = -\frac{2}{x}$ (3)

6.1.2 $f(x) = 4x - 3x^2$ (3)

6.2 Determine $\frac{dy}{dx}$ if $y = \frac{3x - \sqrt[3]{x^2} - 4}{x}$. (4)

[10]

QUESTION 7

$f(x) = -x^3 + 5x^2 + 8x - 12$.

7.1 Show that $(x - 1)$ is a factor of $-x^3 + 5x^2 + 8x - 12$. (2)

7.2 Hence, find the x and y intercepts of f. (5)

7.3 Calculate the turning points of f. (3)

7.4 Show that f has a point of inflection at $x = \frac{5}{3}$. (4)

7.5 If $y = 15x + c$ is a tangent to f at $(a; b)$, determine the value of c if $a, b \in \mathbb{Z}$. (6)

[20]

QUESTION 8

Tito was standing on the sports ground when he threw a ball into the air. The height, in metres, of the ball above the ground after t seconds was given by $h(t) = -3t^2 + 20t + 7$.

8.1 What was the average speed of the ball during the first three seconds? (3)

8.2 How long did it take the ball to hit the ground? (2)

8.3 After how many seconds did the ball reach its maximum height? (3)

8.4 Calculate the maximum height reached. (2)

[10]

QUESTION 9

Consider the code "MAGIC".

9.1 How many different codes are possible if all the letters are used? (2)

9.2 What is the probability that the first letter in any arrangement will be M and the last letter will be C? (3)

9.3 How many different arrangements are possible if the letters M and C must follow each other, in any order? (3)

[8]

QUESTION 10

A survey was carried out to investigate the relationship between Maths results and extra Maths lessons. Some of the results have been recorded in the table below.

	80% or more for Maths	**Less than 80% for Maths**	**Total**
Extra Maths lessons	240	(e)	(c)
No extra Maths lessons	(a)	(d)	200
Total	300	(b)	1 000

10.1 Write down the values of (a), (b), (c), (d) and (e). (3)

10.2 Extra Maths teachers claim that learners who take extra Maths lessons are more likely to get more than 80% for Maths than those who don't. Are they correct? Justify your answer with the necessary calculations to test for independence. (3)

[6]

TOTAL: [150]

PAPER 2A A maximum of 6 marks for bookwork		
Description	**Weighting of marks**	
Statistics	20 ± 3	19
Analytical geometry	40 ± 3	43
Trigonometry	40 ± 3	42
Euclidean geometry and measurement	50 ± 3	46
TOTAL	150	150

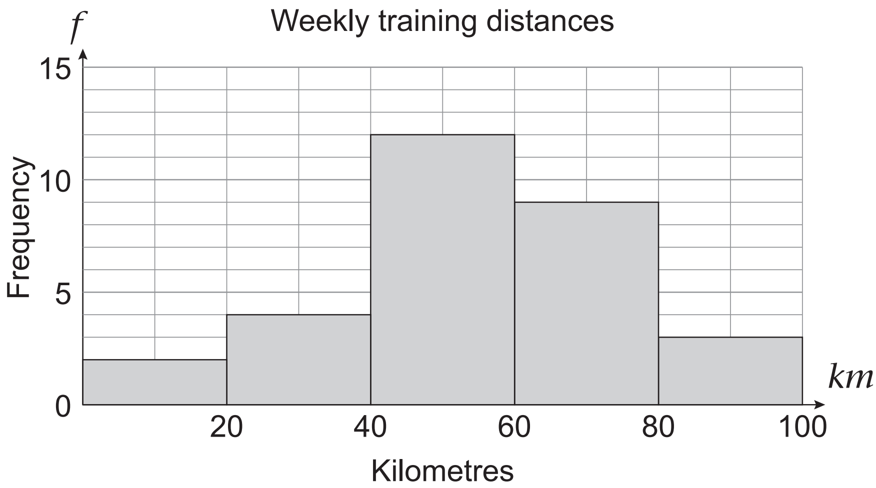
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams and graphs which you have used in determining the answer.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. FOUR diagram sheets for Questions 2, 3, 7, 8, 9, 10 and 11 are attached at the end of the question paper.
8. Number the questions correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

A survey was conducted amongst a group of runners to gather information about their weekly training distances. The information obtained is recorded in the histogram below.



1.1 How many runners participated in this survey? (1)

1.2 Identify the modal class. (1)

1.3 Calculate the approximate mean running distance. (3)

[5]

QUESTION 2

Ten Grade 12 learners take Maths and Science. Their results are recorded below.

	A	B	C	D	E	F	G	H	I	J
Maths (x)	65	59	76	54	80	44	78	56	73	84
Science (y)	59	65	61	48	76	56	71	52	68	78

2.1 Draw a scatter plot on DIAGRAM SHEET 1 to represent each learner's Maths mark versus his or her Science mark. (3)

2.2 Calculate the equation of the least squares regression line for the above data. (3)

2.3 Draw the least squares regression line on the grid with the scatter plot. (2)

2.4 Calculate the correlation coefficient for the data. (2)

2.5 Comment on the correlation. (2)

2.6 Predict the mark of a learner who missed the Science exam, but got 88% for the Maths exam. (2)

[14]

QUESTION 3

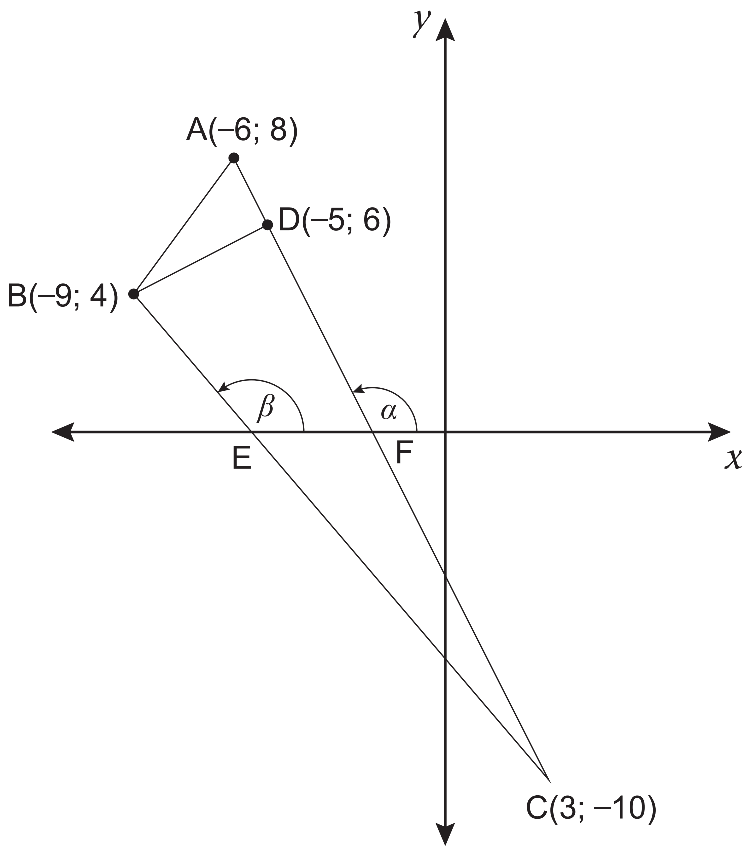
In the figure below, A(-6; 8), B(-9; 4) and C(3; -10) are the vertices of $\triangle$ABC.

D(-5; 6) lies on AC.

BC and AC intersect the x-axis at E and F respectively.

The angle of inclination of BC with the x-axis at E is β.

The angle of inclination of AC with the x-axis at F is α.



3.1 Calculate the length of AC. Leave your answer in simplest surd form. (3)

3.2 Prove that BD $\perp$ AC. (4)

3.3 Calculate the area of $\triangle$ABC. (4)

3.4 Calculate the size of α. (2)

3.5 Calculate the size of $A\hat{C}B$. (5)

3.6 Determine the coordinates of G if ABCG is a kite. (2)

[20]

QUESTION 4

P(−5; −6) and Q(1; 2) are the endpoints of the diameter of a circle.

4.1 Determine the equation of the circle. (5)

4.2 Determine the equation of the tangent to the circle at Q. (4)

[9]

QUESTION 5

Given the points A(−6; 0) and B(2; 4), determine:

5.1 the equation of the perpendicular bisector of AB (5)

5.2 the equation(s) of the circle(s) with a radius of 5 which pass(es) through A and B. (9)

[14]

QUESTION 6

6.1 Show that: $\frac{\cos(180° + x)\sin(90° - x) + \tan x\cos x\cos(90° + x)}{\sin(x - 20°)\cos(50° - x) + \cos(x - 20°)\sin(50° - x)} = -2$ (7)

6.2 If $\cos 48° = p$, determine the following in terms of p:

6.2.1 $\sin 42°$ (2)

6.2.2 $\tan 48°$ (2)

6.2.3 $\cos 96°$ (3)

6.3 Prove that $\frac{\cos 3A}{\sin A} + \frac{\sin 3A}{\cos A} = \frac{2}{\tan 2A}$ for all $A \neq 0° + n \times 90; n \in \mathbb{Z}$. (4)

[18]

QUESTION 7

Consider the functions $f(x) = \sin(2x + 60°)$ and $g(x) = -\cos 3x$.

7.1 Determine the general solution of $f(x) = g(x)$. (8)

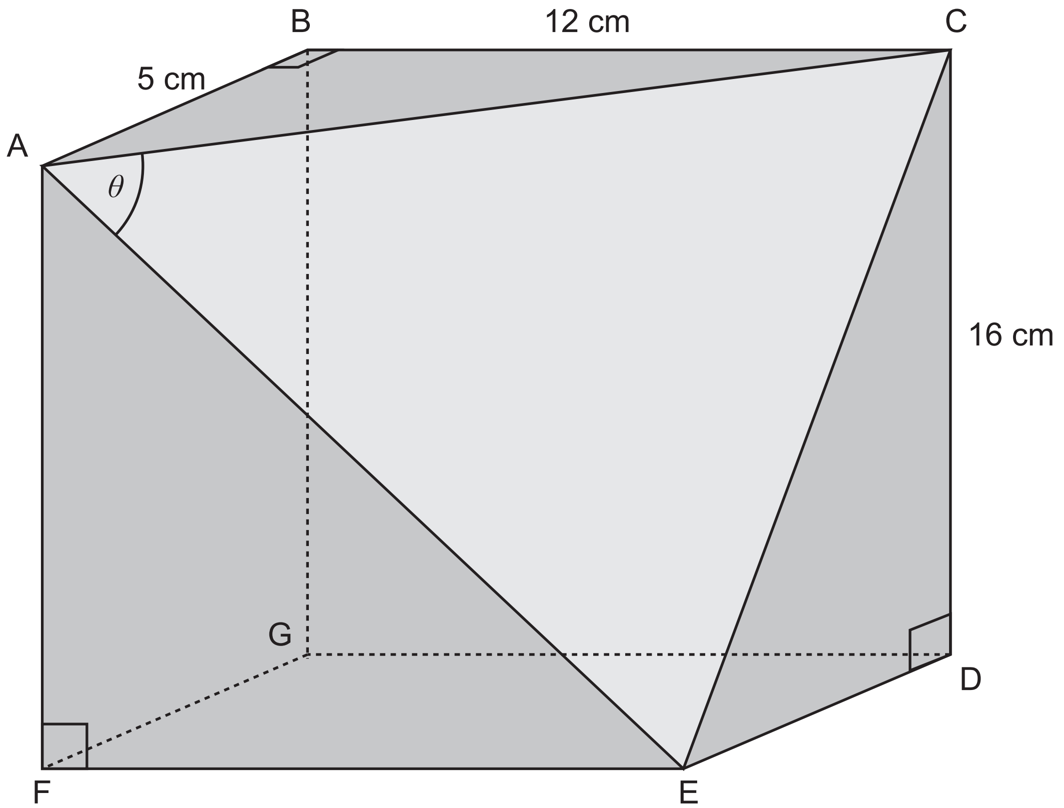
7.2 Sketch f and g on the same set of axes for $x \in [-30°; 150°]$.

A set of axes has been provided for you on DIAGRAM SHEET 2. (6)

7.3. Use your answers to 7.1 and 7.2 to solve for x if $f(x) \leq g(x)$ for $x \in [-30°; 150°]$. (2)

[16]

QUESTION 8



The rectangular prism sketched above has dimensions 5 cm by 12 cm by 16 cm.

A wedge is cut off, exposing a new triangular surface.

8.1 Calculate, without giving reasons, the lengths of:

8.1.1 AC (1)

8.1.2 AE (1)

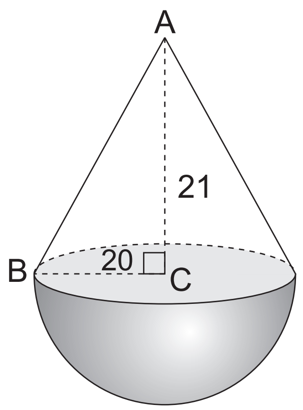
8.1.3 CE (leave your answer in surd form). (1)

8.2 Calculate the size of θ correct to one decimal place. (3)

8.3 Calculate the area of the $\triangle$ACE. (2)

[8]

QUESTION 9



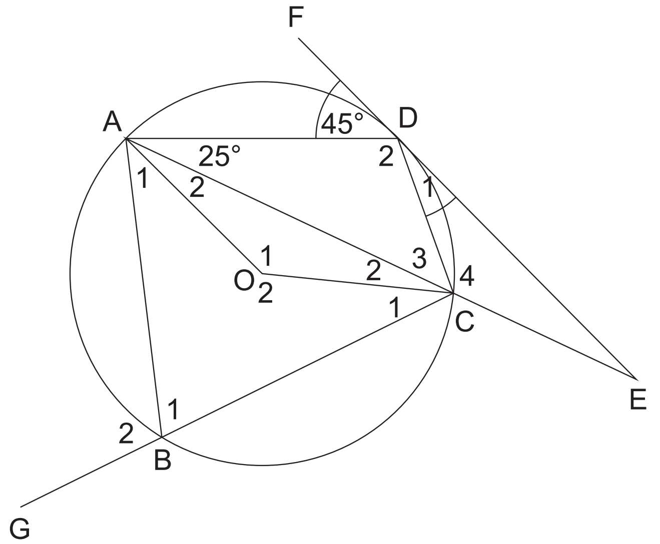
The buoy sketched above has a hemispherical end with a radius of 20 units and a conical end with a height of 21 units.

9.1 Calculate the ratio of the volume of the hemisphere to the volume of the cone. (3)

9.2 Calculate the surface area of the cone. Give your answer in terms of π. (4)

[7]

QUESTION 10



A, B, C and D lie on the circumference of the circle. FDE is a tangent to the circle at D.

AC is produced to E and CB is produced to G. $F\hat{D}A = 45°$ and $C\hat{A}D = 25°$.

10.1 Calculate, with reasons, the size of the following angles:

10.1.1 $\hat{C}_3$ (2)

10.1.2 $\hat{D}_1$ (2)

10.1.3 $\hat{B}_2$ (3)

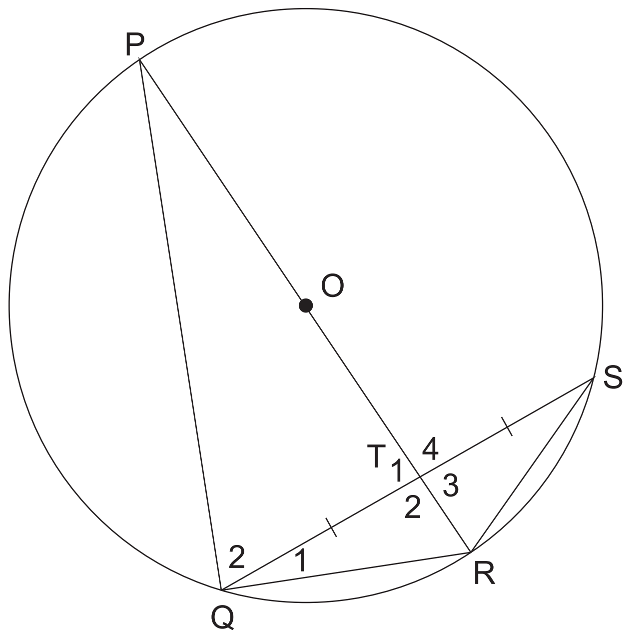
10.1.4 $\hat{O}_2$ (2)

10.2 Prove that $ED^2 = EC.EA$. (5)

[14]

QUESTION 11

11.2



P, Q, R and S lie on the circumference of the circle with centre O.

PR passes through O, QT = TS, PQ = 40 units, QR = 30 units and TR = x units.

Reasons must be given for all statements.

11.2.1 Prove that $P\hat{Q}R = P\hat{T}S$. (4)

11.2.2 Prove that $\hat{Q}_1 = \hat{S}$. (4)

11.2.3 Determine the length of PR. (2)

11.2.4 Express PT in terms of x. (1)

11.2.5 Express QT^2 in terms of x. (3)

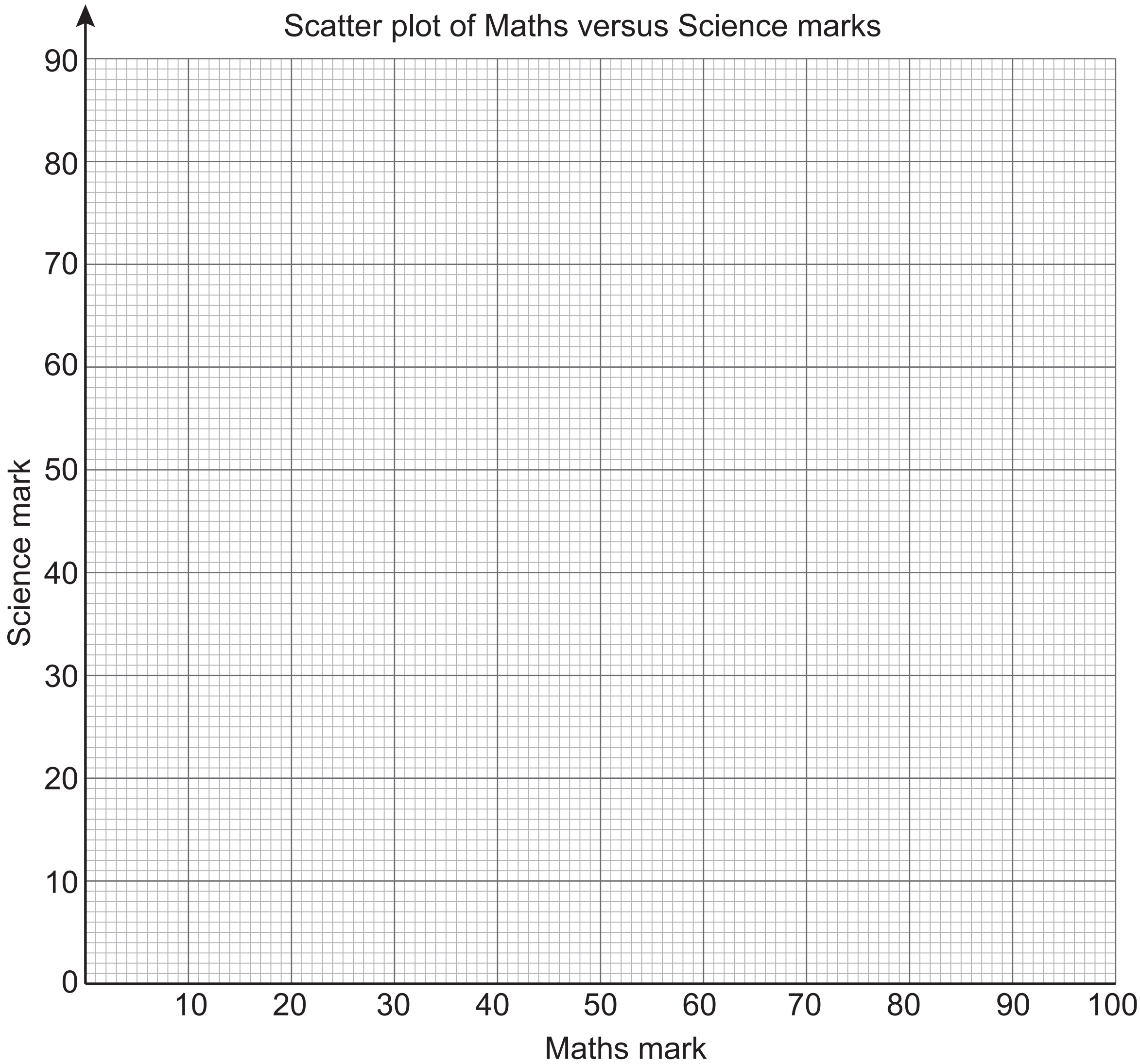
11.2.6 Solve for x. (3)

[25]

TOTAL: [150]

DIAGRAM SHEET 1

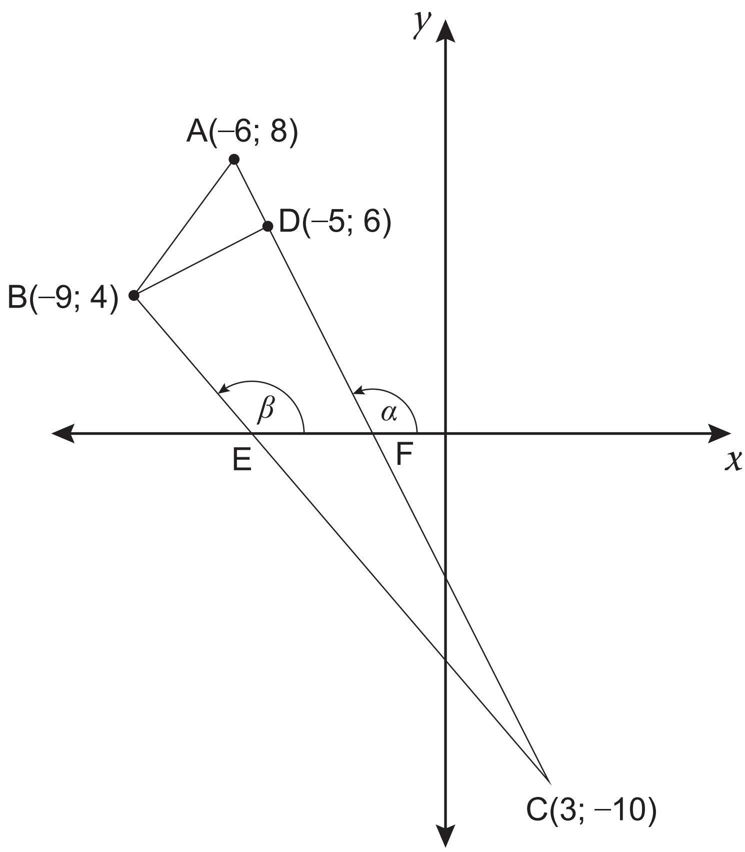
QUESTION 2.1



PAPER 2A

DIAGRAM SHEET 2

QUESTION 3



QUESTION 7

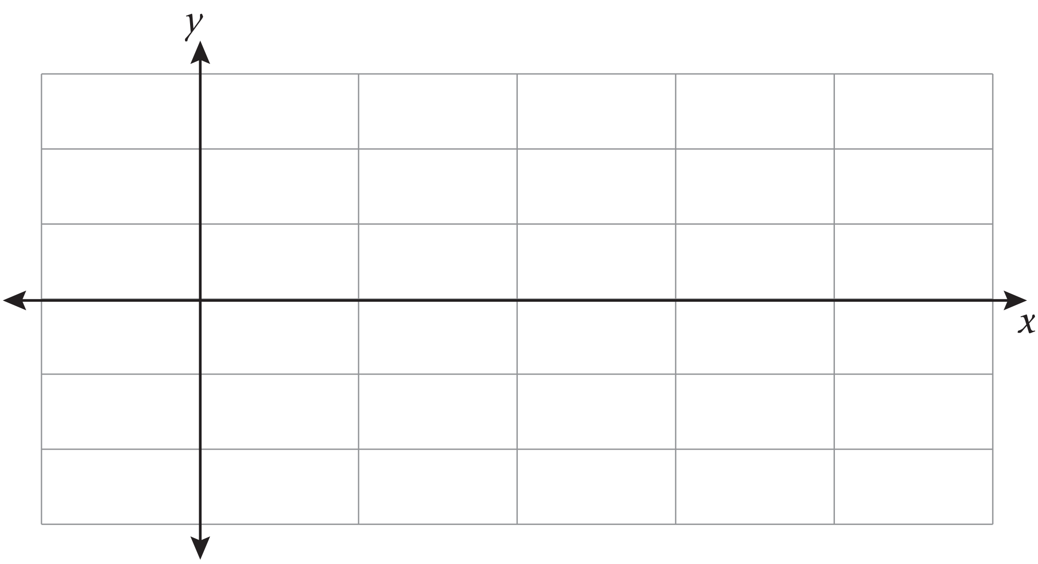
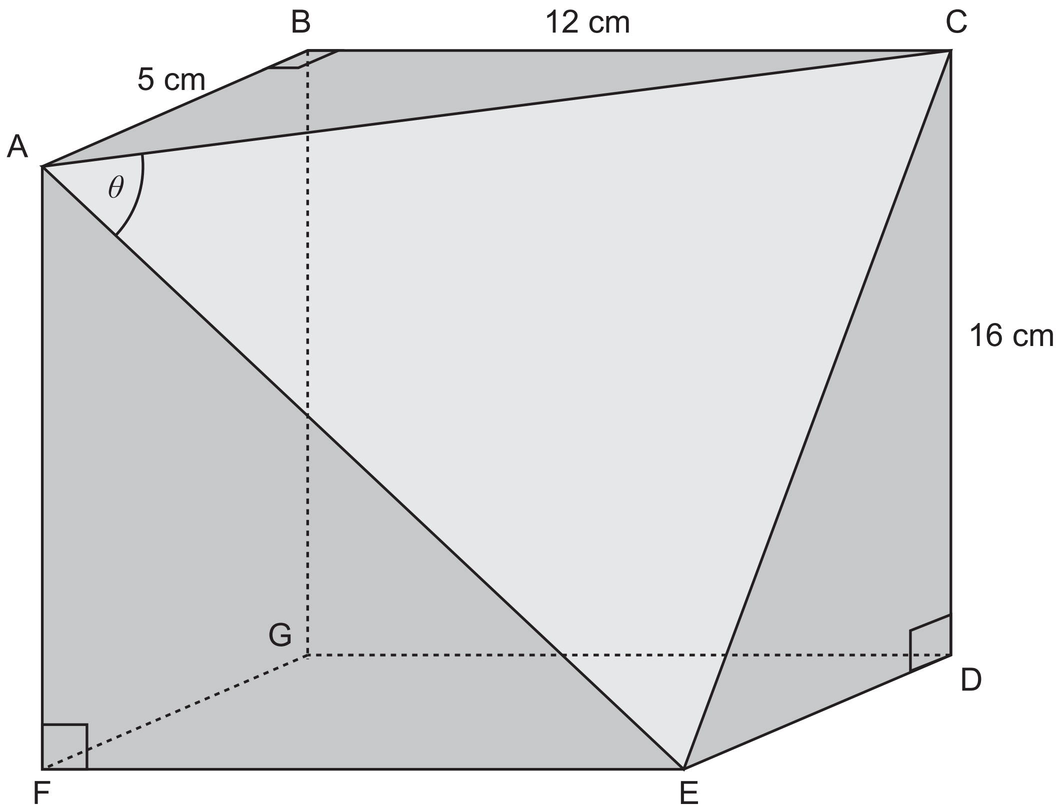


DIAGRAM SHEET 3

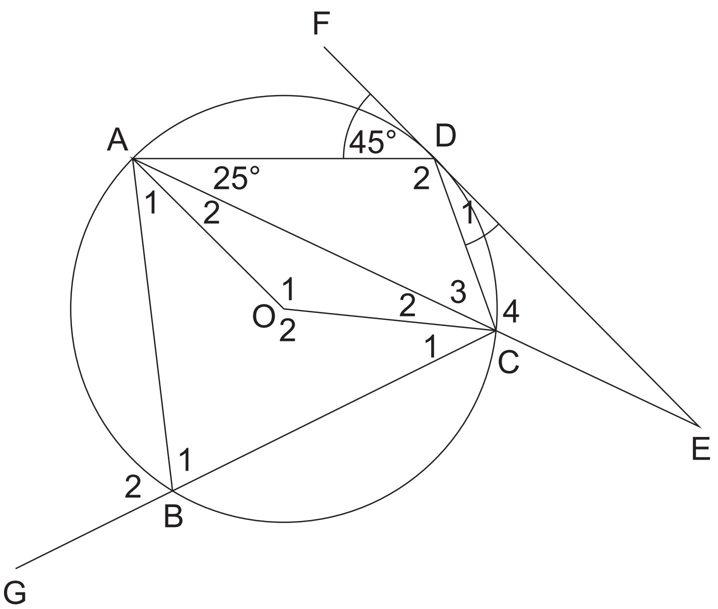
QUESTION 8



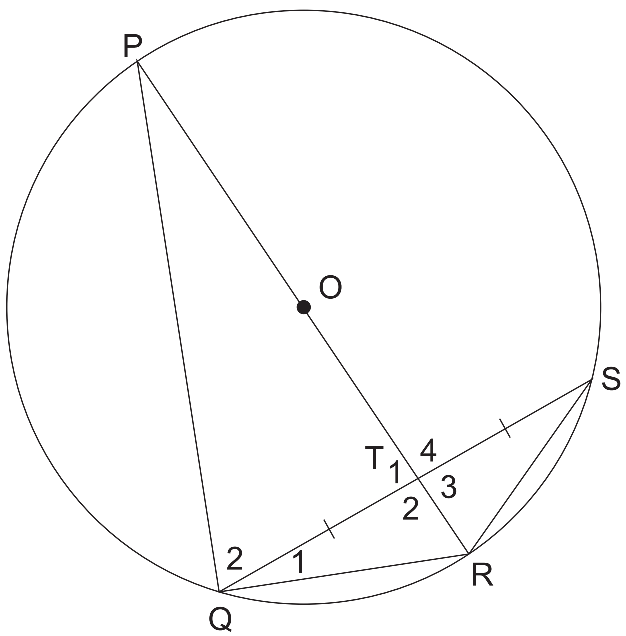
PAPER 2A

DIAGRAM SHEET 4

QUESTION 10



QUESTION 11.2



GRADE 12 MATHEMATICS PAPER 1A MEMORANDUM

QUESTION 1

1.1.1 $(3x+1)(x-5)=0$

$x=-\frac{1}{3}\checkmark$ or $x=5\checkmark$ (2)

1.1.2 $\sqrt{2x-1}+x=2$

$2x-1=x^2-4x+4\checkmark$

$x^2-6x+5=0$

$(x-1)(x-5)\checkmark=0$

$x=1\checkmark;\ x\neq 5\checkmark$

Check:

If $x=1$: LHS $=2=$ RHS

If $x=5$: LHS $=8\neq$ RHS $=2$ (4)

1.2 $\frac{8^x}{4^y}=32$

$\frac{2^{3x}}{2^{2y}}=2^5\checkmark$

$2^{3x-2y}=2^5$

$3x-2y=5\checkmark$ ①

$x-y=3$

$x=3+y\checkmark$ ②

Substitute ② into ① $x=-1\checkmark$

$3(3+y)-2y=5\checkmark$

$9+3y-2y=5$

or

$x=3+y\checkmark$

$\frac{2^{3x}}{2^{2y}}=2^5\checkmark$

$\frac{2^{3(3+y)}}{2^{2y}}=32\checkmark$

$2^{9+3y-2y}=2^5\checkmark$

$9+y=5$

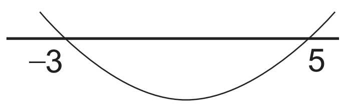
$y=-4$

$y=-4$ and $\overset{\checkmark}{x}=-1$ (5)

1.3 $x^2<15+2x$

$x^2-2x-15<0$

$(x-5)(x+3)\checkmark<0$



✓

$-3<\checkmark x<5\checkmark$

or

$x\in(-3;\checkmark 5)\checkmark$ (4)

1.4 $a=1$, $b=-3$ and $c=4$

$b^2-4ac=(-3)^2-4(1)(4)\checkmark=-7\checkmark$

Roots are non-real✓(or imaginary) (3)

1.5.1 For equal roots, $\triangle=b^2-4ac\checkmark=0$.

$\therefore 12-r=0\checkmark\ \therefore r=12.\checkmark$ (3)

1.5.2 For rational roots, T is a perfect square.✓

$\therefore 12-r=p^2$

$\therefore r=3\checkmark$ (2)

[20]

QUESTION 2

2.1.1 $a=2$ and $d=7$

$T_n=7n-4\checkmark$

$7n-4=332\checkmark$

$7n=336\checkmark\ n=48\checkmark$ and so there are 48 terms (3)

2.1.2 ✓$\sum_{n=1}^{48}(7n-4)$✓ (2)

2.1.3 $S_{30} = \frac{30}{2}[2(3) + 29(7)]$✓ $= 3\ 135$✓ (2)

2.2.1 $\frac{x+2}{3x} = \frac{x-1}{x+2}$✓

$3x^2 - 3x = x^2 + 4x + 4$

$2x^2 - 7x - 4 = 0$✓

$(2x+1)(x-4) = 0$✓

$x = -\frac{1}{2}$✓ or $x = 4$✓ (5)

2.2.2 A sequence is convergent if $-1 < r < 1$.✓

If $x = 4$, then $r = \frac{1}{2}$✓and the sequence is convergent.

If $x = -\frac{1}{2}$, then $r = -1$✓and the sequence is not convergent. (3)

2.2.3 $a = 12$; ✓ $r = \frac{1}{2}$

$S_\infty - S_6 = \frac{12}{1-\frac{1}{2}}$✓ $- \frac{12\left(1-\left(\frac{1}{2}\right)^6\right)}{1-\frac{1}{2}}$✓

$= 24 - 24\left(\frac{63}{64}\right)$

$= \frac{3}{8}$✓ [4]

2.3.1 $S_1 = 12$; $S_2 = 16$; $S_3 = 12$✓

$T_1 = S_1 = 12$✓

$T_2 = S_2 - S_1 = 16 - 12 = 4$✓

$T_3 = S_3 - S_2 = 12 - 16 = -4$✓ (3)

2.3.2 Arithmetic, since common difference✓, $d = -8$. (1)

2.4 4; p; 14; 25

$p - 4$ $\quad 14 - p$ $\quad 11$ ✓ (first differences)

$18 - 2p$ $\quad p - 3$ ✓ (second differences)

$18 - 2p$✓$= p - 3$

$3p = 21$ and $p = 7$✓ (4)

[27]

QUESTION 3

3.1 $p = 1$✓ (1)

3.2 Substitute (2; 4) into $g(x) = \frac{k}{x-p} + q$.

$4 = \frac{k}{2-1} + q$ ✓ $q = 4 - k$ ✓ ①

Substitute (3; 2,5) into $g(x) = \frac{k}{x-p} + q$.

$2{,}5 = \frac{k}{3-1} + q$ ✓ $5 = k + 2q$ ✓ ②

Substitute ① into ②:

$5 = k + 2(4 - k)$

$k = 3$✓ and $q = 1$✓ (4)

3.3 A(0; 4) ✓ (1)

3.4 $\frac{3}{x-1} + 1 = 0$✓

$3 + x - 1 = 0$

$x = -2$✓ $\Rightarrow$ D(−2; 0) (2)

3.5 H(4; 0) (by symmetry)

$y = a(x - x_1)(x - x_2)$

$y = a(x + 2)(x - 4)$ ✓

Substitute (0; 4):

$4 = a(-8)$ ✓ $\Rightarrow a = -\frac{1}{2}$ ✓

$y = -\frac{1}{2}(x + 2)(x - 4)$

$= -\frac{1}{2}(x^2 - 2x - 8)$

$= -\frac{1}{2}x^2 + x + 4$ ✓

$\Rightarrow a = -\frac{1}{2}$, $b = 1$ ✓ and $c = 4$ (5)

3.6.1 $-2 \leq$ ✓ $x \leq 4$ ✓ or $x \in [-2;$ ✓ $4]$ ✓ (2)

3.6.2 $x < -2$ or $1 < x < 2$ ✓ or ✓ $x > 3$ ✓ or: $x \in (1;$ ✓ $2)$ ✓ $\cup (3; \infty)$ ✓ (3)

3.6.3 This is $f(x) = g(x)$; $x > 0$

$\therefore x = 2$; $x = 3$. ✓ (1)

3.7 $n(x) = f(x + 1)$ ✓ $- 4\frac{1}{2}$; for $x < 0$ ✓ or $x > 0$. (2)

[21]

QUESTION 4

4.1 Domain: $x > 0$; $x \in \mathbb{R}$ ✓ of: $x \in (0; \infty)$ ✓

Range: $y \in \mathbb{R}$ ✓ (2)

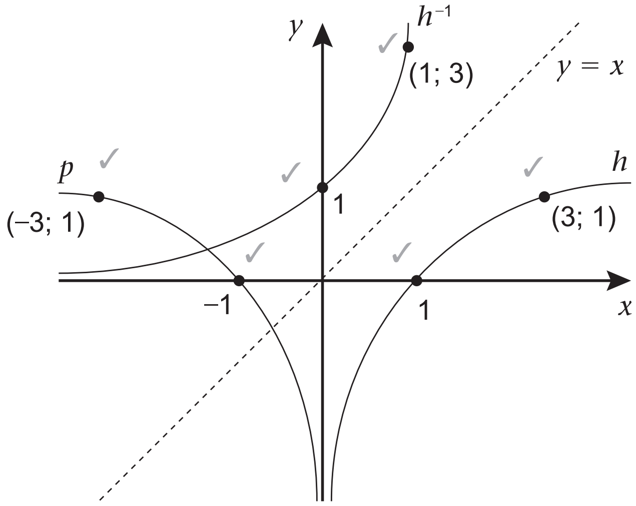
4.2 $h^{-1}(x) = 3^x$ ✓ (1)

4.3 $y = x$ (1)

4.4 $p(x) = \log_3($ ✓ $-x)$, $x <$ ✓ 0 (2)

4.5 $q(x) =$ ✓ $- \log_3 x$ (1)

4.6



(6)

[13]

QUESTION 5

5.1 $A = P(1 - in)$; $P = 1\ 800\ 000$; $i = 0{,}087$; $n = 8$

$A =$ ✓ $1\ 800\ 000(1 - (0{,}087)(8))$ ✓ $=$ R547 200 ✓ (2)

5.2 $A = P(1 + i)^n$; $P = 2\ 400\ 000$; $i = 0{,}075$; $n = 8$

$A = 2\ 400\ 000(1 + 0{,}075$ ✓ $)^{8}$ ✓

$=$ R4 280 346,78 ✓ (3)

5.3 R4 280 346,78 − R547 200 = R3 733 146,78 ✓ (1)

5.4 $F = \frac{x[(1+i)^n - 1]}{i}$; $F =$ R3 735 000; $i = \frac{0,095}{12}$✓; $n = 96$✓

$$3\,735\,000✓ = \frac{x\left[\left(1 + \frac{0,095}{12}\right)^{96} - 1\right]}{\frac{0,095}{12}}✓$$

$$x = 3\,735\,000 \div \frac{\left(1 + \frac{0,095}{12}\right)^{96} - 1}{\frac{0,095}{12}} = \text{R}26\,123{,}41✓$$ (5)

5.5 $60\,000\left(1 + \frac{0,095}{12}\right)^{36}$✓ $=$ R79 696,24✓

The withdrawal of R60 000 will reduce the sinking fund by R79 696,24.

The amount saved will be R3 735 000 − R79 696,24 = R3 655 303,76.✓

The amount needed will be R3 733 146,78.

The shortfall will be R3 733 146,78 − R3 655 303,76 = R77 843,02.✓ (4)

[15]

QUESTION 6

6.1
$$f'(x) = \lim_{h\to 0}\left(\frac{-2}{x+h}\right)✓ - \left(-\frac{2}{x}\right)$$
$$= \lim_{h\to 0}\left[\frac{-2x + 2(x+h)}{x(x+h)}\right]✓ \times \frac{1}{h}$$
$$= \lim_{h\to 0}\left[\frac{2}{x(x+h)}\right] = \frac{2}{x^2}✓$$ (3)

6.1
$$f'(x) = \lim_{h\to 0}\frac{4(x+h) - 3(x+h)^2 - (4x - 3x^2)}{h}✓$$
$$= \lim_{h\to 0}\frac{4x + 4h - 3x^2 - 6xh - 3h^2 - 4x + 3x^2}{h}$$
$$= \lim_{h\to 0}\frac{h(4 - 6x - 3h)}{h}✓$$
$$= \lim_{h\to 0}(4 - 6x - 3h)$$
$$= 4 - 6x✓$$ (3)

6.2 $y = 3 - x^{-\frac{1}{3}} - 4x^{-1}$✓

$$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{4}{3}}✓ + 4x^{-2}✓ = \frac{1}{3x^{\frac{4}{3}}} + \frac{4}{x^2}✓$$ (4)

[10]

QUESTION 7

7.1 $f(1) = -1 + 5 + 8 - 12 = 0$

$\therefore (x - 1)$ is a factor (2)

7.2 $f(x) = (x - 1)(-x^2 + 4x + 12)$

$= -(x - 1)(x - 6)(x + 2)$

$\Rightarrow (x - 1)(x - 6)(x + 2) = 0$

$\therefore x = 1$; $x = -2$; $x = 6$

x-intercepts are $(-2; 0)$; $(1; 0)$ and $(6; 0)$

y-intercepts at y-intercept, $x = 0$

hence $f(0) = -(0)^3 + 5(0)^2 + 8(0) - 12$

$= -12$

$\therefore y$-intercept is $(0; -12)$ (5)

7.3 At turning points, $f'(x) = 0$

$f'(x) = -3x^2 + 10x + 8$

$\Rightarrow -3x^2 + 10x + 8 = 0$

$(x - 4)(-3x - 2) = 0$

$\therefore x = 4$ and $x = -\frac{2}{3}$

$\therefore$ turning points are (4; 36) and $\left(-\frac{2}{3}; -\frac{400}{27}\right)$ (3)

7.4 At the point of inflection, $f''(x) = 0$

$f''(x) = -6x + 10$

$\Rightarrow -6x + 10 = 0$

$f''\left(\frac{5}{3}\right) = -6\left(\frac{5}{3}\right) + 10$

$= -2(5) + 10$

$= 0$

$\therefore x = \frac{5}{3}$ is a point of inflection. (4)

7.5 At turning point, $f'(x) = 15$

$\Rightarrow \quad f'(x) = -3x^2 + 10x + 8 = 15$

$\Rightarrow -3x^2 + 10x + 7 = 0$

$(3x - 7)(x - 1) = 0$

$x \neq \frac{7}{3}$: $x = 1$, since $x \in \mathbb{Z}$

$f(1) = 0 \Rightarrow (1; 0)$

$c = 0 - 15(1)$

$\therefore \; c = -15$ (6)

[20]

QUESTION 8

8.1 Average speed $= \frac{h(3) - h(0)}{3} = \frac{40✓ - 7✓}{3} = 11$ m.s^{-1}✓ (3)

8.2 $3t^2 - 20t - 7 = 0$

$(t - 7)(3t + 1) = 0$

$t = 7$ or $t = -\frac{1}{3}$✓

The ball hit the ground after 7 seconds.✓ (2)

8.3 At maximum height, $h'(t) = 0$✓

$h'(t) = -6t + 20$✓

$\Rightarrow -6t + 20 = 0$

$\therefore t = \frac{10}{3}$s✓ (3)

8.4 At maximum height, $t = \frac{10}{3}$s

$\Rightarrow h\left(\frac{10}{3}\right) = -3 \times \left(\frac{10}{3}\right)^2 + 20 \times \frac{10}{3} + 7$✓

$= 40{,}33$ m✓ (2)

[10]

QUESTION 9

9.1 5 different letters, so 5!✓ = 120✓ arrangements (2)

9.2 $\frac{1 \times 3 \times 2 \times 1 \times 1✓}{120✓} = \frac{1}{20}$✓

or:

$\frac{1}{5}$✓ $\times \frac{1}{4}$✓ $= \frac{1}{20}$✓ (3)

9.3 Group M and C so that there are four arrangements; then arrange M and C.

4!✓ × 2✓ = 48✓ (3)

[8]

QUESTION 10

10.1 **(a)** 60

(b) 700

(c) 800 ✓✓✓ (−1 per error)

(d) 140

(e) 560 (3)

10.2 Let A be the event that a learner gets more than 80% for Maths.

Let E be the event that a learner takes extra Maths lessons.

If A and E are independent, then $P(A) \times P(E) = P(A \cap E)$.

LHS $= P(A) \times P(E) = 0{,}8 \times 0{,}3 = 0{,}24$✓ RHS $= P(A \cap E) = 0{,}24$✓

RHS $= P(A \cap E) = 0{,}24$✓

LHS $\neq$ RHS

Achieving more than 80% for Maths and taking extra Maths lessons are independent.✓ (3)

[6]

TOTAL: [150]

GRADE 12 MATHEMATICS PAPER 2A MEMORANDUM

QUESTION 1

1.1 $2 + 4 + 12 + 9 + 3 = 30$✓runners (1)

1.2 $40 \leq x < 50$ or $40 < x \leq 50$✓ (1)

1.3 Approximate mean running distance

$= \frac{10 \times 2 + 30 \times 4 + 50 \times 12 + 70 \times 9 + 90 \times 3✓}{30✓}$

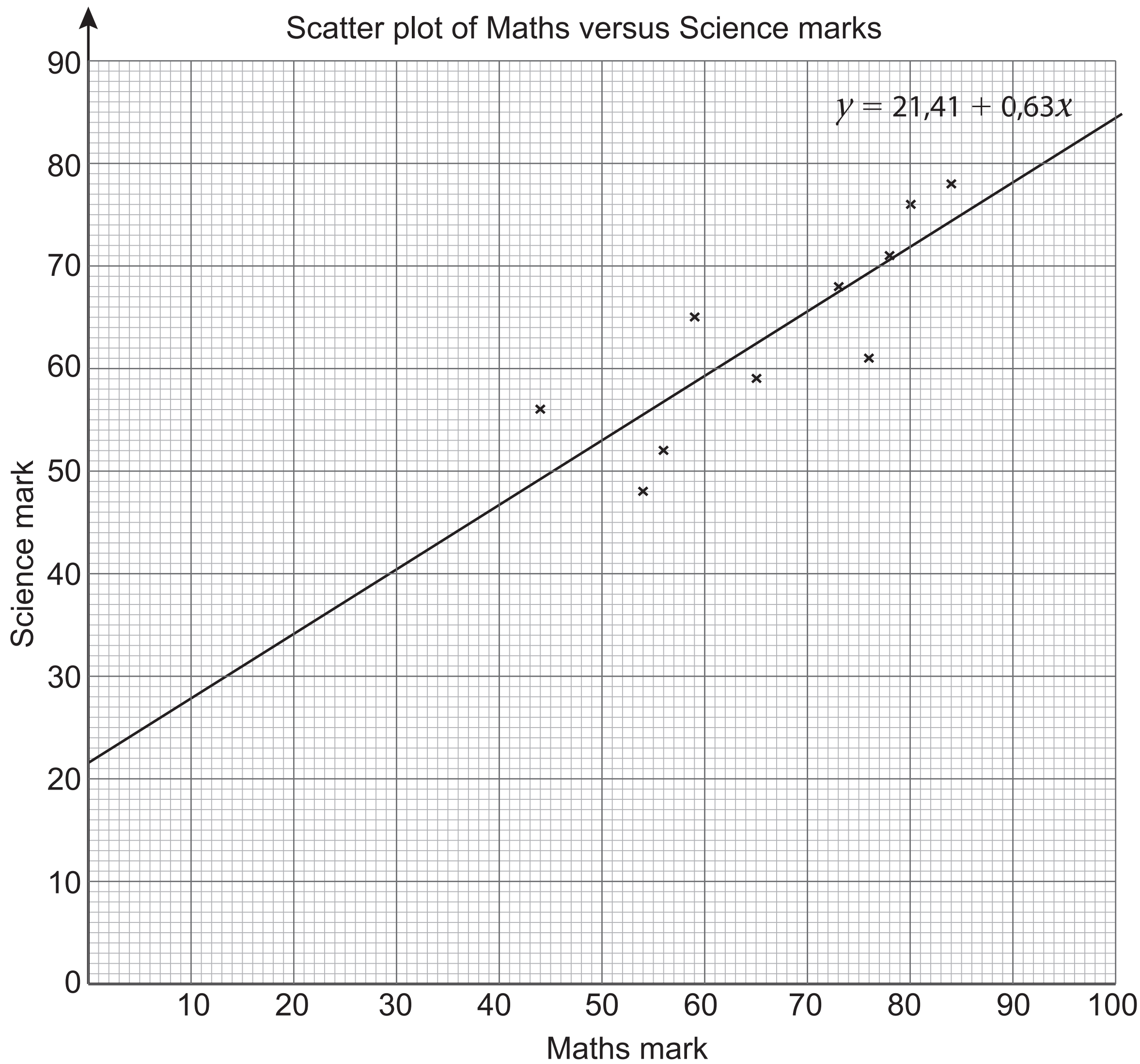
$= \frac{164}{3}$

$= 54\frac{2}{3}$ km✓ (or 54,67) (3)

[5]

QUESTION 2

2.1



All points correct✓✓✓

7 or more points correct✓✓

3 to 6 points correct✓ (3)

2.2 A = 21,41✓

B = 0,63✓

$y = 21,41 + 0,63x$ ✓ (3)

2.3 y-intercept: 21,41✓ (accept reasonable accuracy)

Accuracy of line✓ (2)

2.4 $r = 0{,}83$✓✓ (2)

2.5 There is a strong✓positive✓correlation between the Maths and Science marks. (2)

2.6 $\hat{y} = 76{,}64$✓✓ (using a calculator)

or: $y = 21{,}41 + 0{,}63(88)$✓ $= 76{,}85$✓ (2)

[14]

QUESTION 3

3.1 $AC = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2}$

$= \sqrt{(-6 - 3)^2✓ + (8 - (-10))^2✓}$

$= \sqrt{81 + 324}$

$= \sqrt{405}$

$= 9\sqrt{5}$✓ (3)

3.2 $m_{BD} = \frac{6 - 4}{-5 - (-9)}$✓ $= \frac{1}{2}$✓

$m_{AC} = \frac{8 - (-10)}{-6 - 3} = -2$✓

$m_{BD} \times m_{AC} = -1$,✓so $BD \perp AC$. (4)

3.3 $BD = \sqrt{(-9 - (-5))^2 + (4 - 6)^2}$✓

$= \sqrt{20}$

$= 2\sqrt{5}$✓

Area $\triangle ABC = \frac{1}{2}AC \times BD$

$= \frac{1}{2}(9\sqrt{5})(2\sqrt{5})$✓

$= 45$✓ units^2 (4)

3.4 $\tan\alpha = -2$✓ $\Rightarrow \alpha = 116{,}57°$✓ (2)

3.5 $\tan\beta = \frac{14}{-12} = -\frac{7}{6}$✓ $\Rightarrow \beta = 130{,}60°$✓

$E\hat{F}C = 116{,}57°$✓ (vertically opposite angles)

$A\hat{C}B = 130{,}67° - 116{,}57°$✓ $= 14{,}03°$✓ (exterior angle $\triangle CEF$) (5)

3.6 G(−1;✓ 8✓) (by inspection; BD = DG)

or: $\frac{-9 + x_G}{2} = -5 \Rightarrow x_G = -1$✓ and $\frac{4 + y_G}{2} = 6 \Rightarrow y_G = 8$✓ (2)

[20]

QUESTION 4

4.1 Centre of circle is midpoint PQ = (−2;✓−2✓)

$r^2 = (-5 + 2)^2 + (-6 + 2)^2 = 25$

or: ✓✓

$r^2 = (1 + 2)^2 + (2 + 2)^2 = 25$

$(x - a)^2 + (y - b)^2 = r^2$

$(x + 2)^2 + (y + 2)^2 = 25$✓ (5)

4.2 $m_{\text{radius}} = \frac{2-(-2)}{1-(-2)} = \frac{4}{3}$✓

$m_{\text{tangent}} = -\frac{3}{4}$✓ (radius $\perp$ tangent)

$y - y_Q = m(x - x_Q)$ or $y = -\frac{3}{4}x + c$

$y - 2 = -\frac{3}{4}(x - 1)$✓ $2 = -\frac{3}{4}(1) + c$✓

$4y - 8 = -3x + 3$ $c = 2\frac{3}{4} = \frac{11}{4}$

$3x + 4y = 11$✓ $y = -\frac{3}{4}x + \frac{11}{4}$✓ [4]

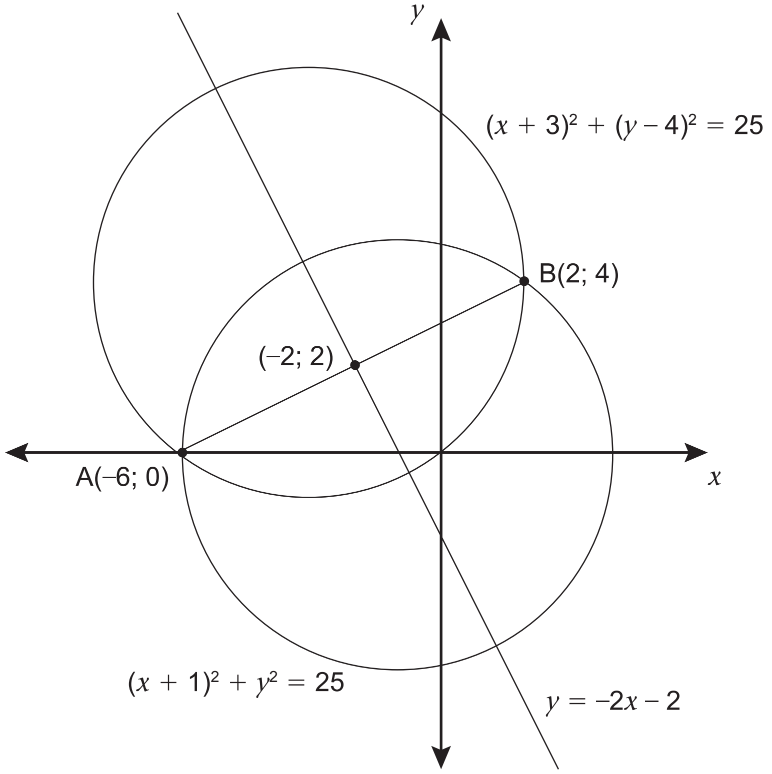
[9]

QUESTION 5

5.1 $m_{AB} = \frac{4}{8} = \frac{1}{2}$✓

$m_{\text{perp. bisector}} = -2$✓

Midpoint AB $= (-2; 2)$✓



$y - y_1 = m(x - x_1)$

$y - 2 = -2(x + 2)$✓

$y = -2x - 2$✓ (5)

5.2 The circle $(x - a)^2 + (y - b)^2 = 25$ passes through A$(-6; 0)$ and B$(2; 4)$.

The perpendicular bisector of AB✓ passes through the point $(a; b)$, the circle centre.

Substitute $(a; b)$ into $y = -2x - 2 \Rightarrow b = -2a - 2$✓

Substitute $b = -2a - 2$ and $(-6; 0)$:

$$(-6 - a)^2 + (-(-2a - 2))^2 = 25 \checkmark$$
$$36 + 12a + a^2 + 4a^2 + 8a + 4 = 25$$
$$5a^2 + 20a + 15 = 0$$
$$a^2 + 4a + 3 = 0 \checkmark$$
$$(a + 1)(a + 3) = 0 \checkmark$$
$$a = -1 \checkmark \quad \text{or} \quad a = -3 \checkmark$$
$$(-1; 0) \quad \text{or} \quad (-3; 4) \checkmark$$

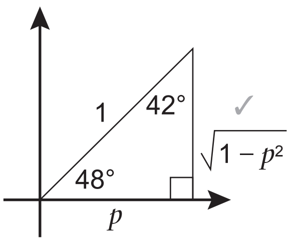
Circle centre

Equations: $(x + 1)^2 + y^2 = 25$ or $(x + 3)^2 + (y - 4)^2 = 25$✓ (9)

[14]

QUESTION 6

6.1 This is a proof and therefore must be set out correctly.



$$\text{LHS} = \frac{\cos(180° + x)\sin(90° - x) + \tan x \cos x \cos(90° + x)}{\sin(x - 20°)\cos(50° - x) + \cos(x - 20°)\sin(50° - x)}$$
$$= \frac{(-\cos x \checkmark)(\cos x \checkmark) + \left(\frac{\sin x}{\cos x}\checkmark\right)(\cos x)(-\sin x \checkmark)}{\sin((x - 20°) + (50° - x))}$$
$$= \frac{-\cos^2 x - \sin^2 x}{\sin 30° \checkmark}$$
$$= \frac{-(\cos^2 x + \sin^2 x)\checkmark}{\frac{1}{2}\checkmark}$$
$$= -2$$
$$= \text{RHS}$$ (7)

6.2.1 $\sin 42° = \cos 48° \checkmark = p \checkmark$ (2)

6.2.2 $\tan 48° = \frac{\sqrt{1 - p^2}}{p} \checkmark$ (2)

(one mark for the sketch)

6.2.3 $\cos 96° = \cos 2(48°) \checkmark$

$= 2\cos^2 48° - 1 \checkmark$

$= 2p^2 - 1 \checkmark$ (3)

6.2.4 $\text{LHS} = \frac{\cos 3A \cos A + \sin 3A \sin A \checkmark}{\sin A \cos A \checkmark}$

$$= \frac{\cos 2A \checkmark}{\sin A \cos A}$$
$$= \frac{\cos 2A}{\frac{1}{2}\sin 2A \checkmark}$$
$$= \frac{2}{\tan 2A}$$
$$= \text{RHS}$$ (4)

[18]

QUESTION 7

7.1 $\sin(2x + 60°) = -\cos 3x$

$-\sin(2x + 60°) = \cos 3x$

$\sin(-2x - 60°)$✓ $= \sin(90° - 3x)$✓

$-2x - 60° = 90° - 3x + n \times 360°;\ n \in \mathbb{Z}$✓

$x = 150° + n \times 360°$✓

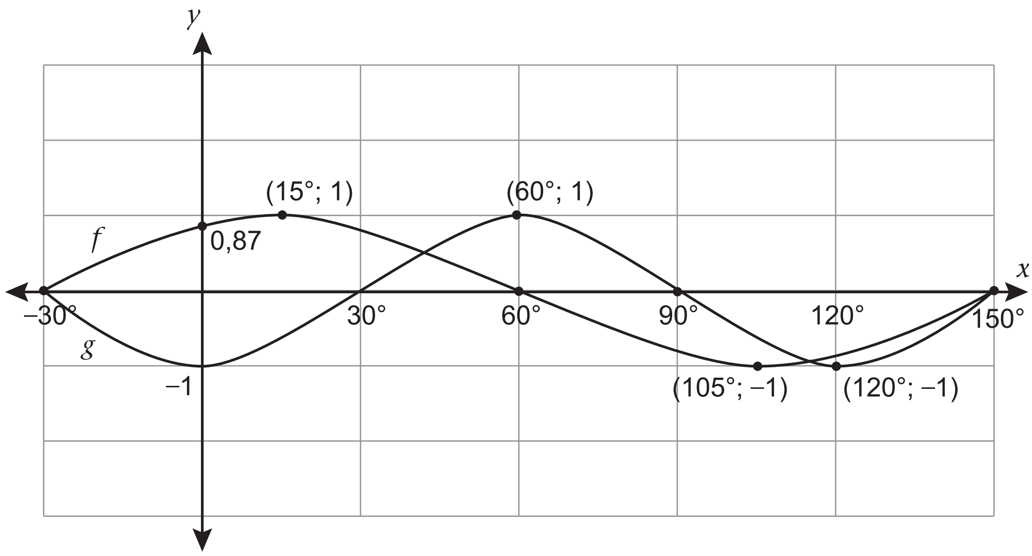
or

$-2x - 60° = 180° - (90° - 3x) + n \times 360°;\ n \in \mathbb{Z}$✓

$-5x = 150° + n \times 360°$✓

$x = -30°$✓ $- n \times 72°$✓ (8)

7.2



	$f(x) = \sin(2x + 60°)$	$g(x) = -\cos 3x$
TPs:	$(15°; 1); (105°; -1)$✓	$(0°; -1); (60°; 1); (-120°; -1)$✓
x-intercepts:	$(-30°; 0); (60°; 0); (150°; 0)$✓	$(-30°; 0); (30°; 0); (90°; 0); (150°; 0)$✓
y-intercept:	$(0°; 0{,}87)$✓	$(0°;\ 1)$✓

(6)

7.2.3 $x \in [42°;$✓ $114°]$✓ or ✓$42° \leq x \leq 114°$✓ (2)

[16]

QUESTION 8

8.1.1 AC = 13 cm✓ (Pythagoras in $\triangle$ABC) (1)

8.1.2 AE = 20 cm✓ (Pythagoras in $\triangle$AFE) (1)

8.1.3 CE = $\sqrt{281}$ cm✓ (1)

8.2 $\cos\theta = \dfrac{13^2 + 20^2 - 281}{2(13)(20)}$✓ $= \dfrac{36}{65}$✓

$\theta = 56{,}4°$✓ (3)

8.3 Area $\triangle$ACE $= \frac{1}{2}(13)(20)\sin 56{,}4°$✓ $= 108{,}28\text{ cm}^2$✓ (2)

[8]

QUESTION 9

9.1 Volume hemisphere : volume cone $= \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^2 h$✓

$= 2r : h$✓

$= 40 : 21$✓ (3)

9.2	AB = 29 units	(Pythagoras in $\triangle$ABC)	
	TSA $= 2\pi r^2 + \pi rs$ ✓		
	$= 2\pi(20)^2$ ✓ $+ \pi(20)(29)$ ✓		
	$= 1\ 380\pi$ ✓		(4)
			[7]

QUESTION 10

10.1.1	$\hat{C}_3 = 45°$ ✓	(tangent FD, chord AD) ✓	(2)
10.1.2	$\hat{D}_1 = 25°$ ✓	(tangent DE, chord DC) ✓	(2)
10.1.3	$\hat{D}_2 = 110°$	(straight line) ✓	
	$\hat{B}_2 = 110°$ ✓	(exterior $\angle$ cyclic quad.) ✓	(3)
10.1.4	$\hat{O}_2 = 220°$ ✓	($\angle$ at centre) ✓	(2)
10.2	In $\triangle$EDC and $\triangle$EAD: ✓		
	1) $\hat{E}$ is common. ✓		
	2) $\hat{D}_1 = D\hat{A}C = 25°$	(proved in 10.1.2) ✓	
	3) $E\hat{C}D = A\hat{D}E$	(sum of $\angle$s of $\triangle$ = 180°)	
	$\therefore \triangle EDC \ \vert\vert\vert\ \triangle EAD$	(AAA) ✓	
	$\frac{ED}{EA} = \frac{EC}{ED}$ ✓	($\triangle EDC \ \vert\vert\vert\ \triangle EAD$)	
	$ED^2 = EC.EA$		(5)
			(14)

QUESTION 11

11.2.1	$P\hat{Q}R = 90°$ ✓	($\angle$ in semi-circle) ✓	
	$P\hat{T}S = 90°$ ✓	(radius bisects chord) ✓	(4)
	$P\hat{Q}R = P\hat{T}S = 90°$		
11.2.2	Let $\hat{Q}_1 = x$.		
	$Q_2 = 90° - x$	(adjacent complementary $\angle$s) ✓	
	$P\hat{R}S = 90° - x$ ✓	($\angle$s in same segment; chord PS) ✓	
	$\hat{S} = x$	(exterior $\angle$ of $\triangle$STR) ✓	(4)
11.2.3	PR = 50 units ✓	(Pythagoras in $\triangle$PQR) ✓	(2)
11.2.4	$PT = 50 - x$ ✓		(1)
11.2.5	$QT^2 = TR \times PT$ ✓ $= x(50 - x)$ ✓		
	$= 50x - x$	($\triangle$PQR with $P\hat{Q}R = 90°$ and QT $\perp$ PR) ✓	(3)
11.2.6	$30^2 = x^2 + x(50 - x)$ ✓	(Pythagoras in $\triangle$TQR)	
	$900 = x^2 + 50x - x^2$ ✓		
	$x = 18$ ✓ units		(3)
			[25]

TOTAL: [150]

PAPER 1B