Western Cape Government
Education

Directorate: Curriculum FET

MATHEMATICS

REVISION BOOKLET
2020 TERM 2

Grade 12

This revision program is designed to assist you in revising the critical content and skills covered during the 2nd term. The purpose is to prepare you to understand the key concepts and to provide you with an opportunity to establish the required standard and the application of the knowledge necessary to succeed in the NCS examination.

The revision program covers the following topics:

- Analytical Geometry(± 40 of 150 marks in Paper 2)
- Functions and Inverse Functions (± 35 of 150 marks in Paper 1)
- Differential Calculus (± 35 of 150 marks in Paper 1)

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Dear Learner

In the Term 2 revision material you will be revising:

- Analytical Geometry: ± 40 of 150 marks of Paper 2
- Functions and Graphs (inverse functions): ± 35 of 150 marks of Paper 1
- Calculus: ± 35 of 150 marks of Paper 1

Each of the above topics is compiled as follows:

- Summary of the key concepts in the topic.
- Section A, essentially questions which you must ensure you are able to do, to at least obtain 30% in Mathematics.
- Section B, these are questions at a higher level
- Section C, where it is available like with Functions and graphs are question from the grade 12 examination papers, focusing on content on the topic done in grade 11.

Please ensure that you work thoroughly through the summary of key concepts at the beginning of each topic. This is essential before embarking on section A, B and C. If anything in the notes are not understood it is important that you get support with it. Any challenges experienced indicates some shortcoming in the understanding of certain concepts which will hamper the completion of questions on the topic. Ensure that you get support from a tutor, teacher, consult your textbook or search online for support. You could also use the Tutonic concept videos to support with your understanding of certain concepts.

https://bit.ly/tutonic_AG

By clicking on the link, https://bit.ly/tutonic_AG you will access a Tutonic concept mindmap with videos on Analytical Geometry.

By clicking on the link, https://bit.ly/tutonic_functions you will access a Tutonic concept mindmap with videos of Functions and Graphs. This mindmap covers Functions and Graphs, Inverse Functions and Calculus.

https://bit.ly/tutonic_functions

By clicking on the little balls on the above two mindmaps you will access a video explaining the concept listed next to it.

After having worked through the topic summary start doing the questions on the topic. Please check your answers as you progress through the material. It is important to know whether your answers are correct before progressing to the next question. You will learn a lot by looking at the solutions also look at how solutions are set out. If you are not able to even start a particular question, look at the memorandum, it is all part of the learning process. Mark the question and make sure you attempt it again a day later. Please spend at least an 1 hour every day on revising Mathematics.

Thank you for working and revising your Mathematics daily in preparing for the examination. Working regularly at Mathematics is extremely important to realising your success in this subject.

All of the best!!!!

ANALYTICAL GEOMETRY

SUMMARY:

1. SUMMARY AND KNOWLEDGE FROM PREVIOUS GRADES

FORMULAE/CONCEPTS	Summary of procedure	EXAMPLE
Consider any two points $(x_1;y_1)$ and $(x_2;y_2)$ in a cartesian plane		Suppose $A(-3;\ -5)$ and $B\ (2;\ 3)$ are points in a cartesian plane.
Distance or Length of line Formula: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$	Label one point as point one and the other as point 2 and then substitute into the formula.	Determine the length AB: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $d_{AB} = \sqrt{((2)-(-3))^2+((3)-(-5))^2}$ $d_{AB} = \sqrt{(2+3))^2+(3+5)^2}$ $d_{AB} = \sqrt{(5))^2+(8)^2} = \sqrt{89} = 9,\ 43$ units
Midpoint of a line: $(\frac{x_2+x_1}{2};\frac{y_2+y_1}{2})$	Label one point as point one and the other as point 2 and then substitute into the formula.	Determine the coordinates of E the midpoint of AB. $\left(\frac{x_1+x_2}{2};\frac{y_1+y_2}{2}\right)$ $E\left(\frac{-3+2}{2};\frac{-5+3}{2}\right)$ $E\left(\frac{-1}{2};\frac{-2}{2}\right)$ $= E\left(\frac{-1}{2};-1\right)$
Gradient: $m = \frac{y_2-y_1}{x_2-x_1}$	Label one point as point one and the other as point 2 and then substitute into the formula.	Determine the gradient of AB. $m = \frac{y_2-y_1}{x_2-x_1}$ $m_{AB} = \frac{(3)-(-5)}{(2)-(-3)} = \frac{3+5}{2+3} = \frac{8}{5}$
Parallel Lines: **The gradients of the lines are equal i.e.** $\boldsymbol{m_1 = m_2}$	Determine the gradient of one line then the parallel lines gradient is the same.	Determine the gradient of line CD if $CD \parallel AB$. $m_{AB} = \frac{8}{5}$ $\therefore m_{CD} = \frac{8}{5}$ because $CD \parallel AB$
Perpendicular Lines **the product of the gradients is -1 i.e.** $\boldsymbol{m_1 \times m_2 = -1}$	Take the reciprocal of the gradient of the given line and change the sign. Given $m_1 = +\frac{2}{3}$ then $m_2 = -\frac{3}{2}$	Determine the gradient of line CD if $CD \perp AB$. $m_{AB} = +\frac{8}{5}$ $\therefore m_{CD} = -\frac{5}{8}$ because $CD \perp AB$
Co-linear Points A, B and C $m_{AB} = m_{BC}$	Calculate the gradient of segments one point must be common. i.e. $m_{AB} = m_{BC}$	Determine whether the points $A(-3;\ -5)$, $B\ (2;\ 3)$ and C(7; 11) are collinear. $m_{AB} = \frac{8}{5}$ $m_{BC} = \frac{y_3-y_2}{x_3-x_2}$ $m_{BC} = \frac{(11)-(3)}{(7)-(2)}$ $m_{BC} = \frac{(11-3)}{(7-2)}$ $m_{BC} = \frac{8}{5}$ Hence A, B and C are collinear.

<table>
<tr>
<td>Gradient (inclination) of a line

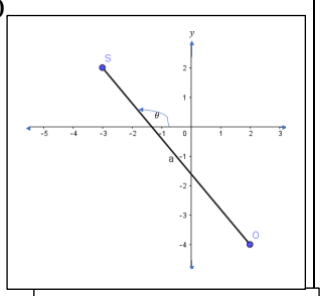
$tan\ \theta = m_{AB}$

θ is the angle of inclination

$m > 0 \rightarrow \quad \theta < 90°$

$tan\ \theta = m_{AB}$

$m < 0 \rightarrow 90° < \theta < 180°$</td>
<td>Use calculator to determine $\tan^{-1}(m)$.
Use the positive value of m. If m is negative, the angle you obtained must be subtracted from 180°.</td>
<td>(a) Determine the inclination of AB
$m_{AB} = \frac{8}{5}$ and $\tan\theta = \boldsymbol{m_{AB}}$
$\therefore \tan\theta = \frac{8}{5}$ To get θ , arc(shift) tan $\frac{8}{5}$,
$\theta = 57{,}99°$

(b) Given S (-3; 2) and O (2; - 4) determine the inclination of SO
 
$m_{SO} = \frac{y_2 - y_1}{x_2 - x_1}$
$m_{SO} = \frac{(-4) - (2)}{(2) - (-3)}$
$m_{SO} = \frac{(-4 - 2)}{(2 + 3)}$
$m_{SO} = \frac{-6}{5}$
$\tan\theta = \boldsymbol{m_{SO}}$
NB! Do not use the (–) sign when you calculating the angle
$\therefore \tan\theta = -\frac{6}{5}$ To get θ , arc(shift) tan $\frac{6}{5}$,

Reference $angle = 50{,}19°$
$\therefore \theta = 180° - 50{,}19° = 129{,}8$</td>
</tr>
<tr>
<td colspan="3">Equation of a line</td>
</tr>
<tr>
<td>$y = mx + c$</td>
<td>Use when m and c is known.</td>
<td>Determine the equation of the line with gradient $\frac{8}{5}$ and y-intercept -3.

Equation of the line is: $y = \frac{8}{5}x - 3$</td>
</tr>
<tr>
<td>$y - y_1 = m(x - x_1)$</td>
<td>Use when 1 point and a gradient is given.</td>
<td>Determine the equation of the line through $P(4; 3)$ with gradient -1.

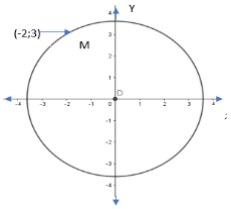
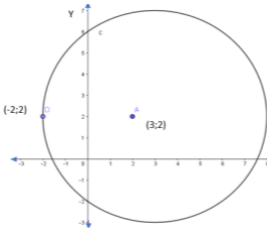
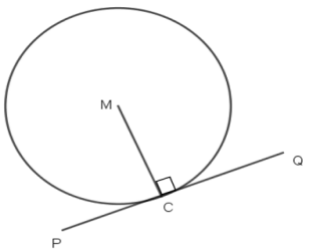
$y - y_1 = m(x - x_1)$
$\therefore \quad y - 3 = -1(x - 4)$
$\therefore \quad y - 3 = -x + 4$
$\therefore \quad y = -x + 7$</td>
</tr>
<tr>
<td>$\boldsymbol{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$</td>
<td>Use when 2 points are given.</td>
<td>Determine the equation of the line passing through two points $A(-3; -5)$ and $B(2; 3)$

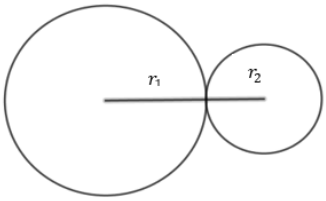
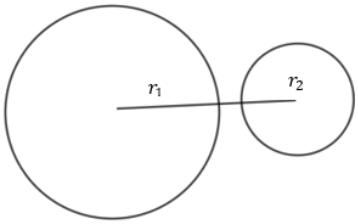
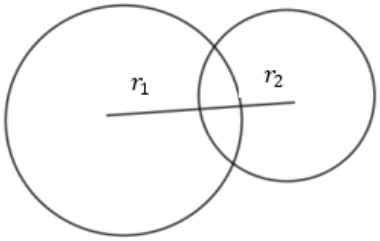
$\boldsymbol{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$
$y - (-5) = \frac{3-(-5)}{2-(-3)}(x - (-3))$
$y + 5 = \frac{3+5}{2+3}(x + 3)$
$y + 5 = \frac{8}{5}(x + 3)$
$y = \frac{8}{5}x + \frac{24}{5} - 5$
$y = \frac{8}{5}x - \frac{1}{5}$</td>
</tr>
</table>

In Analytical Geometry questions can be asked which require you to know the properties of the various quadrilaterals. The properties of the various quadrilaterals you have learnt in grade 10 and is given in the table below. The table below captures the various properties you have to show a particular quadrilateral has for it to be a Parallelogram, Rectangle, Rhombus, Square, Trapezium or Kite.

Quadrilaterals	Used to prove whether a four-sided figure is the indicated quadrilateral that is Parallelogram Rectangle, Rhombus, Square, Trapezium or Kite.
A. Parallelogram	• 1 pair of opposite sides are = and // • 2 pairs of opposite sides are = • 2 pairs of opposite sides are // • 2 pairs of opposite ∠'s are = Diagonals bisect each other **any one**
B. Rectangle	• prove that the figure is a parallelogram + 1 angle = 90° + diagonals are = **+ any one**
C. Rhombus	• prove that the figure is a parallelogram + diagonals cut ⊥ + all sides are = + diagonals bisect ∠'s **+ any one**
D. Square	• prove that the figure is a parallelogram + 1 angle = 90° + diagonals are = **+ any one of these** and + diagonals cut ⊥ + all sides are = **+ any one of these**
E. Trapezium	1 pair opposite sides are //
F. Kite	• 1 pair of opposite sides are = • diagonals cut ⊥ **any 1**

2. SUMMARY OF GRADE 12 CONCEPTS

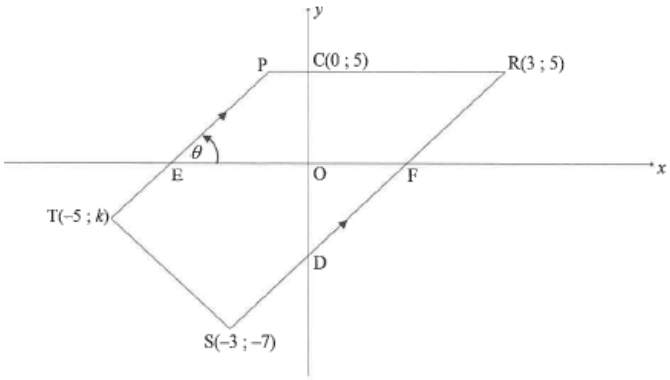
FORMULA/CONCEPTS	SUMMARY OF PROCEDURE	EXAMPLE
Equation of a circle with centre $(0;0)$ and radius r is: $x^2 + y^2 = r^2$	**Substitute the point on the circle to determine r^2.**	Determine the equation of the circle with centre D (0; 0) passing through (-2; 3)  $x^2 + y^2 = r^2$ Circle formula $r^2 = 13$ $x^2 + y^2 = 13$
Equation of a circle with centre $(a;b)$ and radius r $(x-a)^2 + (y-b)^2 = r^2$	Need to determine the centre of the circle if it is not given as well as the radius r.	Determine the equation of the circle with centre at A(3; 2) and D (-2; 2) on the circumference.  $(x-a)^2 + (y-b)^2 = r^2$ Circle formula $r = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $r = \sqrt{(-2-3)^2 + (2-2)^2}$ $r = \sqrt{(-5)^2 + (0)^2}$ $r = \sqrt{25} = 5$ $(x-3)^2 + (y-2)^2 = 25$
Tangent to a circle : $y = mx + c$	You require the gradient of the line joining the centre of the circle and the point of contact. Use $m_{tangent}.m_{radius} = -1$ to calculate the gradient of the tangent. Substitute $m_{tangent}$ and the coordinates of the point of contact into the equation of a straight line 	Find the equation of the tangent to the circle with Centre M(-2; 1) and C (3; -1) MC ⊥PQ. **Solution:** $m_{MC} = \frac{y_2 - y_1}{x_2 - x_1}$ $m_{MC} = \frac{(-1)-(1)}{(3)-(-2)} = \frac{-1-1}{3+2} = \frac{-2}{5}$ $m_{PQ} = +\frac{5}{2}$ because MC ⊥PQ. and C(3; -1) is on the tangent $\therefore \quad y - y_1 = m(x - x_1)$ $y - (-1) = (\frac{5}{2})(x-3)$ $y + 1 = \frac{5}{2}x - \frac{15}{2}$ $y = \frac{5}{2}x - \frac{15}{2} + 1$ $y = \frac{5}{2}x - \frac{13}{2}$

Algebraic manipulation to finding the centre of a circle.	Determine the centre of the circle • Group similar variables together, constant term on the RHS • Add a third term to each bracket on LHS so that a perfect square can be obtained. • What you added on LHS you also add on the RHS. • On the LHS each trinomial is written as a perfect square, while the RHS is simplified	Given the equation of a circle: $x^2 + y^2 + 2x - 12y - 12 = 0$ • Group similar variables together, constant term on the RHS $(x^2 + 2x) + (y^2 - 12y) = 12$ • Add a third term to each bracket on LHS so that a perfect square can be obtained. On the RHS add the terms included in each bracket. $(x^2 + 2x+1) + (y^2 - 12y +36) = 12+1+36$ • On the LHS each trinomial is written as a perfect square, while the RHS is simplified $(x + 1)^2 + (y - 6)^2 = 49$ Centre of the circle $(a; b)$ = (-1; 6)
To Determine if two circles **a) touches** **b) do not touch** **c) intersect** **Distance (D)between centres**	Determine the radius of each circle. Then the sum of the radiu i.e $r_1 + r_2$.	a) If two circles are touching then. $D = r_1 + r_2$  b) If two circles do not touch then $D > r_1 + r_2$  c) If two circles overlap then $D < r_1 + r_2$ 

SECTION A

QUESTION 1 **DBE NOV 2019**

1. In the diagram P, $R(3;5)$, , $S(-3;-7)$ and $T(-5;k)$ are vertices of trapezium PRST and PT ∥ RS. RS and PR cut the $y-$ axis at D and $C(0;5)$ respectively. PT and RS cut the $x-$ axis at E and F respectively. P$\hat{E}$F = θ.



1.1 Write down the equation of PR. (1)

1.2 Calculate the:

1.2.1 Gradient of RS. (2)

1.2.2 Size of θ (3)

1.2.3 Coordinates of D (3)

1.3 If it is given that TS $= 2\sqrt{5}$, calculate the value of k. (4)

1.4 Parallelogram TDNS, with N in the 4th quadrant, is drawn. Calculate the coordinates of N. (3)

1.5 ΔPRD is reflected in the $y-$ axis to form ΔP′R′D′. Calculate the size of $\angle RDR'$. (3)

[19]

QUESTION 2 **DBE Feb-Mar 2018**

In the diagram P, $Q(-7;-2)$, R and $S(3;6)$ are vertices of a quadrilateral. R is a point on the $x-$axis. QR is produced to N such that QR $= 2$RN. SN is drawn. $\angle PTO = 71{,}57°$ and $\angle SRN = \theta$.

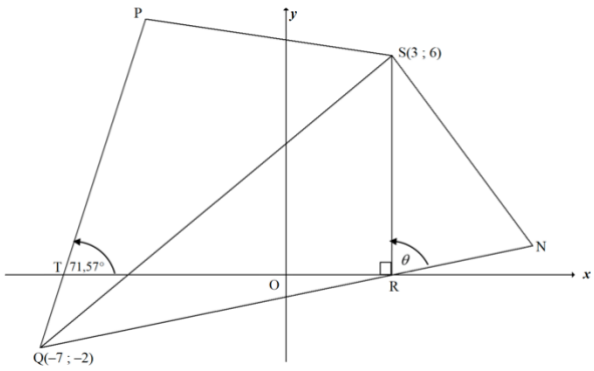
Determine:

2.1 The equation of SR (1)

2.2 The gradient of QP to the nearest integer. (2)

2.3 The equation of QP in the form $y = mx + c$ (2)

2.4. The length of QR. Leave your answer in **surd form.** **(2)**



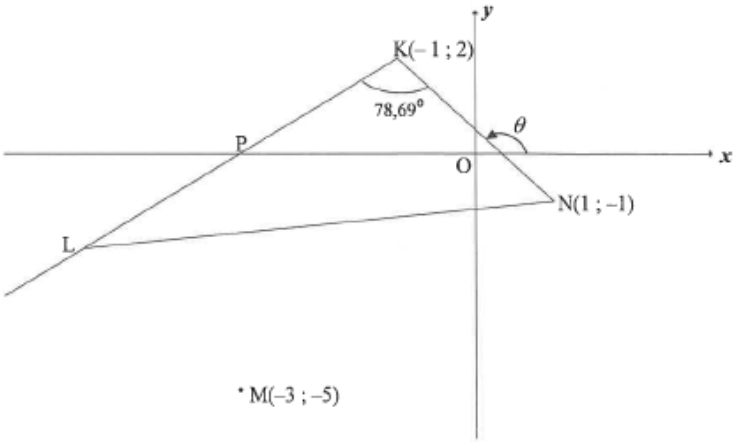
2.5 $\tan(90° - \theta)$ (3)

2.6 The area of Δ RSN, **without using a calculator.** (6)

[16]

QUESTION 3 **DBE NOV 2018**

In the diagram K$(-1;2)$, L and N$(1;-1)$ are vertices of Δ KLN such that $\angle LKN = 78{,}69°$. KL intersects the $x-$ axis at P. KL is produced. The inclination of KN is θ. The coordinates of M are $(-3;-5)$.



3.1 Calculate

3.1.1 The gradient of KN. (2)

3.1.2 The size of θ, the inclination of KN (2)

3.2 Show that the gradient of KL is equal to -1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

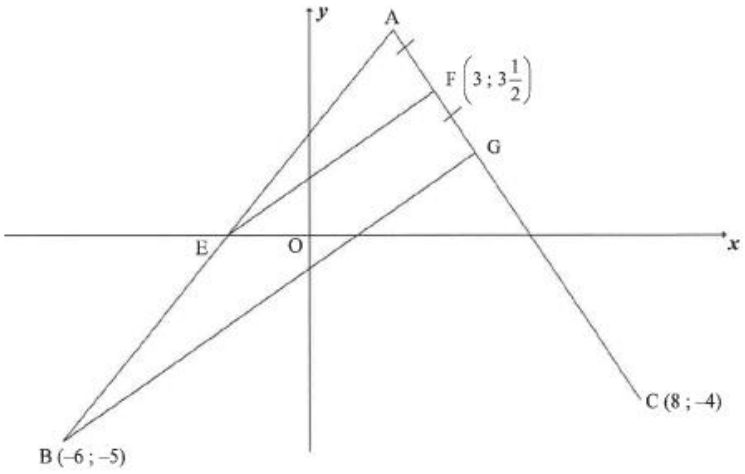
3.4 Calculate the length of KN. (2)

3.5 It is further given KN = LM.
3.5.1 Calculate the possible coordinates of L. (5)
3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram (3)

3.6 T is a point on KL produced. TM is drawn such that TM = LM.
Calculate the area of ΔKTN. (4)
[22]

QUESTION 4 **DBE NOV 2017**

In the diagram, A, B$(-6;-5)$ and C$(8;-4)$ are points in the Cartesian plane. F$(3;3\frac{1}{2})$ and G are points on the line AC such that AF = FG. E is the $x-$ intercept of AB.



4.1 Calculate:

4.1.1 The equation of AC, in the form $y = mx + c$. (4)

4.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$ (3)

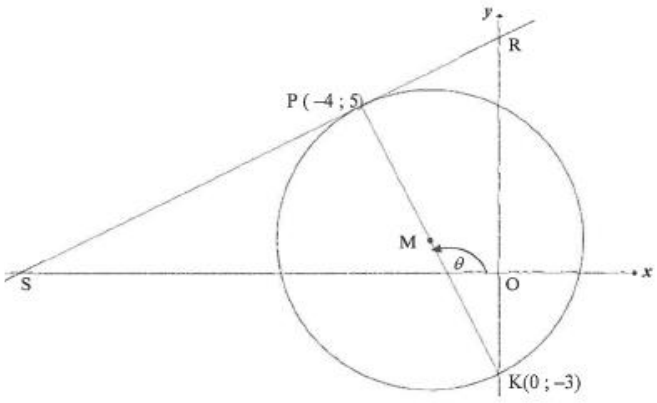
4.2 Show by calculation that the coordinates of A is $(2;5)$. (2)

4.3 Prove that $EF \parallel BG$. (4)

4.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)
[17]

QUESTION 5 **DBE NOV 2017**

In the diagram, P$(-4;5)$ and K$(0;-3)$ are the end points of the diameter of the circle with centre M. S and R are respectively the $x-$ and $y-$ intercepts of the tangent to the circle at P. θ is the inclination of PK with the positive $x-$axis.



5.1 Determine

5.1.1 The The gradient of SR (4)

5.1.2 The equation of SR in the form $y = mx + c$ (3)

5.1.3 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

5.1.4 The size of ∠PKR (3)

5.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$. (2)

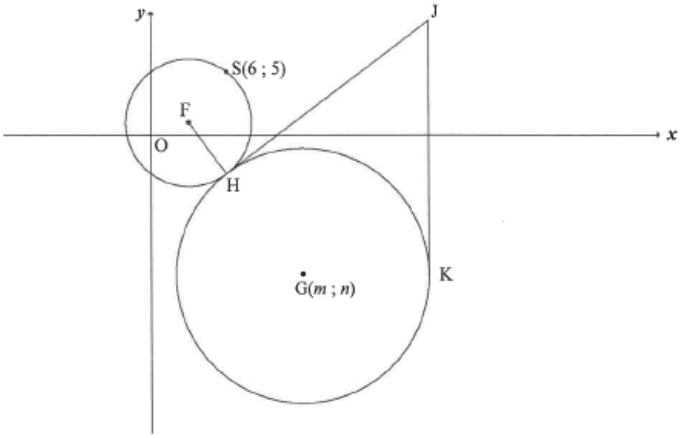
5.2 Determine the value(s) of t such that the line $y = \frac{1}{2}x + t$ cuts the circle at two different points. (3)

5.3 Calculate the area of ΔSMK. (5)

[23]

QUESTION 6 **DBE NOV 2018**

In the diagram, the equation of the circle with centre F is $(x-3)^2 + (y-1)^2 = r^2$.
S$(6;5)$, is a point on the circle with centre F. Another circle with centre G$(m;n)$ in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2 The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



6.1 Write down the coordinates of F. (2)

6.2. Calculate the length of FS. (2)

6.3 Write down the length of HG. (1)

6.4 Give a reason why JH = JK. (1)

6.5 Determine:

6.5.1 The distance FJ, with reasons, if it is given that JK $= 20$. (4)

6.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2 + (y-b)^2 = r^2$ (1)

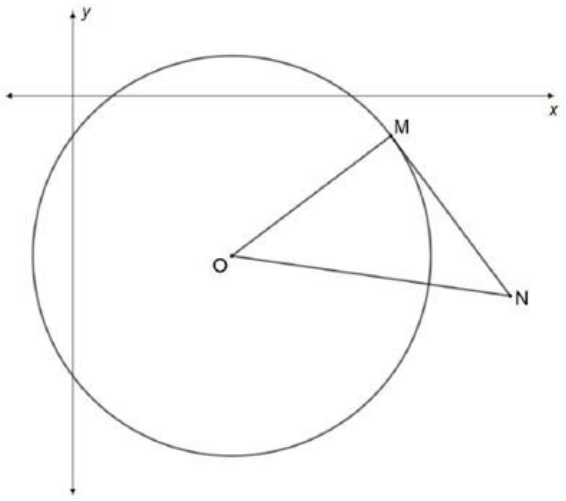
6.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x = 22$ (7)

SECTION B

QUESTION 1 **IEB NOV. 2018 (Question 2)**

In the diagram below, circle with centre O is drawn in the Cartesian plane.

- MN is a tangent to the circle at M.
- N is a point outside the circle with co-ordinates $N(11;-5)$



1.1 Write down the size of $\angle OMN$.
Give a reason for your answer. (3)

1.2 If the equation of the circle is. $x^2 - 8x + (y+4)^2 = 9$ then:

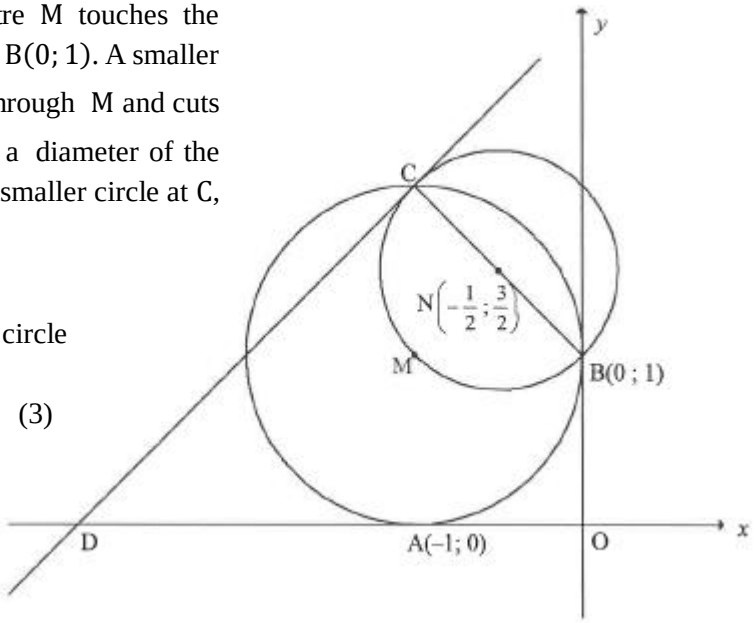
1.2.1. Determine the coordinates of O. (2)

1.2.2. Determine the length of OM. (2)

1.3. Calculate the length of MN. (4)

QUESTION 2 **DBE NOV 2019(Question 4)**

In the diagram, a circle having centre M touches the x −axis at $A(-1;0)$ and the y −axis at $B(0;1)$. A smaller circle, centred at $N\left(-\frac{1}{2};\frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C. BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C, cuts the x −axis at D.



2.1 Determine the equation of the circle centred at M in the form of $(x-a)^2 + (y-b)^2 = r^2$ (3)

2.2 Calculate the coordinates of C. (2)

2.3 Show that the equation of the tangent CD is $y - x = 3$. (4)

2.4 Determine the values of t for which the line $y = x + t$ will NOT touch or cut the smaller circle. (3)

2.5 The smaller circle centred at N is transformed such that point C is translated along the tangent to D. Calculate the coordinates of E, the new centre of the smaller circle. (3)

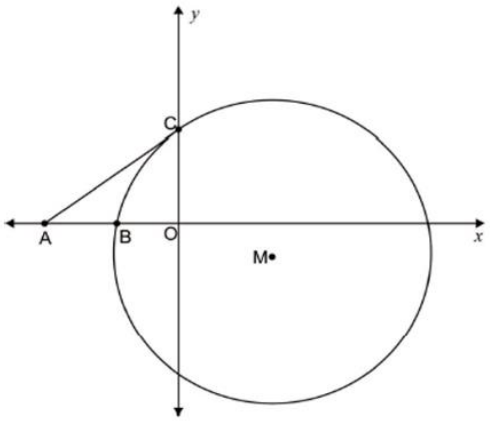
2.6 If it is given the area of quadrilateral OBCD is $2a^2$ square units and $a > 0$, show that $a = \frac{\sqrt{7}}{2}$ units. (5)

[20]

QUESTION 3 **IEB NOV 2017 (Question 4)**

In the Cartesian plane below, circle centre M is drawn.

- A is a point on the x −axis.
- Point at B lies on the circle and the x −axis.
- Point at C lies on the circle and the y −axis.
- The equation of the circle is. $(x-3)^2+(y+1)^2=25$.
- The line AC is a tangent to the circle at C.



3.1 Write the coordinates of M. (1)

3.2 Determine the coordinates of point C (3)

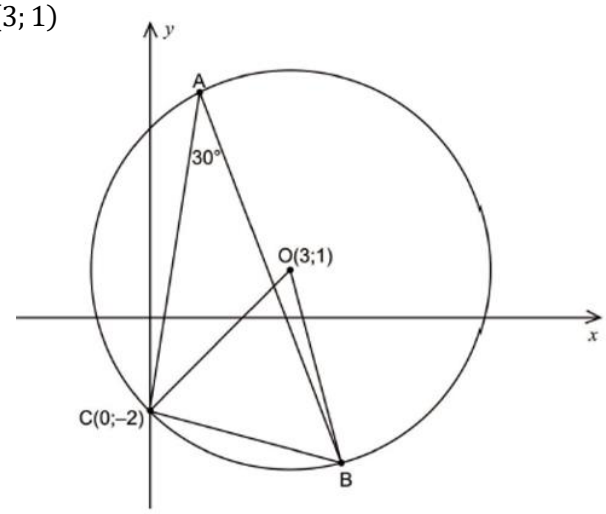
3.3 Determine the equation of the tangent AC. (3)

3.4. Determine the length of AB. Leave your answer correct to one decimal place. (4)
[11]

QUESTION 4 **IEB NOV 2017 (Question 8)**

In the Cartesian plane below, circle having centre at $O(3;1)$

- A and C(0; -2) are fixed points on the circle.
- $\angle CAB = 30°$.
- B is a variable point on the circle.



4.1 4.1.1 Determine the length of OC. (3)

4.1.2 BC moves along the circle until BC is parallel to the $x-$ axis. Write down the new coordinates of B. (4)

4.1.3 Calculate the size of $\angle CAB$ for this new position of B. Give all the reasons. (4)

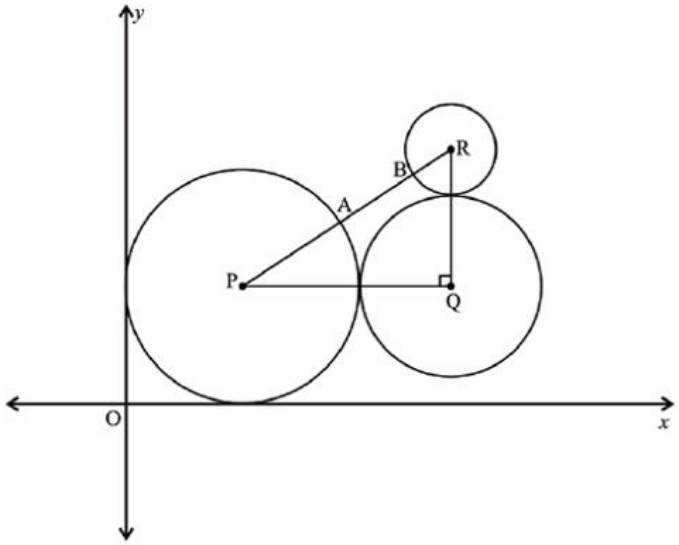
4.2 B moves from its original position along the circle in an anti-clockwise direction until the area Δ OBC $= \frac{9}{2}$ square units.
Find the shortest distance that B has to move along the circle for the above to occur. (6)
[17]

QUESTION 5 **IEB NOV 2016 (Question 3)**

In the diagram below:

- Circle P touches the x −axis and the y −axis.
- Circle P touches circle Q at one point.
- Circle Q touches circle R at one point.
- Circle P has a radius of 5 units.
- PQ is parallel to the x −axis and RQ is parallel to the y −axis

PR cuts the circles with centres P and R at A and B respectively.



5.1 If PQ = 9 units, determine the equation of the circle centre Q. (4)

5.2 Find the length of line QR if the equation of the circle centre R is $(x-p)^2 + y^2 - 22y = -117$. (3)

5.3 Determine the length of AB, correct to two decimal places. (4)

[11]

FUNCTIONS

SUMMARY:

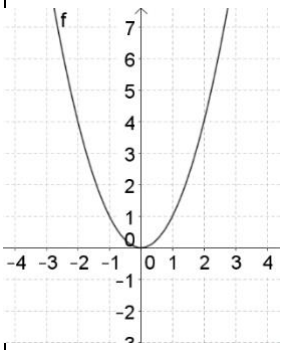
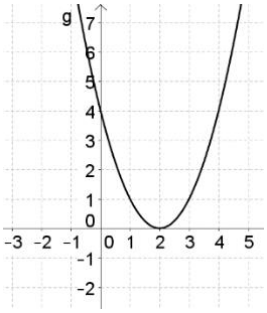
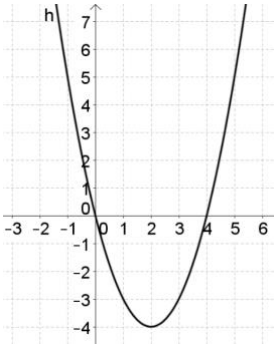
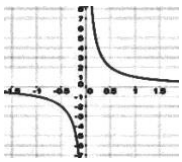
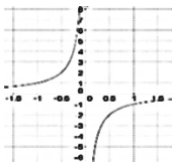
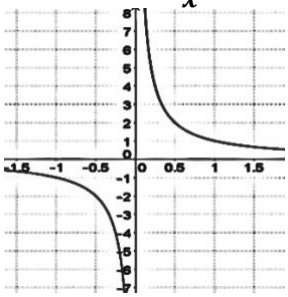
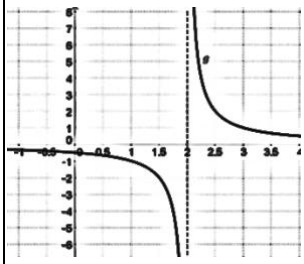
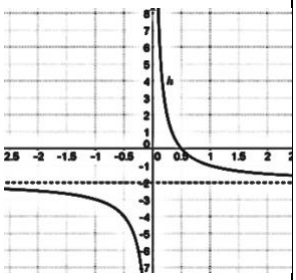
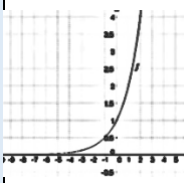
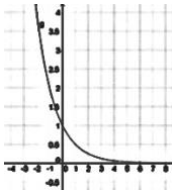
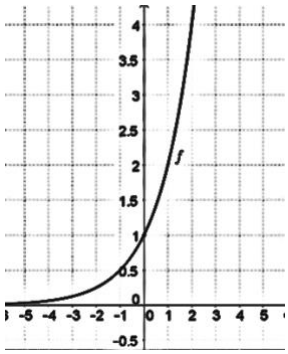
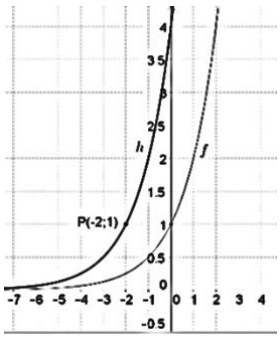
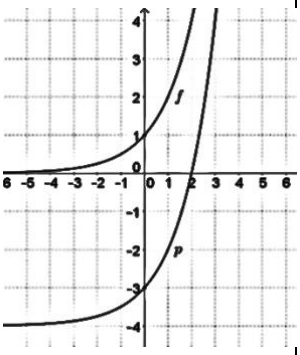
1. CAPS EXTRACTION INDICATING PROGRESSION FROM GRADES 10-12

Grade 10	Grade 11	Grade 12
1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations. Note: that the graph defined by $y = x$ should be known from Grade 9. 2. Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable). 3. Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0, b \neq 1$.	1. Revise the effect of the parameters *a* and *q* and investigate the effect of *p* on the graphs of the functions defined by: 1.1. $y = f(x) = a(x + p)^2 + q$ 1.2. $y = f(x) = \frac{a}{x + p} + q$ 1.3. $y = f(x) = ab^{x + p} + q$ where $b>0$, $b\neq1$ 2. Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point.	1. Definition of a *function*. 2. General concept of the *inverse of a function* and how the domain of the function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function. 3. Determine and sketch graphs of the inverses of the functions defined by $y = ax + q$; $y = ax^2$ $y = b^x; (b > 0, b \neq 1)$ Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average gradient (average rate of change), intervals on which the function increases /decreases.

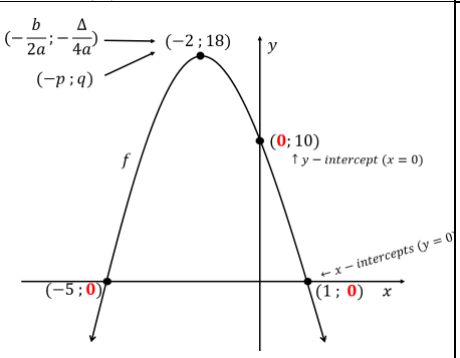
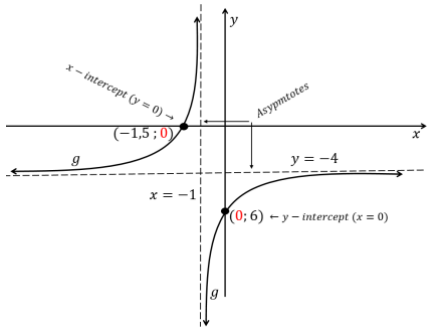
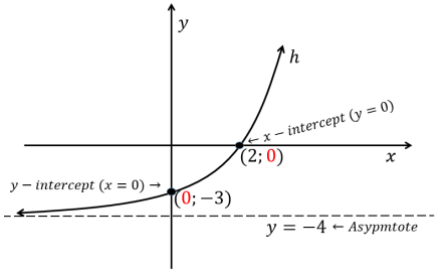
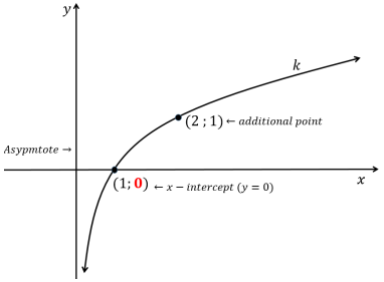
2. FUNCTIONS: TERMINOLOGY

- A **function** is a relation in which each input value $(x - value)$ has only one output value $(y - value)$.
  The vertical line test can be used to determine if a relation is a function or not.
  If the vertical line cuts the graph at only one point, the graph represents a function; if it cuts at more than one point then the graph does not represent a function.
- **Functional notation:** The $f\ (x)$ **notation** is another way of representing the $y-$value in a **function**, $y\ =\ f\ (x)$.
- The **inverse** of a **function** is obtained by reflecting the graph f about the line $y = x$.
  The inverse of f, is written as f^{-1}. To find the inverse of a function **swop** the $x-$ and the $y-$ values and make y the subject of the formula.
- The **domain** is the set of all possible x-values which will make the **function** "work", and will output real $y-$values.
- The **range** of a function is the possible values of y.

3. MOTHER FUNCTION AND INFLUENCE OF PARAMETERS

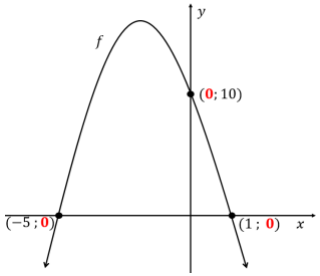
	Influence of	Mother function	Influence of p and q	
PARABOLA	$a > 0$  $a < 0$ 	$f(x) = x^2$ 	$g(x) = (x-2)^2$  Moved to 2 units to the right ($p < 0$)	$h(x) = (x-2)^2 - 4$  Moved 4 units down ($q < 0$)
HYPERBOLA	$a > 0$ **1st and 3rd quadrant**  $a < 0$ **2nd and 4th quadrant** 	$f(x) = \frac{1}{x}$ 	$g(x) = \frac{1}{x-2}$  Moved to 2 units to the right ($p < 0$)	$h(x) = \frac{1}{x} - 2$  Moved 2 units down ($q < 0$)
EXPONENTIAL GRAPH $a > 0$ **Above** $x-$**axis** $a < 0$ **Below** $x-$**axis**	$b > 1$ **increasing**  $0 < b < 1$ **decreasing** 	$f(x) = 2^x$ 	$h(x) = 2^{x+2}$  Moved to 2 units to the left ($p > 0$)	$p(x) = 2^x - 4$  Moved 4 units down ($q < 0$)

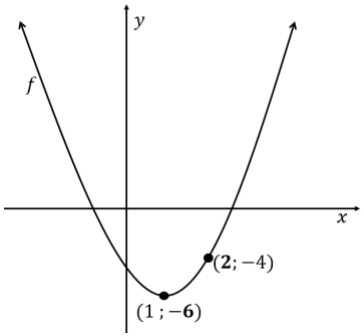
4. SKETCH OF GRAPHS

	Parabola	Hyperbola	Exponential Graph	Logarithmic graph
Standard form	$y = ax^2 + bx + c$ $y = a(x + p)^2 + q$ $y = a(x - x_1)(x - x_2)$	$y = \frac{a}{x+p} + q$	$y = a.b^{x+p} + q$	$y = log_a x$
To sketch the graphs, you need: y −intercept $(x = 0)$ x −intercept $(y = 0)$ Turning point $\left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right)$ **OR** $(-p; q)$ Asymptote(s) • $x = -p$ • $y = q$	Example 1 Sketch $f(x) = -2x^2 - 8x + 10$ **OR** $f(x) = -2(x + 2)^2 + 18$	Example 2 Sketch $g(x) = \frac{-2}{x+1} - 4$	Example 3 Sketch $h(x) = 2^x - 4$	Example 4 Sketch $k(x) = log_2 x$
				
Domain	$x \in R$	$x \in R\ ; x \neq -1$	$x \in R$	$x \in R\ ; x > 0$
Range	$y \in R\ ; y \leq 18$	$y \in R\ ; y \neq -4$	$y \in R\ ; y > -4$	$y \in R$
Symmetry axis	$x = -\frac{b}{2a}$ $x = -2$	$y = x + c$ Substitute $(-p; q) \rightarrow (-1; -4)$ $\therefore -4 = -1 + c$ $c = -3$ $y = x - 3$ OR $y = x - +c$ Substitute $(-p; q) \rightarrow (-1; -4)$ $\therefore -4 = -(-1) + c$ $c = -5$ $y = -x - 5$		

5. TO DETERMINE THE EQUATION OF A GRAPH

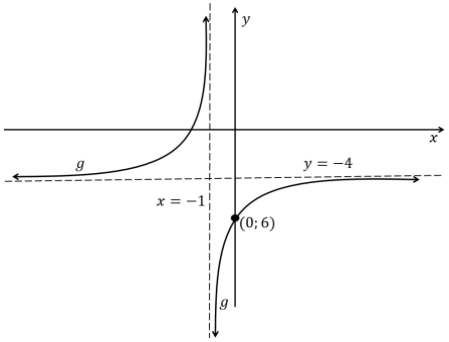
<table>
<tr><th colspan="2">Parabola</th><th>Hyperbola</th><th>Exponential Graph</th><th>Logarithmic graph</th></tr>
<tr><td colspan="2">$y = ax^2 + bx + c$
$y = a(x + p)^2 + q$
$y = a(x - x_1)(x - x_2)$</td><td>$y = \frac{a}{x+p} + q$</td><td>$y = a.b^{x+p} + q$</td><td>$y = log_a x$</td></tr>
<tr><td colspan="2"></td><td></td><td></td><td></td></tr>
<tr><td>x –intercepts given:
$y = a(x - x_1)(x - x_2)$

For a:
Substitute another point given on the graph into the equation.
 

$y = a(x + 5)(x - 1)$
Subst. $(0; 10)$
$10 = a(0 + 5)(0 - 1)$
$10 = -5a$
$a = -2$
$y = -2(x + 5)(x - 1)$
$y = -2x^2 - 8x + 10$</td><td>Turning point given:
$y = a(x + p)^2 + q$
For a:
Substitute a point given on the graph into the equation.
 

$y = a(x - 1)^2 - 6$
Subst. $(2; -4)$:
$-4 = a(2 - 1)^2 - 6$
$2 = a(1)^2$
$a = 2$
$y = 2(x - 1)^2 - 6$

OR
$y = 2x^2 - 4x - 4$</td><td>For p and q:
Start by substituting the asymptotes into the equation.

For a:
Substitute a point given on the graph into the equation.
 

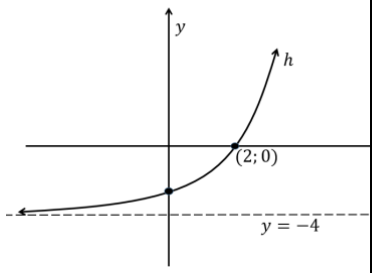
$y = \frac{a}{x+1} - 4$

subst. $(0; -6)$
$-6 = \frac{a}{0+1} - 4$

$-2 = a$

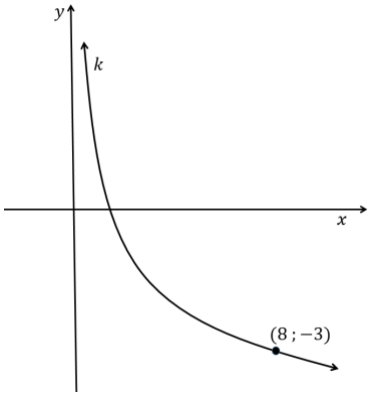
$y = \frac{-2}{x+1} - 4$</td><td>For q:
Start by substituting the asymptote into the equation.

For any other:
Substitute a point given on the graph into the equation.

For this example, given $y = a^x + q$
 

$y = a^x - 4$

$subst.(2; 0)$:
$0 = a^2 - 4$
$a^2 = 4$
$a = 2$

$y = 2^x - 4$</td><td>For a:
Substitute a point given on the graph into the equation.
 

$y = log_a x$
Subst. $(8; -3)$

$-3 = log_a 8$

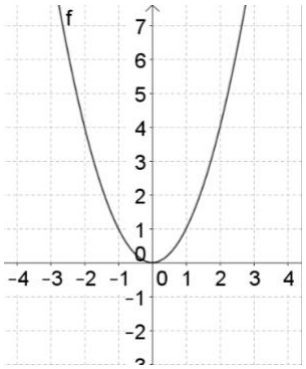
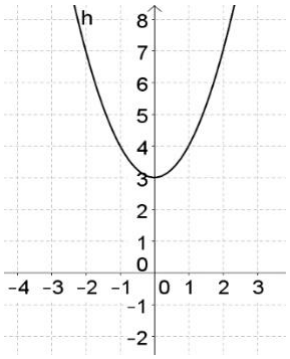
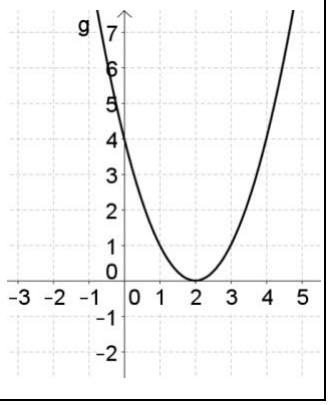
Write in exponential form:
$a^{-3} = 8$
$a = \frac{1}{2}$

$y = log_{\frac{1}{2}} x$</td></tr>
</table>

6. TRANSFORMATION

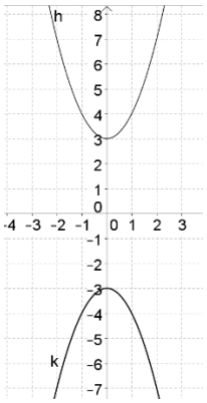
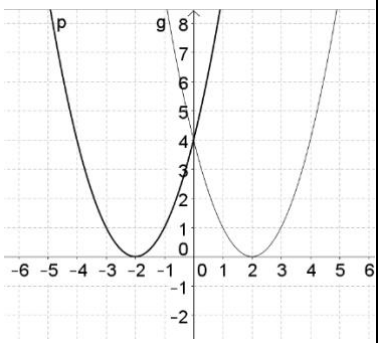
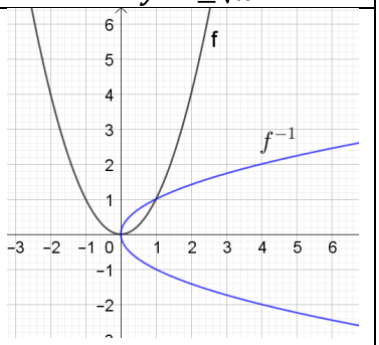
6.1 TRANSLATION

- $p > 0$ horizontal shift **left**
- $p < 0$ horizontal shift **right**
- $q > 0$ vertical shift **up**
- $q < 0$ vertical shift **down**

EXAMPLE:		**Translate 3 units up.** Notation: $f(x) + 3$	**Translate 2 units to the right.** Notation: $f(x-2)$
PARABOLA	$f(x) = x^2$	$h(x) = x^2 + 3$	$g(x) = (x-2)^2$
			

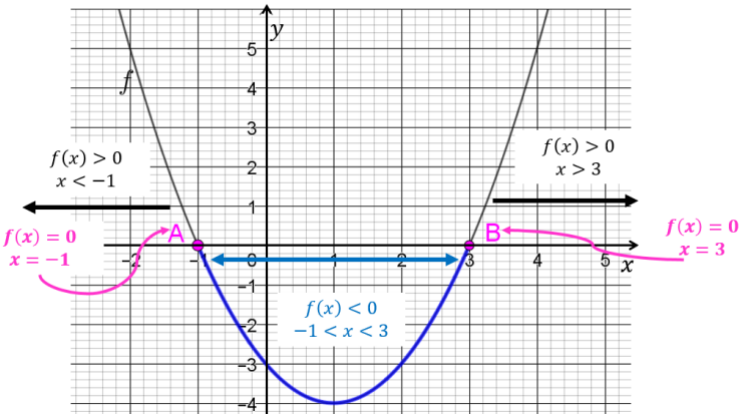
6.2 REFLECTION:

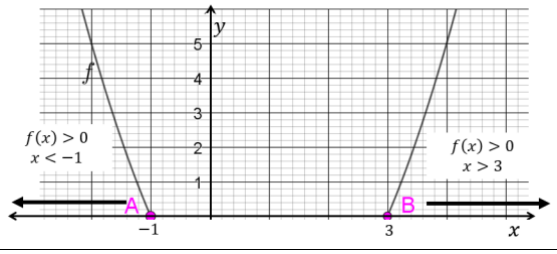
- **REFLECTION in the $(x-axis)$ OR in the line $y = 0$** Notation: $f(x) \rightarrow -f(x)$
- **REFLECTION in the $(y-axis)$ OR in the line $x = 0$** Notation: $f(x) \rightarrow f(-x)$
- **REFLECTION in the line $y = x$** Notation: $f(x) \rightarrow f^{-1}(x)$

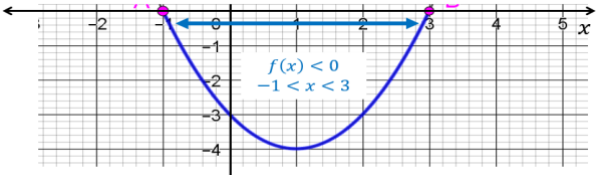
EXAMPLE:	$h(x) = x^2 + 3$ $\rightarrow k(x) = -x^2 - 3$	$g(x) = (x-2)^2$ $\rightarrow p(x) = (-x-2)^2$	f: $y = x^2$ f^{-1}: $x = y^2$ $\therefore y = \pm\sqrt{x}$
PARABOLA			
EXPONENTIAL FUNCTION	$f(x) = 2^x + 3$ $\rightarrow -f(x) = -(2^x) - 3$	$f(x) = 2^x - 2$ $\rightarrow f(-x) = 2^{-x} - 2$	$f(x) = 2^x$ $\rightarrow f^{-1}(x) = log_2 x$

7. GRAPHS AND INEQUALITIES: EXAMPLES

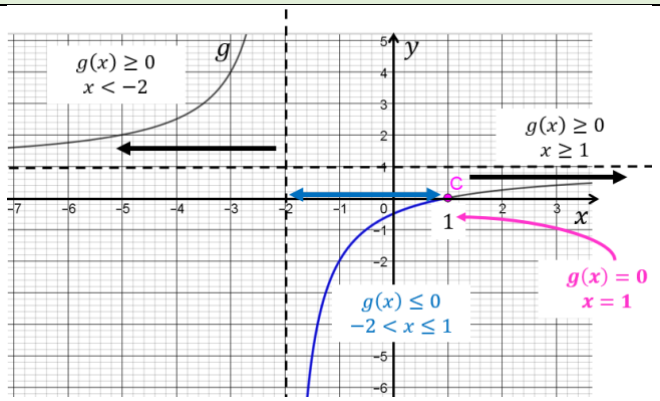
PARABOLA



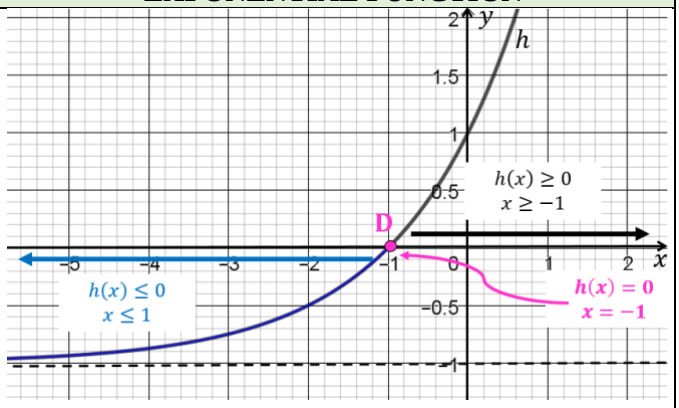




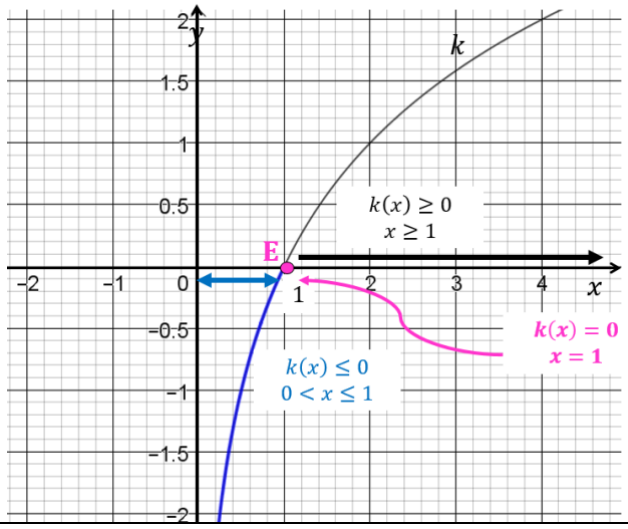
HYPERBOLA



EXPONENTIAL FUNCTION



LOGARITMIC FUNCTION



8. INVERSE FUNCTIONS

8.1 STRAIGHT LINE

Example 1: Given: $f(x) = 2x - 4$

1.1 Determine f^{-1}, that is the inverse of f.

1.2 Sketch the graphs of f , $f^{-1}$and $y = x$ on the same system of axes.

Solution:

1.1 So if, $f(x) = 2x - 4$

f: $y = 2x - 4$

f^{-1}: $x = 2y - 4$ (interchange x and y)

$\therefore -2y = -x - 4$

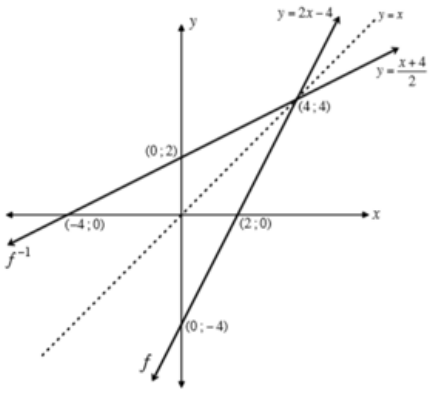
$\therefore 2y = x + 4$

$\therefore y = \frac{1}{2}x + 2$

We then say, inverse function of f is:

$f^{-1}(x) = \frac{1}{2}x + 2$

1.2



8.2 PARABOLA

Example 2: Given: $f(x) = 2x^2$

2.1 Determine f^{-1}, that is the inverse of f.

2.2 Sketch the graphs of f , $f^{-1}$and $y = x$ on the same system of axes.

Solution: 2.1

So if, $f(x) = x^2$

f: $y = x^2$

f^{-1}: $x = y^2$ (interchange x and y)

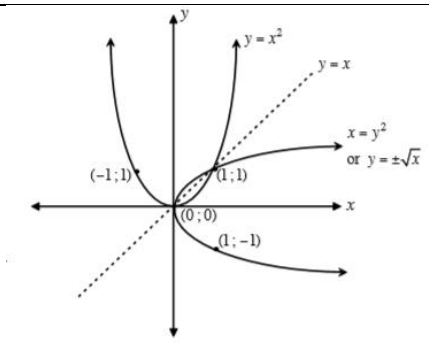
$\therefore y^2 = x$

$\therefore y = \pm\sqrt{x}$

We then say, inverse function of f is:

$f^{-1}(x) = \pm\sqrt{x}$

2.2



Example 3: (parabola with restricted domain).

Given: $f(x) = -\frac{1}{2}x^2$; $x \leq 0$

3.1 Determine f^{-1}, that is the inverse of f.

3.2 Sketch the graphs of f , f^{-1} and $y = x$ on the same system of axes.

Solution:

3.1

So, if, $f(x) = -\frac{1}{2}x^2$ $x \leq 0$

f: $y = -\frac{1}{2}x^2$

- interchange x and y

f^{-1}: $x = -\frac{1}{2}y^2$ $y \leq 0$

$\therefore y^2 = -2x$

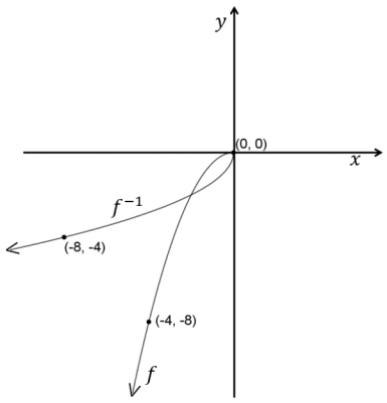
$\therefore y = -\sqrt{-2x}$

Choose only negative y

We then say, inverse function of f is:

$f^{-1}(x) = -\sqrt{-2x}$

3.2

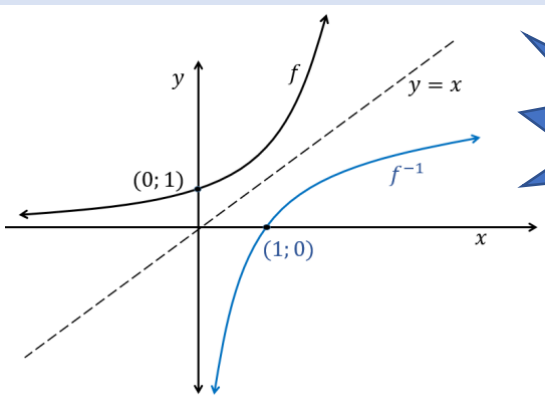


8.3 EXPONENTIAL AND LOGARITHMIC GRAPH

The inverse of the exponential function $f(x) = a^x$ ($a > 0$; $a \neq 1$) is the logarithmic function $f^{-1}(x) = \log_a x$.

$f(x) = a^x$ and $f^{-1}(x) = \log_a x$

$a > 1$

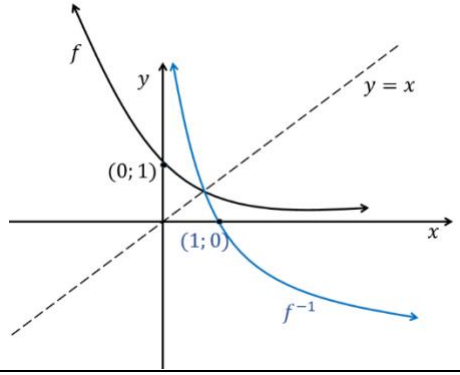


increasing

Note:

- *The exponential graph f is an **increasing** graph* $(a > 1)$
- *The logarithmic graph* f^{-1} *is an **increasing*** $(a > 1)$

$0 < a < 1$



decreasing

Note:

- *The exponential graph f is a **decreasing*** $(0 < a < 1)$
- *The logarithmic graph f is a **decreasing*** $(0 < a < 1)$

Domain and Range

f	*Domain:* $x \in R$	*Range:* $y > 0$
f^{-1}	*Range:* $y \in R$	*Domain:* $x > 0$

Inverse functions:

Graph f^{-1} is obtained by reflecting graph f about the line $y = x$. (the inverse of f)

- If $y = a^x$ then the inverse function is given by $x = a^y$ which can also be written as $y = \log_a x$
- If $y = \log_a x$ then the inverse function is given by $x = \log_a y$ which can also be written as $y = a^x$

Asymptotes:

- *The exponential graph f has an **asymptote*** $y = 0$ $(x - axis)$
- *The logarithmic graph* f^{-1} *has an **asymptote*** $x = 0$ $(y - axis)$

SECTION A – GR 12 INVERSE FUNCTIONS (Routine questions)

- **Note that some of the questions in this section is taken from MIND THE GAP.**

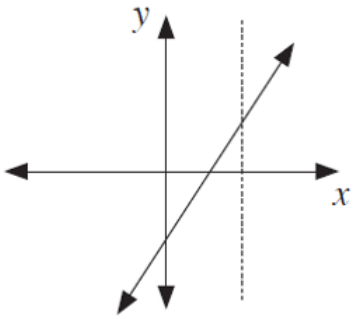
QUESTION 1

1.1

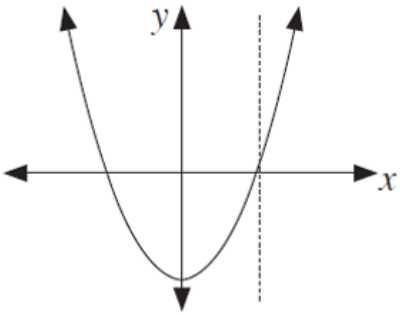
- A function is a relationship between x and y, where for every x-value there is only one y-value.
- One way to decide whether or not a graph represents a function is to use the vertical line test.

Say if the following graphs are functions or not.

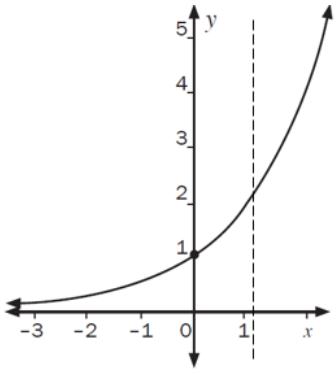
GRAPH A



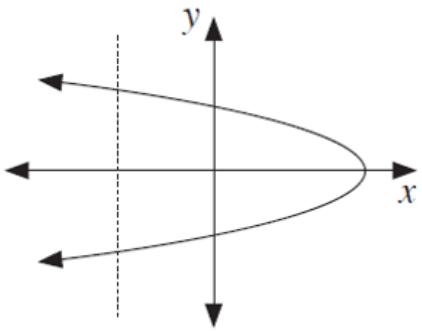
GRAPH B



GRAPH C



GRAPH D



1.2 Complete the following sentences:

(i) A function must be reflected along the line _____to form the inverse of the function.

(ii) The notation for the inverse of a function f is written as _____

QUESTION 2

Given: $f(x) = 2x + 6$.

2.1 Determine $f^{-1}(x)$.

2.2 Sketch the graphs of f, f^{-1} and $y = x$ on the same set of axes.
Indicate clearly the intercepts with the axes of the graph of each of f and f^{-1}.

2.3 Determine the domain and range for $f \ and \ f^{-1}$

QUESTION 3

3.1 Sketch $f(x) = 2x^2$
Indicate clearly the intercepts with the axes, as well as another point on the graph of f.

3.2 Determine the inverse of $f(x)$.

3.3 Sketch the graphs of f^{-1} and $y = x$ on the same set of axes.
Indicate clearly the intercepts with the axes, as well as the coordinates of another point on the graph of f^{-1}.

3.4 Determine the domain and range for $f \ and \ f^{-1}$

QUESTION 4

Given $f(x) = -3x^2$

4.1 Determine the inverse of $f(x)$ in the form $y =$……..

4.2 Determine the domain and range for $f \ and \ f^{-1}$

QUESTION 5

Given: $f(x) = 2^x$

5.1 Determine $f^{-1}(x)$ in the form $y =$……..

5.2 Sketch the graphs of f, f^{-1} and $y = x$ on the same set of axes.
Indicate clearly the intercepts with the axes, as well as the coordinates of another point on the graph of each of f and f^{-1}.

5.3 Determine the domain and range for $f \ and \ f^{-1}$

SECTION B – GR 12 INVERSE FUNCTIONS

(Complex questions)

- Note that the questions in this section is taken from past exam papers. (The year from which the question is, is indicated in brackets next to the question.)

QUESTION 1 (March 2016 – Question 6)

Given: $f(x) = \frac{1}{4}x^2, x \leq 0$

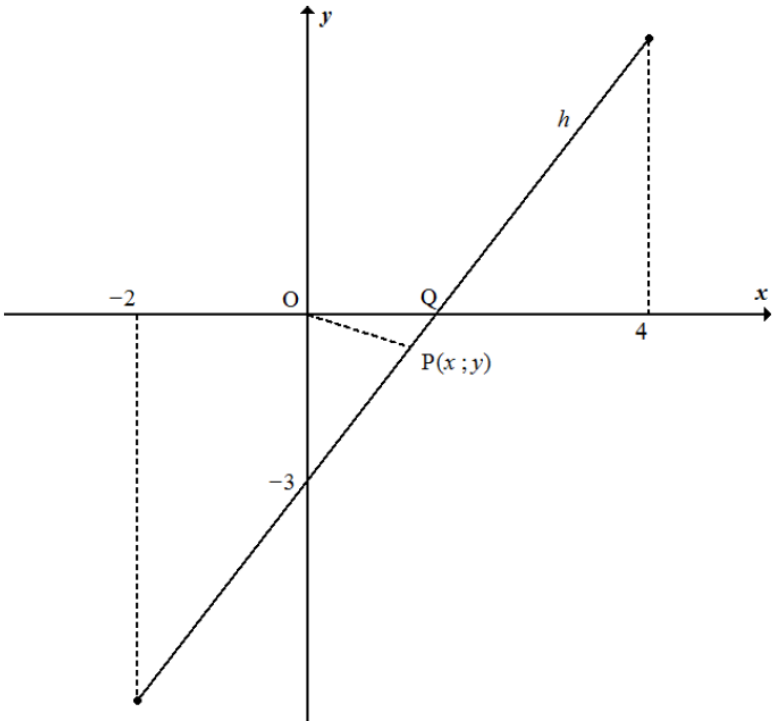
1.1 Determine the equation of f^{-1} in the form $f^{-1}(x) =$ ……

1.2 On the same axes, sketch the graphs of f , f^{-1} , indicate clearly the intercepts with the axes, as well as the coordinates of another point on the graph of each of f and f^{-1}.

1.3 Is f^{-1} a function? Give a reason for your answer.

QUESTION 2 (Nov 2015 – Question 5)

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x –intercept of h is Q.



2.1 Determine the coordinates of Q.

2.2 Write down the domain of h^{-1}.

2.3 Sketch the graphs of h^{-1} , clearly indicating the $y-$ intercept and the coordinates of the end points.

2.4 For which value(s) of x will $h(x) = h^{-1}(x)$?

QUESTION 3 (DBE 2015 – Question 5)

Given: $f(x) = \left(\frac{1}{4}\right)^x$

3.1 Determine the value of $f(-2)$.

3.2 Write down the equation of $f^{-1}(x)$ in the form $y =$……...

3.3 Andrew has no idea how to draw the graph of f^{-1}. Explain to Andrew how he may use the graph of f to draw the graph of f^{-1}.

3.4 Hence or otherwise, sketch the graph of f^{-1}. Clearly indicate ALL intercepts with the axes.

3.5 Write down the domain of f^{-1}.

3.6 For which value(s) of x will $f^{-1}(x) \geq -2$?

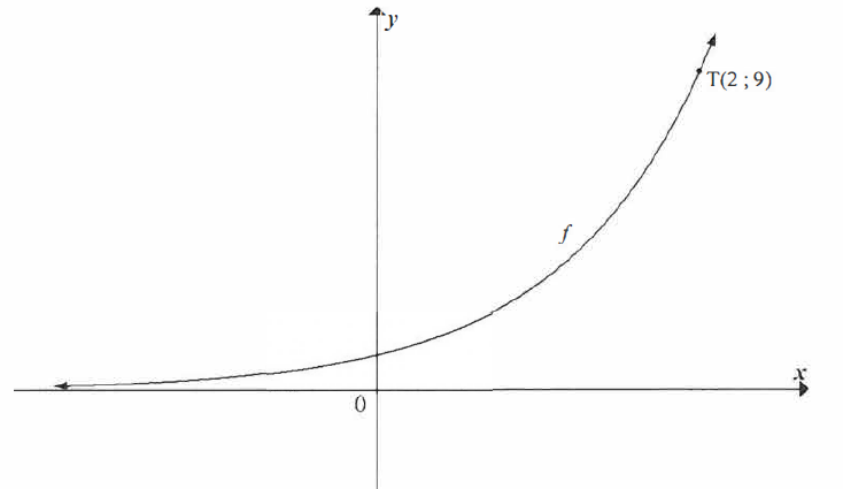
3.7 Given $q = log_{\frac{1}{4}} \frac{1}{2}$

3.7.1 Determine the value of q.

3.7.2 Hence or otherwise, determine the coordinates of the point of intersection of f and f^{-1}.

QUESTION 4 (March 2015-Question 5)

The graph of $f(x) = a^x$, $a > 1$ is shown below. T (2 ; 9) lies on f.



4.1 Calculate the value of a.

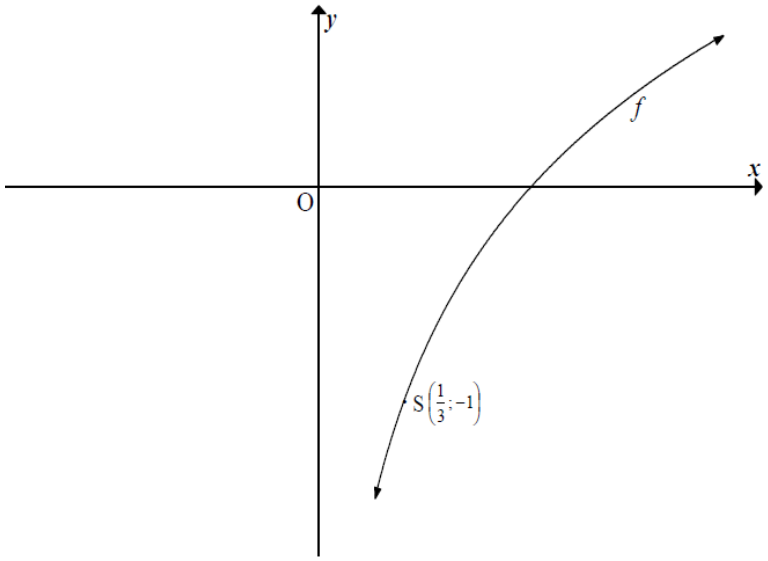
4.2 Determine the equation of $g(x)$ if $g(x) = f(-x)$

4.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$

4.4 Is the inverse of f a function? Explain your answer.

QUESTION 5 (Nov 2014−Question 5)

Given: $f(x) = log_a x$ where $a > 0$. S $\left(\frac{1}{3}\ ;\ -1\right)$ is a point on the graph of f.



5.1 Prove that $a = 3$.

5.2 Write down the equation of h, the inverse of f, in the form $y =$……..

5.3 If $g(x) = -f(x)$, determine the equation of g.

5.4 Write down the domain of g.

5.5 Determine the value(s) of x for which $f(x) \geq -3$

QUESTION 6 (DBE 2014−Question 6)

The graph of g is defined by the equation $g(x) = \sqrt{ax}$. The point $(8\ ;\ 4)$ lies on g.

6.1 Calculate the value of a.

6.2 If $g(x) > 0$, for which values of x will g be defined?

6.3 Determine the range of g.

6.4 Write down the equation of g^{-1}, the inverse of g, in the form $y =$……..

6.5 If $h(x) = x - 4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of h and g.

6.6 Hence, or otherwise, determine the value(s) of x for which $g(x) > h(x)$

SECTION C– GR 11 FUNCTIONS **(from past exam papers)**

NOTE:

Questions in this revision were compiled from past exam National papers for GR 11 and GR 12.

- Some questions have been altered.

QUESTION 1 (GR11 DBE NOV 2017)

Given $f(x) = \frac{-3}{x+2} + 1$ and $g(x) = 2^{-x} - 4$

1.1 Determine $f(-3)$.

1.2 Determine x if $g(x) = 4$.

1.3 Write down the asymptotes of f.

1.4 Write down the range of g.

1.5 Determine the coordinates of the x –and y –intercepts of f.

1.6 Determine the equation of the axis of symmetry of f which has a negative gradient. Leave your answer in the form $y = mx + c$.

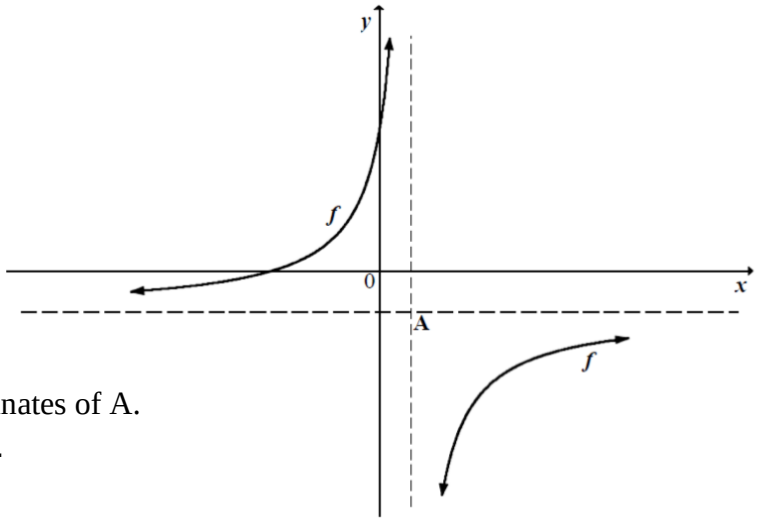
1.7 Sketch the graphs of f and g on the same axes. Clearly show ALL the intercepts with the axes and the asymptotes.

QUESTION 2 (GR11 DBE NOV 2015)

The sketch below represents the graph of

$$f(x) = \frac{-9}{x-1} - 2$$

A is the point of intersection of the asymptotes of f.



2.1 Write down the coordinates of A.

2.2 Give the domain of f.

2.3 Give the equation of the horizontal asymptote for $f(x) - 1$.

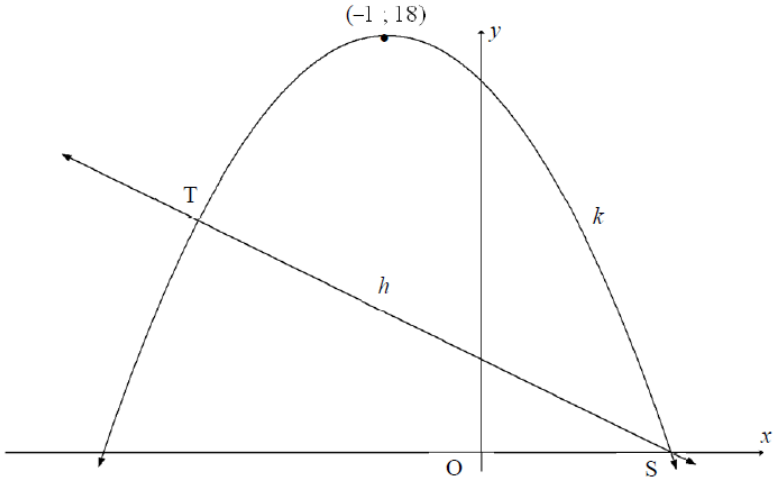
2.4 Determine the coordinates of the $x -$ and y –intercepts of f.

2.5 Write down the equation of the axis of symmetry of f that has a negative gradient.

2.6 Determine the values of x if $f(x) \geq 0$

QUESTION 3 (GR 12 DBE 2019 Question 5)

Sketched below are the graphs of $k(x) = ax^2 + bx + c$ and $h(x) = -2x + 4$.
Graph k has a turning point at $(-1\ ;\ 18)$. S is the x −intercept of h and k.
Graphs h and k also intersect at T



3.1 Calculate the coordinates of S.

3.2 Determine the equation of k in the form $y = a(x + p)^2 + q$

3.3 If $k(x) = -2x^2 - 4x + 16$, determine the coordinates of T.

3.4 Determine the value(s) of x for which $k(x) < h(x)$

3.5 It is further given that k is the graph of $g'(x)$

(a) For which values of x will the graph of g be concave up?

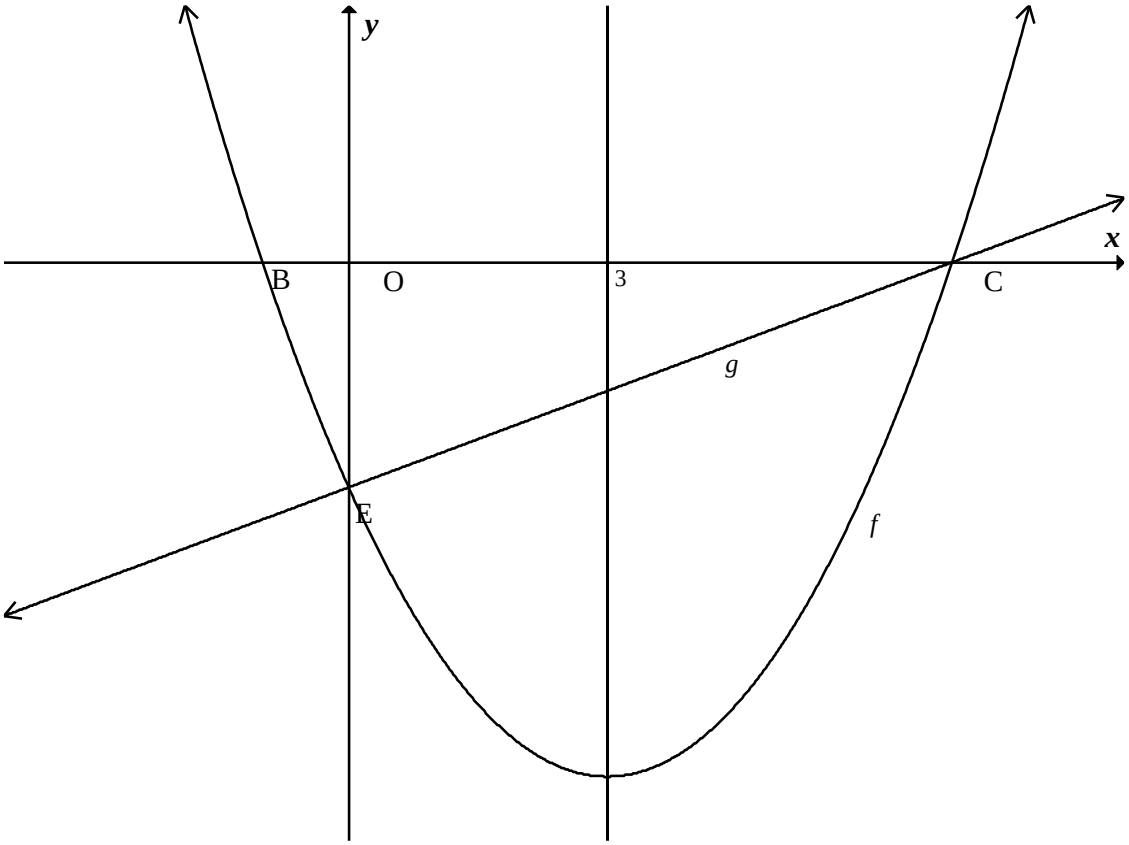
(b) Sketch the graph of g, showing clearly the x −values of the turning points and the point of inflection.

3.6 Determine the values of k such that $f(x) = k$ has equal roots.

3.7 If the graph f is shifted TWO units to the right and ONE unit upwards to form h, determine the equation h in the form $y = a(x + p)^2 + q$.

QUESTION 4 (NSC March 2011)

A parabola f intersects the *x*-axis at B and C and the *y*-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



4.1 Show that the coordinates of C are (7 ; 0).

4.2 Calculate the *x*-coordinate of B.

4.3 Determine the equation of f in the form $y = a(x - p)^2 + q$.

4.4 Write down the equation of the graph of h, the reflection of f in the *x*-axis.

4.5 Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$.

4.6 Determine the values of x if $f(x) \geq 0$

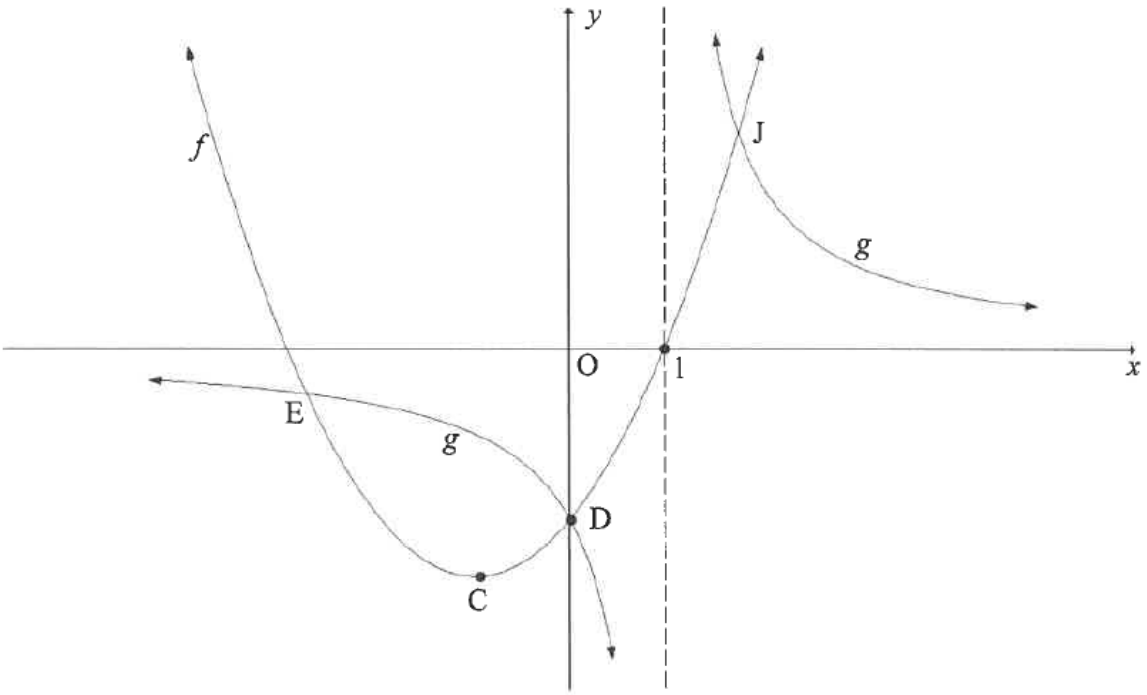
4.7 Determine the values of x if $f'(x) < 0$

4.8 Determine the values of x if $f(x) < g(x)$

QUESTION 5 (GR12 NOV 2019 Question 4)

Below are the graphs of $f(x) = x^2 + bx - 3$ and $g(x) = \frac{a}{x+p}$

- f has a turning point at C and passes through the $x-$ axis at $(1;0)$
- D is the $y-$intercept of both f and g. The graphs f and g also intersect each other at E and J.
- The vertical asymptote of g passes through the $x-$intercept of f.



5.1 Write down the values of p.

5.2 Show that $a = 3$ and $b = 2$.

5.3 Calculate the coordinates of C.

5.4 Write down the range of f.

5.5 Determine the equation of the line through C that makes an angle of $45°$ with the positive $x-$axis. Write your answer in the form $y =$……..

5.6 Is the straight line, determined in QUESTION 5.5, a tangent to f?
Explain your answer.

5.7 The function $h(x) = f(m - x) + q$ has only one $x-$iintercept at $x = 0$.
Determine the values of m and q.

GR 12 – CALCULUS

CALCULUS – SUMMARY

Question Type	Summary of procedure	Example question
1. Find the limits of simple functions	Clear the unknown in denominator and substitute the value being approached in the expression.	$\lim_{h\to 0}\frac{4h-h^2}{h}$; $\lim_{a\to 6}\frac{a^2-36}{a-6}$
2. Find the average gradient between two points	Need the coordinates of two points, find gradient using formula	If $f(x)=2x^3$, find the gradient between $x=1$ and $x=2$
3. Find the gradient at a point	Find the derivative of the expression, substitute x value of point	If $f(x)=2x^3$, find the gradient where $x=2$
4. Find the derivative from first principles	Have $f(x)$, find $f(x+h)$ and use derivative formula	Find $f'(x)$ if $f(x)=2x^2$ from first principles
5. Find derivatives using the rules	Simplify the expression and then differentiate. Be careful of layout of the expression.	Find $\frac{dy}{dx}$ if $y=\frac{2}{x}-\sqrt{x}=2(x+1)$
6. Use different notations for differentiation	Be careful of layout of the expression.	$f'(x)$, $\frac{dy}{dx}$, D_x , $\frac{dA}{dx}$, $\frac{ds}{dt}$
7. Find turning points, horizontal points of inflection of curves	Make derivative equal to zero, substitute back into original equation.	$y=x^2-3x-4$
8. Finding x intercepts of cubic curves	Make $y=0$, solve equation. Guess 1^{st} factor, factorize further , may need quadratic equation formula.	Determine where $y=x^3-5x^2-8x+12$ cuts the x-axis
9. Sketch cubic graphs	y cut, x cuts using factor theorem, turning points using derivative = 0, sketch	Sketch the curve of $y=x^3-5x^2-8x+12$
10. Find points of inflection	Let 2^{nd} derivative = 0 and substitute x value back into the original equation	Find the point of inflection between the 2 stationary points of $y=x^3-5x^2-8x+12$
11. Find the equation of a tangent to a graph	$y=mx+c$, Substitute x value into the derivative to get m. Substitute x value into original equation to get a point on the curve. Use point to find c	Find the equation of a tangent to the curve $y=35x^2-4x+1$ at the point where $x=1$
12. Maximize and minimize lengths, areas, volumes etc.	Find expression to represent situation described, make the derivative = 0, substitute x value back into original equation.	

NOTE:

- Questions in this revision were compiled from past exam National papers for GR 12.

SECTION A

QUESTION 1. Determine $f'(x)$ from first principles if

1.1 $f(x) = 3x^2 - 5$ (5)

1.2 $f(x) = -x^2 + 4$ (5)

1.3 $f(x) = \frac{4}{x}$ (5)

1.4 $f(x) = x^3$ (5)

QUESTION 2. Determine :

2.1 $\frac{dy}{dx}$ if $y = 2x^5 + \frac{4}{x^3}$ (3)

2.2 $\frac{dy}{dx}$ if $y = \left(\sqrt{x} - x^2\right)^2$ (4)

2.3 $\frac{dy}{dx}$ if $y = \left(x^2 - \frac{1}{x^2}\right)^2$ (3)

2.4 the derivative of $f(x) = \left(x - \frac{3}{x}\right)^2$ (3)

2.5 the derivative of $f(x) = \left(\frac{1}{x^3} + 4x\right)^2$ (4)

2.6 $D_x\left(\frac{x^3-1}{x-1}\right)$ (3)

2.7 $D_x\left[\sqrt[3]{x^2} - \frac{1}{2}x\right]$ (3)

QUESTION 3.

3.1 If $y = (x^6 - 1)^2$, prove that $\frac{dy}{dx} = 12x^5\sqrt{y}$, if $x > 1$. (3)

3.2 Given: $f(x) = 2x^3 - 2x^2 + 4x - 1$. Determine the interval on which f is concave up. (4)

3.3 $g(x) = -8x + 20$ is a tangent to $f(x) = x^3 + ax^2 + bx + 18$ at $x = 1$. Calculate the values of a and b. (5)

QUESTION 4

Given $f(x) = 2x^3 - 23x^2 + 80x - 84$

4.1 Prove that $(x - 2)$ is a factor of f. (2)

4.2 Hence, or otherwise, factorise $f(x)$ fully. (2)

4.3 Determine the $x-$ coordinates of the turning points of f. (4)

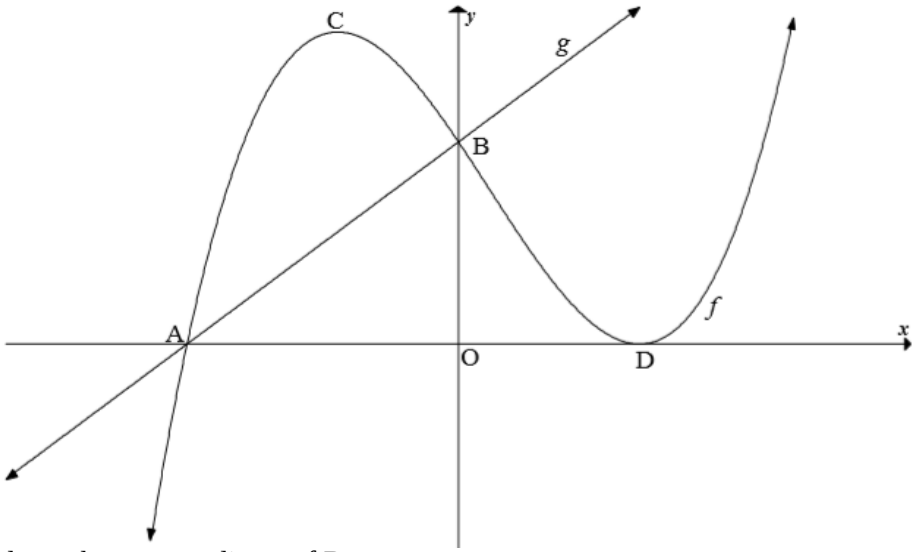
4.4 Sketch the graph of f, clearly labelling ALL intercepts with the axes and turning points. (3)

4.5 Determine the coordinates of the $y-$ intercept of the tangent to f that has a slope of 40 and touches f at a point where the $x-$ coordinate is an integer. (6)

QUESTION 5

The sketch below are the graphs of $f(x) = (x-)^2(x - k)$ and $g(x) = mx + 12$.

- A and D are the $x-$ intercepts of f.
- B is the common $y-$ intercepts of f and g.
- C and D are turning points of f.
- The straight line g passes through A.



5.1 Write down the $y-$coordinate of B. (1)

5.2 Calculate the $x-$ coordinates of A. (3)

5.3 If $k = -3$, calculate the coordinates of C. (6)

5.4 For which values of x will f be concave down. (3)

QUESTION 6

Given : $h(x) = -x^3 + ax^2 + bx$ and $g(x) = -12x$. P and Q(2; 10) are the turning points of h. The graph of h passes through the origin.

6.1 Show that $a = \frac{3}{2}$ and $b = 6$. (5)

6.2 Calculate the average gradient of h between P and Q, if it is given that $x = -1$ at P? (4)

6.3 Show that the concavity of h changes at $x = \frac{1}{2}$. (3)

6.4 Explain the significance of the change in QUESTION 6.3 with respect to h. (1)

6.5 Determine the value of x, given $x < 0$, at which the tangent to h is parallel to g. (4)

QUESTION 7

Given : $f(x) = (x + 2)(x^2 - 6x + 9)$
$= x^3 - 4x^2 - 3x + 18$

7.1 Calculate the coordinates of the turning points of the graph of f (6)

7.2 Sketch the graph of , clearly indicating the intercepts with the axes and the turning points. (4)

7.3 For which value(s) of x will $x.f'(x) < 0$ (3)

QUESTION 8

Given : $f(x) = 3x^3$

8.1 Solve $f(x) = f'(x)$ (3)

8.2 The graphs f, f' and f'' , all passes through the point (0 ; 0)

8.2.1 For which of the graphs will (0;0) be a stationary point. (1)

8.2.2 Explain the difference , if any , in the stationary points referred to in QUESTION 8.2.1 (2)

8.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$ (3)

8.4 For which value(s) of x is $f(x) - f'(x) < 0$ (4)

SECTION B

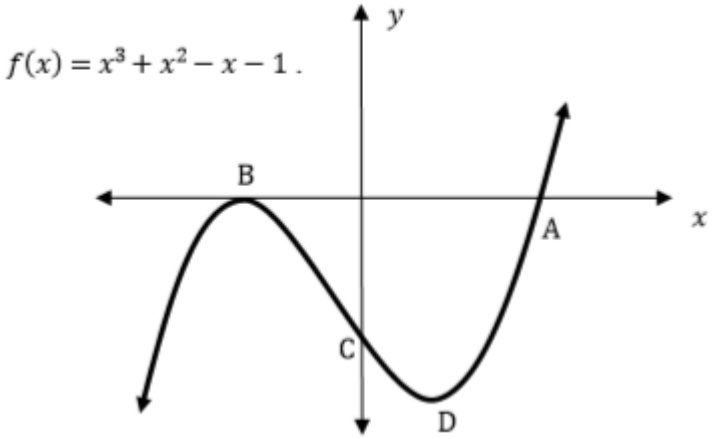
QUESTION 1.

1.1 Use first principles to determine the value of (8)
$D_x\left(\frac{4x^3-x^4}{2x^2}\right)$, if $x = 2$

1.2 $f(x) = 4^x - 2^{x+3} - 128$ and $g(x) = 16x^{\frac{1}{2}}$ (9)

Determine the value of $g(x)$, if $f(x) = 0$

1.3 The diagram below shows the graph of $f(x) = x^3 + x^2 - x - 1$



1.3.1 Calculate the distance between A and B, the x- intercepts (5)

1.3.2 Calculate the coordinates of D, a turning point of f. (3)

1.3.3 Show that the concavity of f changes at $x = -\frac{1}{3}$ (3)

1.3.4 For which values of x is:

a) $f(x) > 0$ (1)

b) $f(x).f'(x) < 0$ (3)

QUESTION 2.

2.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ where a, b and c are real constants has the following properties:

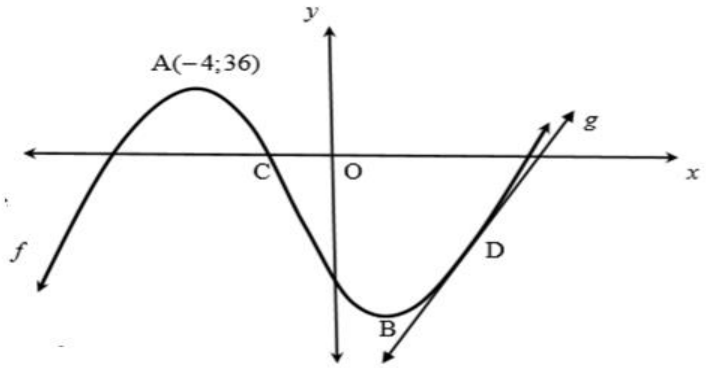
$a > 0$, $d = 10$

$f(-2) = 0$, $f(2) = 0$ and $f(4) = 0$

$f'(0) = 0$, $f'(3) = 0$ and $f(3) = -3$

Using the above information, draw a sketch graph of f. Clearly indicate all intercepts with the axes, as well as the turning points. (6)

2.2 The function defined by $f(x) = x^3 + px + qx - 12$ is sketched below.
A(-4 ;36) and B are turning points of f. g is a tangent to the graph of at D.



2.2.1 Show that $p = 5$ and $q = -8$. (6)

2.2.2 If C(-1 ; 0) is an x- intercept of f, calculate the other x- intercept of f. (4)

2.2.3 Determine the equation of g, the tangent of f at D(1 ;-14). (4)

2.2.4 For which values of k will $f(x) = g(x) + k$ have two positive roots. (2)

QUESTION 3.

Given : $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

3.1 Show that f has a point of inflection at $x = 2$ (5)

3.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

3.3 For which values of x will $y = -f(x)$ be concave down. (2)

3.4 Use your graph to answer the following questions:

3.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2) + 3$ (2)

3.4.2 Claire claims that $f'(2) = 1$. (2)
Do you agree with Claire? Justify your answer

QUESTION 4.
A marathon athlete trains between two towns A and C. He starts at point B which lies between towns A and C. The athlete runs from point B to town C and back to point B. The road between the towns is in a straight line. The displacement S, in kilometres, from point B after t hours, is given by:

$$S(t) = -t^3 + 12t^2 - 32t$$

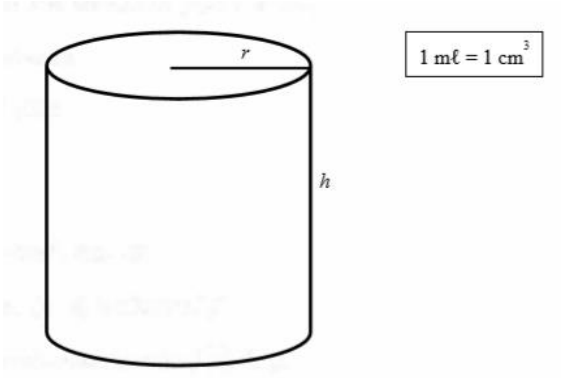
4.1 How many hours will it take the athlete to return to point B. (3)

4.2 Calculate the distance between point B and town C. (5)

4.3 Calculate the maximum speed that the athlete has reached while training (4)

QUESTION 5.

A 340 ml can with height h cm and radius r cm is shown below.

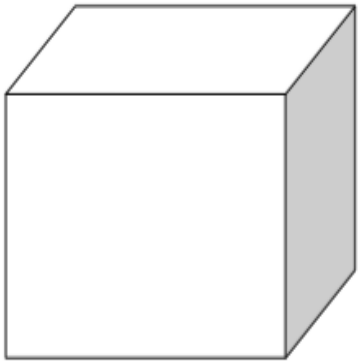


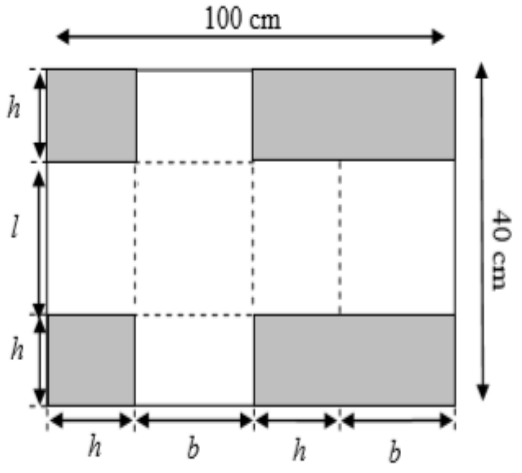
5.1 Determine the height of the can in terms of the radius r. (3)

5.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)

QUESTION 6.

A box is made from a rectangular piece of a cardboard, 100 cm by 40 cm, by cutting out the shaded areas and folding along the dotted lines as shown in the diagram below.





6.1 Express the length l in terms of the height h. (1)

6.2 Hence prove that the volume of the box is given by $V = h(50 - h)(40 - 2h)$ (3)

6.3 For which value of h will the volume of the box be a maximum. (5)