education

Department:
Education
PROVINCE OF KWAZULU-NATAL

KZN DEPARTMENT OF EDUCATION

MATHEMATICS
JUST IN TIME MATERIAL
GRADE 11

TERM 2 – 2020

This document has been compiled by the FET Mathematics Subject Advisors together with Lead Teachers. It seeks to unpack the contents and to give more guidance to teachers.

TABLE OF CONTENTS

FUNCTIONS AND GRAPHS

FROM GR. 11 ANNUAL TEACHING PLAN 2020:

DATES	CURRICULUM STATEMENT
04/5-7/5 (4 days)	Revise the effect of a and q and investigate the effect of p on the graphs of the functions defined by: • $y = f(x) = a(x+p)^2 + q$
8/5-14/5 (5 days)	Revise the effect of the parameters a and q and investigate the effect of p on the graphs of the functions defined by: • $y = f(x) = \dfrac{a}{x+p} + q$
15/5-20/5 (4 days)	Revise the effect of the parameters a and q and investigate the effect of p on the graphs of the functions defined by: • $y = f(x) = a.b^{x+p} + q$, where $b > 0$ and $b \neq 1$
21/5-25/5 (3 days)	• Investigate numerically the average gradient between two points on a curve. • Develop an intuitive understanding of the concept of the gradient of a curve at a point. • Interpretations, applications and Practical problems. **NB: Apply nature of roots with functions**

JUNE COMMON TEST WEIGHTING

Functions & Graphs	35 ±3 marks out of 100 marks in Paper 1

CAPS EXAM GUIDELINE WEIGHTING FOR NOVEMBER EXAMINATION

Functions & Graphs	45 ± 3 marks out of 150marks in Paper 1

CONCEPTS TO BE RE-INFORCED WITH LEARNERS

1. Subsititution

e.g if $f(x) = 2x^3 + 3x - 3$, determine (i) $f(-2)$; (ii) $f(a)$; (iii) $f(p+1)$

2. Graph interpretation

- Interval notation and algebraic notation:

  ( ) ; $<$ or $>$ denotes excluding critical / end values.

  [ ] $\leq$ or $\geq$ denotes including critical / end values.

  On the side where there is $\pm\infty$ we use **only** "open" brackets **( ).**
- $f(x) > 0$: where the graph is above the $x-$axis.
- $f(x) < 0$: where the graph is below the $x-$axis.
- $f(x).g(x) \geq 0$: where both graphs are either below or above the $x-$axis.
- $f(x) = g(x)$: where the graphs of f and g intersect.
- $f(x) > g(x)$: where the graph of f is above the graph of g.
- $f(x) < g(x)$: where the graph of f is below the graph of g.
- $\frac{f(x)}{g(x)} \geq 0$: also, where both graphs are either below or above the $x-$axis. But in this case, always remember that $g(x) \neq 0$.

3. Transformations

- $f(x-2)$: a horizontal shift of the graph of f two units to the right.
- $f(x+2)$: a horizontal shift of the graph of f two units to the left.
- $f(x)+2$: a vertical shift of the graph of f two units upwards.
- $f(x)-2$: a vertical shift of the graph of f two units downwards.
- $f(x-2)+3$: a horizontal shift of the graph of f two units to the right and 3 units upwards.
- $f(-x)$: reflection of the graph of f about the y-axis (the line $x = 0$).
- $-f(x)$: reflection of the graph of f about the x-axis (the line $y = 0$).

BACKGROUND KNOWLEDGE THAT LEARNERS NEED

1. FUNCTION AND MAPPING NOTATION

In Grade 10 learners were introduced to different ways of representing functions.
The different notations are summarised below:

- $y = \ldots$ equation notation
- $f(x) = \ldots$ function notation
- $f : x \rightarrow \ldots$ mapping notation

2. INTERCEPTS WITH THE AXES

To determine the x – intercept(s), substitute $y = 0$.
For example: If $f(3) = 0$, then the function has an x intercept at $(3;\ 0)$.

To determine the y – intercept(s), substitute $x = 0$.
For example: If $f(0) = 4$, then the function has a y intercept at $(0;\ 4)$.

3. GRAPH INTERPRETATION

3.1 Axes of symmetry:

If a function has a line of symmetry, it means that the function is a mirror image of itself about that line. In other words, if the graph was folded along the line of symmetry, it would duplicate itself on the other side of the line.

3.2 Asymptotes:

Asymptotes are imaginary lines that a graph approaches, but never touches or cuts.

3.3 Domain and range:

Domain: The domain refers to the set of possible $\boldsymbol{x} - \textbf{values}$ for which a function is defined.

Range: The range refers to the set of possible $\boldsymbol{y} - \textbf{values}$ that the function can assume.

4. BASELINE ACTIVITY

For each of the following functions
(I) Sketch the graph of the function.
(II) Determine the domain and the range of each function.
(III) Determine the equation of the axes of symmetry and asymptotes, where applicable.

Straight Line

1.1 $y = x + 3$
1.2 $y = (x - 2)$
1.3 $y = -x$
1.4 $y = -2x - 3$

Parabola

2.1 $y = x^2 + 3$
2.2 $y = (x - 2)^2$
2.3 $y = -x^2$
2 .4 $y = -2x^2 - 3$

Hyperbola

3.1 $y = \frac{6}{x} + 3$
3.2 $y = \frac{6}{x-2}$
3.3 $y = -\frac{6}{x}$
3.4 $y = \frac{-2}{x} - 3$

Exponential graph

4.1 $y = 2^x + 3$
4.2 $y = 2^{x-2}$
4.3 $y = 2^{-x}$
4.4 $y = -(2)^x - 3$

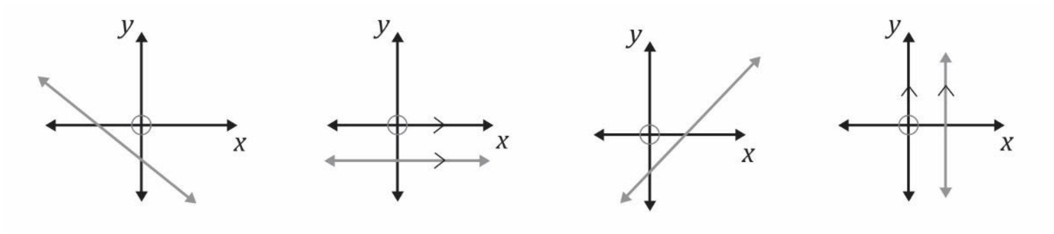
BASIC INFORMATION ON THE DIFFERENT TYPES OF GRAPHS

A. STRAIGHT LINE

General representation or equation:

$y = ax + q$ or $f(x) = mx + c$, a or m is the gradient, and q or c is the $y-$intercept .

Also note the shape of the following linear functions:



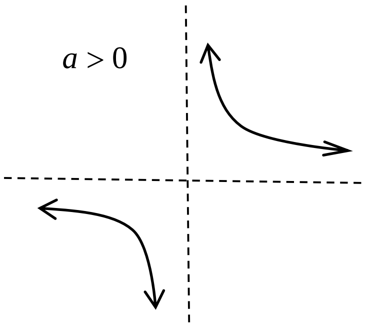
$a < 0$	$a = 0$	$a > 0$	a is undefined
$q < 0$	$y = q$	$q < 0$	there is no q-value

For all the linear functions, except horizontal and vertical lines, the domain is $x \in R$, and the range is $y \in R$.

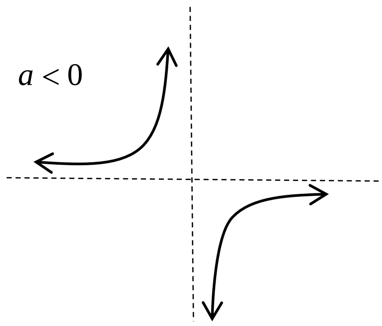
B. HYPERBOLA

General representation or equation:

$$y = \frac{a}{x+p} + q$$



Dotted lines are asymptotes



Dotted lines are asymptotes

- The value of q represents the vertical translation (shift) from the x-axis.
- The value of p represents the horizontal translation (shift) from the y-axis.
- In the case of $y = \frac{a}{x}$, $p = 0$ and $q = 0$. The **vertical** asymptote is the y-axis ($x = 0$) and the **horizontal** asymptote is the x-axis ($y = 0$). The **axes of symmetry** are $y = x$ (+ve gradient) and $y = -x$ (-ve gradient).
  The **domain** is $x \in R,\ x \neq 0$; and the r**ange** is $y \in R, y \neq 0$.
- In the case of $y = \frac{a}{x} + q$, $p = 0$. The **vertical** asymptote is the y-axis ($x = 0$) and the **horizontal** asymptote is $y = q$. The **axes of symmetry** are $y = x + q$ (+ve gradient) and $y = -x + q$ (-ve gradient). The d**omain** is $x \in R,\ x \neq 0$; and the r**ange** is $y \in R,\ y \neq q$.

- In the case of $y=\dfrac{a}{x+p}+q$, the **vertical** asymptote is $x=-p$ and the **horizontal** asymptote is $y=q$. The **axes of symmetry** are $y=\pm(x+p)+q$. The d**omain** is $x\in R,\ x\neq -p$ and the **range** is $y\in R, y\neq q$.
- Alternative method to determine the equations of the axes of symmetry:
  In all cases the one axis of symmetry has a gradient of $+1$ and the other a gradient of -1. Therefore the equations of the axes of symmetry are $y=x+c$ and $y=-x+c$. In all cases the value of c may be determined by simply substituting the coordinates of the point of intersection of the two asymptotes into the above equations – since the axes of symmetry always pass through this point.

Example no. 1:

Given : $f(x)=\dfrac{3}{x-2}+1$

1.1. Write down the equations of the asymptotes of f.

1.2. Determine the coordinates of B, the x-intercept of f.

1.3. Determine the coordinates of D, the y-intercept of f.

1.4. Determine the domain and the range of f.

1.5. Determine the equations of the two axes of symmetry of f.

1.6. Draw a sketch graph of f.

Solution:

1.1 For the vertical asymptote:

$$x-2=0$$
$$x=2$$

Horizontal asymptote:

$$y=1$$

1.2 For the x – intercept, substitute $y=0$:

$$\frac{3}{x-2}+1=0$$
$$\frac{3}{x-2}=-1$$
$$-1(x-2)=3$$
$$x=-1$$

1.3 For the y – intercept, substitute $x=0$:

$$y=\frac{3}{-2}+1=\frac{3-2}{-2}=-\frac{1}{2}$$

1.4 Domain is $x\in R;\ x\neq 2$

Range is $y\in R;\ x\neq 1$

1.5 Point of intersection of asymptotes: (2 ; 1)

Axis of symmetry with positive gradient:

Substitute (2 ; 1) into $y=x+c$:

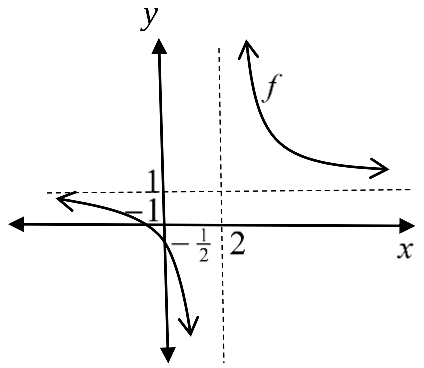
$$1=2+c$$
$$c=-1$$
$$y=x-1$$

Axis of symmetry with negative gradient:

Substitute (2 ; 1) into $y=-x+c$:

$$1=-2+c$$
$$c=3$$
$$y=-x+3$$

1.6)

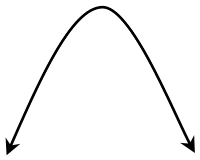
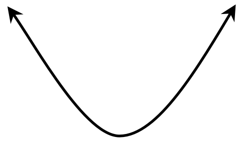


C. **PARABOLA**

Defining Equation:

$y = a(x+p)^2 + q$ or $y = ax^2 + bx + c$ or $y = a(x - x_1)(x - x_2)$

Sketching a parabola:

	for $a < 0$	for $a > 0$
Shape		

For $y = ax^2 + bx + c$, the **turning point** is $\left(\frac{-b}{2a}\ ;\ f\left(\frac{-b}{2a}\right)\right)$ and the ***y*-intercept** is $y = c$.

Given: $y = ax^2 + bx + c$

***y*-intercept** : $(0\ ;\ c)$

Turning point (TP) :

$x = \dfrac{-b}{2a}$ (the axis of symmetry)

Substitute this value into the equation to find the *y*-coordinate of the TP, i.e. the minimum or maximum value.

Given: $y = a(x+p)^2 + q$

Multiply out the expression to get it in the form $y = ax^2 + bx + c$

***y*-intercept**:$(0\ ;\ c)$

Turning Point (TP): $(-p\ ;q)$

If there are ***x*-intercepts**: Let $y = 0$ and solve for x (factorise or use the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$).

- If $a < 0$, the function has a **maximum** value, represented by the ***y*** value of the turning point.
- If $a > 0$, the function has a **minimum** value, represented by the ***y*** value of the turning point.
- The equation of the axis of symmetry is given by $x = \frac{-b}{2a}$, (is the *x* value of the turning point)
- The **domain** is $x \in R$
- The **range: If $\boldsymbol{a > 0}$** then $y \geq$ minimum value ; **If $\boldsymbol{a < 0}$** then $y \leq$ maximum value.

To determine the equation of a parabola:

<u>Given:</u> TP and one other point

Use $y = a(x+p)^2 + q$

- TP is $(-p\ ;q)$; substitute that in above equation.
- Substitute the other point for *x* and *y*.
- Solve for *a*.
- Rewrite the equation with the values for *a*, *p* and *q*.
- If required, rewrite in the form $y = ax^2 + bx + c$.

<u>Given:</u> *x*-intercepts and one other point

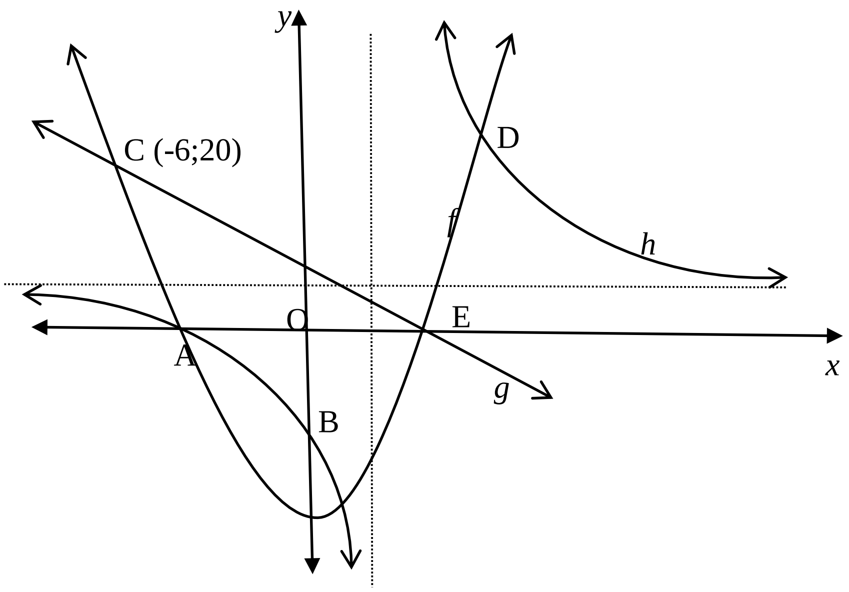
Use $y = a(x - x_1)(x - x_2)$

- Substitute the *x*-intercepts for x_1 and x_2.
- Substitute the other point for *x* and *y*.
- Solve for *a*.
- Rewrite the equation with the values for a, x_1 and x_2.
- If required, rewrite in the form $y = ax^2 + bx + c$.

Example no. 2:

Sketched below are the graphs of: $g(x) = -2x + 8$; $f(x) = x^2 + k$; and $h(x) = \frac{6}{x-2} + 1$.

A is an x-intercept and B a y – intercept of *h*. C (−6; 20) and E are the points of intersection of *f* and *g*.



2.1 Determine the coordinates of A, B and E.

2.2 Show that the value of $k = -16$

2.3 Determine the domain and the range of *f*.

2.4 Write down the values of x for which $g(x) - f(x) \geq 0$.

2.5 Determine the equation of the axis of symmetry of h that has a negative gradient.

2.6 Write down the range of *s*, if $s(x) = f(x) + 2$.

2.7 Write down the range of *t*, if $t(x) = h(x) + 2$.

Solution**:**

2.1 At A, substitute $y = 0$:

$$\frac{6}{x-2} + 1 = 0$$

$$6 = -x + 2$$

$$\therefore x = -4$$

Thus: A$(-4;0)$

At B, substitute $x = 0$:

$$y = \frac{6}{-2} + 1$$

$$y = -3 + 1$$

$$\therefore y = -2$$

Thus: B$(0;-2)$

E is the x-intercept of the straight line and the parabola. It is easy and straight-forward to use the equation of the straight line to get the coordinates of E.

At E, substitute $y = 0$, $\therefore 0 = -2x + 8$

$$x = 4$$

Thus: E$(4;0)$

2.2 C(-6; 20) is on f and g.

Substitute C into $f(x) = x^2 + k$

$$20 = (-6)^2 + k$$

$$k = -16$$

2.3 Domain is $x \in R$

Range is $y \geq -16$; $y \in R$

2.4 These are the values of x for which the graphs of g and f intersect **or** where f is below g.
It occurs from C(-6 ; 20) and E(4 ; 0).
That is $-6 \le x \le 4$.

2.5 Point of intersection of asymptotes: $(2;1)$
For axis of symmetry with negative gradient:
$$y = -x + c$$
Substitute $(2;1)$: $1 = -2 + c$
$$c = 3$$
$$y = -x + 3$$

2.6 The "+ 2" implies a shift vertically upwards by 2 units. The new minimum value will now be -14. The range of s is $y \ge -14$.

2.7 The " + 2" implies a shift vertically upwards by 2 units.
The range of t is $y \ne 1+2;\ y \in R$
$$y \ne 3;\ y \in R$$

D. EXPONENTIAL GRAPH

Defining equation: $y = ab^{x+p} + q$.

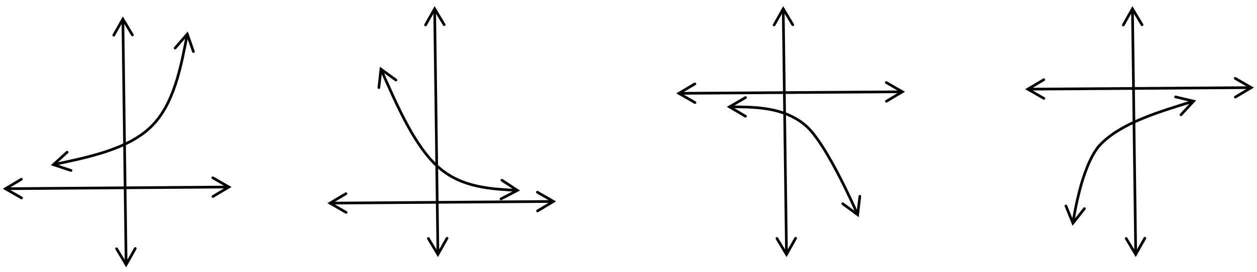
If $q = 0$ and $p = 0$ then $y = ab^x$.

If $p = 0$ then $y = ab^x + q$.

The restriction is $b > 0;\ b \ne 1$

Shape:

for $a > 0$ *and* $b > 1$ for $a > 0$ *and* $0 < b < 1$ for $a < 0$ *and* $b > 1$ for $a < 0$ *and* $0 < b < 1$



- For $y = ab^x$, the **asymptote** is $y = 0$ and the ***y*-intercept** is $y = a$.
- For $y = ab^x + q$, the **asymptote** is $y = q$ and the ***y*-intercept** is $y = a + q$.
- For $y = ab^{x+p} + q$, the **asymptote** is $y = q$ and the ***y*-intercept** is $y = ab^p + q$.

Example no. 3:

Given: $f(x) = 3^{-x+1} - 3$

3.1 Write $f(x)$ in the form $y = ab^x + q$

3.2 Draw the graph of f, showing all the intercepts with the axes and the asymptote.

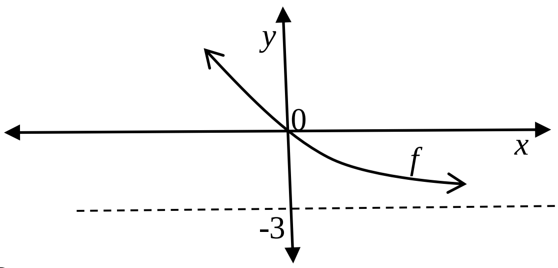
3.3 Write down the domain and the range of f.

<u>Solution</u>**:**

3.1 $y = 3^{-x+1} - 3 = 3^{-x}.3 - 3 = 3.3^x - 3 = 3\left(\frac{1}{3}\right)^x - 3$

3.2 The asymptote is $y = -3$.

For the x-intercept, let $y = 0$: $3\left(\frac{1}{3}\right)^x - 3 = 0$
$$\left(\frac{1}{3}\right)^x = 1$$
$$x = 0$$



3.3. The domain is $x \in R$, and the range is $y > -3;\ y \in R$.

QUESTIONS FROM PAST PAPERS ON FUNCTIONS AND GRAPHS

QUESTION 5 (GR. 12 DBE NOVEMBER 2010)

Consider the function $f(x) = 4^{-x} - 2$

5.1 Calculate the coordinates of the intercepts of *f* with the axes. (4)

5.2 Write down the equation of the asymptote of *f*. (1)

5.3 Sketch the graph of *f*. (3)

5.4 Write down the equation of g if *g* is the graph of *f* shifted 2 units upwards. (1)

5.5 Solve for x if $f(x) = 3$. (You need not simplify your answer.) (3)

[12]

QUESTION 5 (Gr. 12 DBE MARCH 2011)

Consider the function $f(x) = \frac{3}{x-1} - 2$.

5.1 Write down the equations of the asymptotes of *f*. (2)

5.2 Calculate the intercepts of the graph of f with the axes. (3)

5.3 Sketch the graph of *f*. (3)

5.4 Write down the range of $y = -f(x)$. (1)

5.5 Describe, in words, the transformation of f to g if $g(x) = \frac{-3}{x+1} - 2$. (2)

[11]

QUESTION 5 (GR. 12 DBE MARCH 2010)

Given: $f(x) = \frac{2}{x-3} + 1$

5.1 Write down the equations of the asymptotes of *f*. (2)

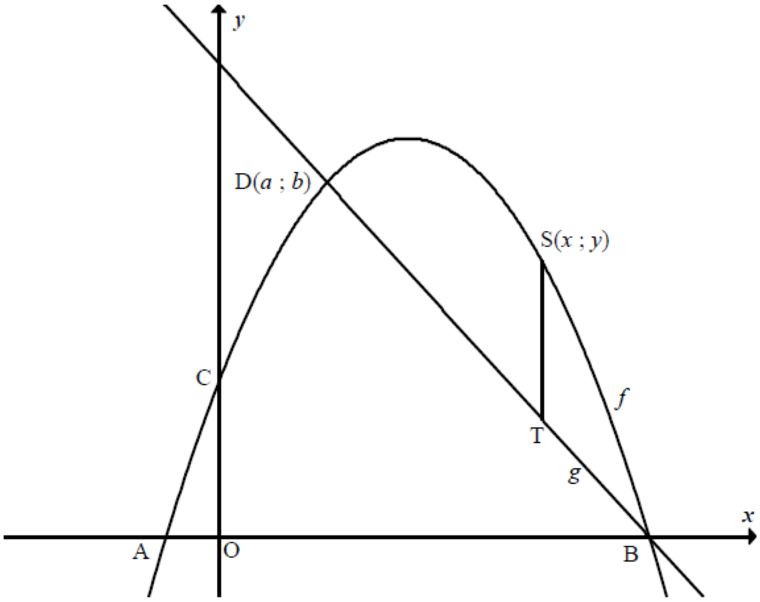
5.2 Calculate the coordinates of the *x*- and *y*-intercepts of *f*. (3)

5.3 Sketch *f*. Show all intercepts with the axes and the asymptotes. (3)

[8]

QUESTION 6 (GR. 12 DBE MARCH 2010)

The graphs of $f(x) = -x^2 + 7x + 8$ and $g(x) = -3x + 24$ are sketched below.
f and g intersect in D and B. A and B are the x-intercepts of f.



6.1 Determine the coordinates of A and B. (4)

6.2 Calculate a, the x-coordinate of D. (4)

6.3 S(x ; y) is a point on the graph of f, where $a \leq x \leq 8$. ST is drawn parallel to the y-axis with T on the graph of g. Determine ST in terms of x. (2)

6.4 Calculate the maximum length of ST. (2)

[12]

QUESTION 4 (GR. 12 DBE MARCH 2015)

Given: $g(x) = \frac{6}{x+2} - 1$

4.1 Write down the equations of the asymptotes of g. (2)

4.2 Calculate:

4.2.1 The y – intercept of g (1)

4.2.2 The x – intercept of g (2)

4.3 Draw the graph of g, showing clearly the asymptotes and the intercepts with the axes. (3)

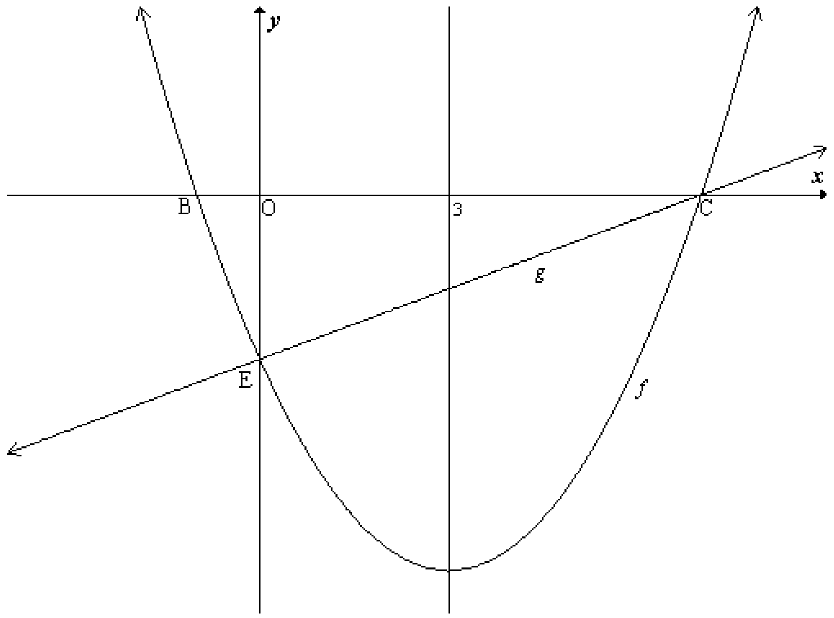
4.4 Determine the equation of the line of symmetry that has a negative gradient, in the Form $y = \ldots$ (3)

4.5 Determine the value(s) of x for which : $\frac{6}{x+2} - 1 \geq -x - 3$ (2)

[13]

QUESTION 6 (GR. 12 DBE MARCH 2011)

A parabola f intersects the x-axis at B and C and the y-axis at E. The axis of symmetry of the parabola has equation $x = 3$. The line through E and C has equation $g(x) = \frac{x}{2} - \frac{7}{2}$.



6.1	Show that the coordinates of C are (7 ; 0).	(1)
6.2	Calculate the x-coordinate of B.	(1)
6.3	Determine the equation of f in the form $y = a(x-p)^2 + q$.	(6)
6.4	Write down the equation of the graph of h, the reflection of f in the x-axis.	(1)
6.5	Write down the maximum value of $t(x)$ if $t(x) = 1 - f(x)$.	(2)
6.6	Solve for x if $f(x^2 - 2) = 0$.	(4)
		[15]

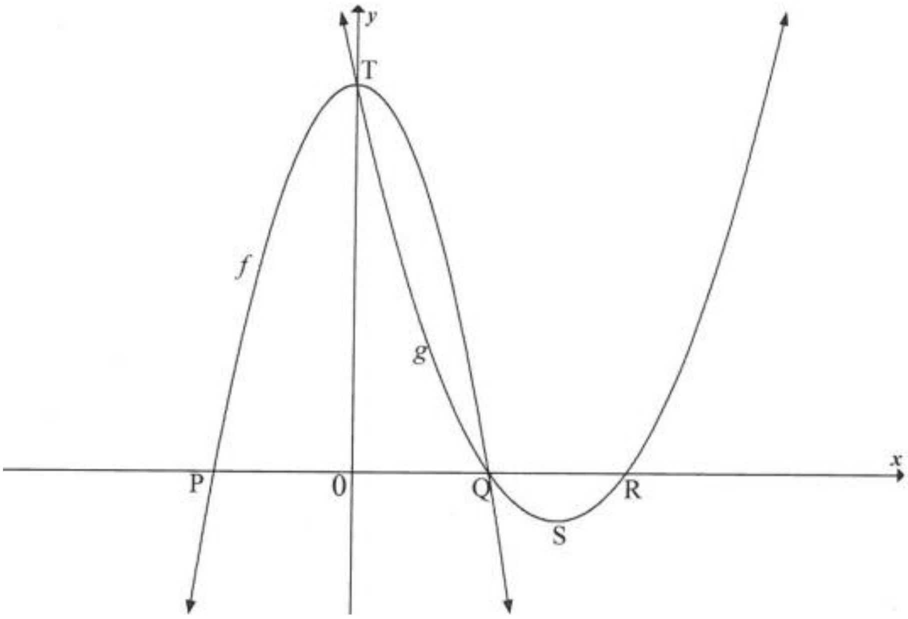
QUESTION 4 (GR. 12 DBE NOVEMBER 2015)

Given: $f(x) = 2^{x+1} - 8$

4.1	Write down the equation of the asymptote of *f*.	(1)
4.2	Sketch the graph of *f*. Clearly indicate ALL intercepts with the axes as well as the asymptote.	(4)
4.3	The graph of g is obtained by reflecting the graph of *f* in the *y* – axis. Write down the equation of *g*.	(1)
		[6]

QUESTION 6 (GR. 12 DBE NOVEMBER 2015)

6.1 The graphs of $f(x) = -2x^2 + 18$ and $g(x) = ax^2 + bx + c$ are sketched below.
Points P and Q are the x – intercepts of f. Points Q and R are the x – intercepts of g.
S is the turning point of g. T is the y – intercept of both f and g.



6.1.1 Write down the coordinates of T. (1)

6.1.2 Determine the coordinates of Q. (3)

6.1.3 Given that x = 4,5 at S, determine the coordinates of R. (2)

6.2 The function defined as $y = \frac{a}{x+p} + q$ has the following properties:

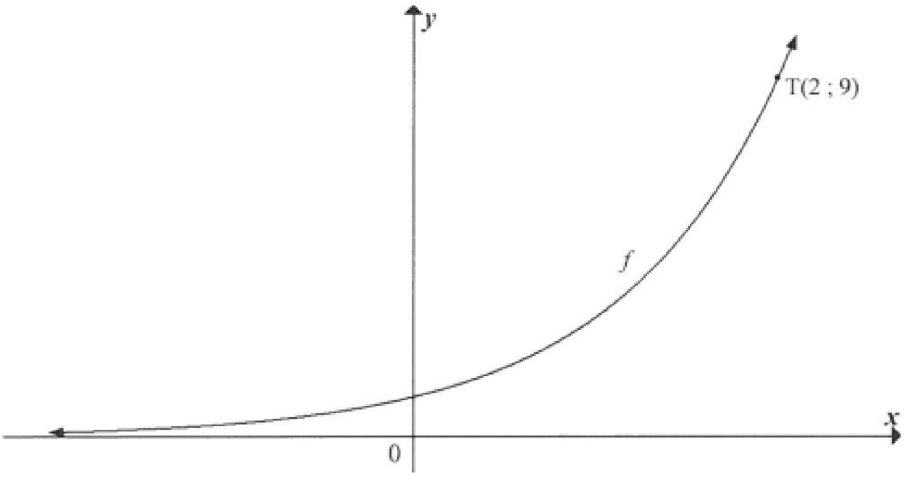
- The domain is $x \in R,\ x \neq 2$.
- $y = x + 6$ is an axis of symmetry.
- The function is increasing for all $x \in R,\ x \neq 2$.

Draw a neat sketch graph of this function. Your sketch must include the asymptotes, if any. (4)

[10]

QUESTION 5 (GR. 12 DBE MARCH 2015)

The graph of $f(x) = a^x, a > 1$ is shown below. T(2 ; 9) lies on f.



5.1 Calculate the value of a. (2)

5.2 Determine the equation of $g(x)$ if $g(x) = f(x)$ (1)

5.3 Determine the value(s) of x for which $f^{-1}(x) \geq 2$. (2)

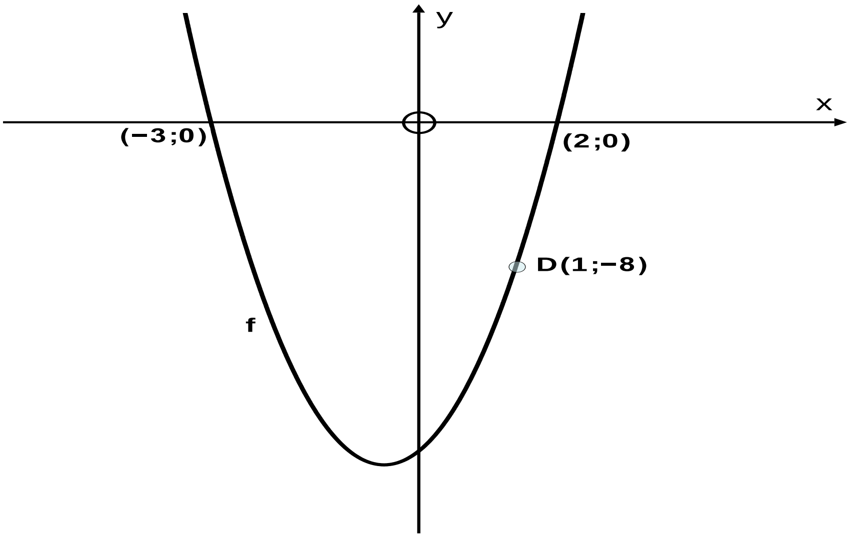
5.4 Is the inverse of f a function? Explain your answer. (2)

QUESTION 5 (GR. 12 DBE MARCH 2015 – adapted for gr. 11)

The graph of $f(x) = ax^2 + bx + c\ ; a \neq 0$ is drawn below.

D(1 ; – 8) is a point on f.

f intersects the $x-$axis at (– 3 ; 0) and (2 ; 0).



6.1 For which value(s) of x is $f(x) \leq 0$? (2)

6.2 Determine the values of a, b and c. (5)

6.3 Determine the coordinates of the turning point of f. (3)

6.4 Write down the equation of the axis of symmetry of h if $h(x) = f(x-7) + 2$. (2)

[12]

QUESTION 5 (GR. 12 DBE NOVEMBER 2011)

5.1 Consider the function: $f(x) = \frac{-6}{x-3} - 1$

5.1.1 Calculate the coordinates of the $y-$intercept of f. (2)

5.1.2 Calculate the coordinates of the $x-$intercept of f. (2)

5.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (4)

5.1.4 For which values of x is $f(x) > 0$? (2)

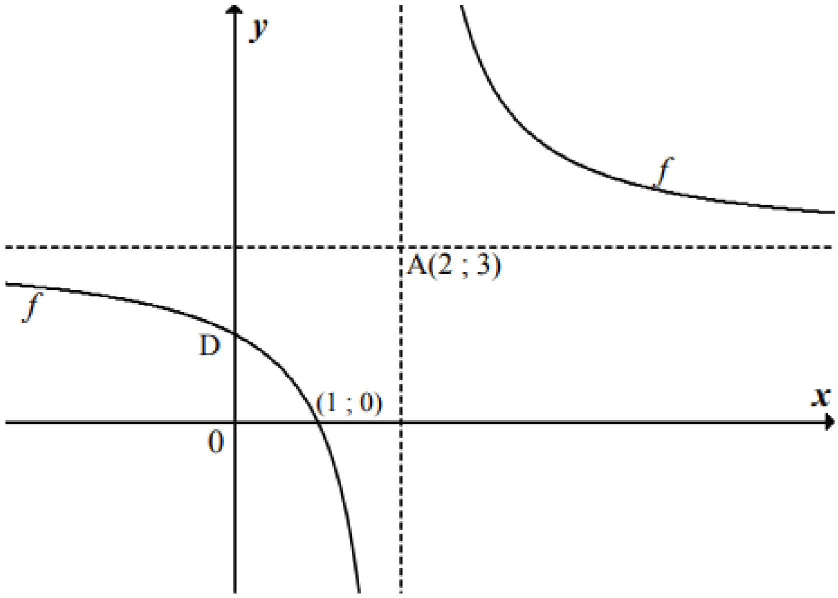
5.1.5 Calculate the average gradient of f between $x = -2$ and $x = 0$. (4)

5.2 Draw a sketch graph of $y = ax^2 + bx + c$, where $a < 0,\ b < 0,\ c < 0$ and $ax^2 + bx + c = 0$ has only ONE solution. (4)

[19]

QUESTION 4 (GR. 12 DBE NOVEMBER 2010)

Given: $f(x) = \frac{a}{x-p} + q$.
The point A(2; 3) is the point of intersection of the asymptotes of f.
The graph of f intersects the $x-$axis at (1; 0).
D is the $y-$intercept of f.



4.1 Write down the equations of the asymptotes of f. (2)

4.2 Determine an equation of f. (3)

4.3 Write down the coordinates of D. (2)

4.4 Write down an equation of g if g is the straight line joining A and D. (3)

4.5 Write down the coordinates of the other point of intersection of f and g. (4)

[14]

PROBABILITY

FROM GR. 11 Annual Teaching Plan 2020:

DATES	CURRICULUM STATEMENT
20/8 – 21/8 (2 days)	✓ Revise the Addition Rule for Mutually Exclusive Events: P (A or B) = P (A) + P (B) , ✓ Complementary Rule: P (not A) = 1 – P (A) ✓ And the identity P (A or B) = P (A) + P (B) – P (A and B)
24/8 – 31/8 (6 days)	✓ Identify Dependent and independent events and the Product Rule for Independent Events. ✓ P (A and B) = P (A) × P (B). ✓ The use of Venn Diagrams (for any 3 events in a sample space) to solve probability problems. ✓ Use Tree Diagrams for the probability of consecutive or simultaneous events which are not necessarily independent.

SEPTEMBER COMMON TEST WEIGHTING

Probability	20 ±3 marks out of 75 marks

CAPS EXAM GUIDELINE WEIGHTING FOR NOVEMBER EXAMINATION

Probability	20 ± 3 marks out of 150 marks in Paper 1

1. REVISION EXERCISE ON GR. 10 PROBABILITY:

1.1 In a staff of 50 teachers a survey was conducted to establish how many drink tea and how many drink coffee during their lunch break. The following was found:

- 15 teachers did not drink either tea or coffee;
- 21 drank tea
- 18 drank coffee

1.1.1 Represent this information in a Venn diagram.
Let T= {teachers who drink tea} and C= {teachers who drink coffee}.
Hint: Let the number of teachers who drink both tea and coffee = x.

1.1.2 Calculate n(C and T)

1.1.3 If a teacher is chosen randomly, calculate the probability that she/he drinks
(a) Tea only
(b) Coffee only
(c) Neither coffee nor tea
(d) Tea and coffee
(e) Tea or coffee

1.2 Numbers 1 to 30 are placed in a bag. One number is drawn at random.

1.2.1 What is the probability that:
(a) It is divisible by 3?
(b) It is not divisible by 3?
(c) It is divisible by 4?
(d) It is not divisible by 4?
(e) It is divisible by 3 and 4?
(f) It is divisible by 3 or 4?

1.2.2 Is the event: divisible by 3 and the event: divisible by 4 mutually exclusive? Give a reason.

1.2.3 Which of the events (a) to (f) are complementary events? Substantiate your answer.

1.3 There are 250 grade 10 learners at a school. 190 learners offer Mathematics and 150 offer Accounting. There are 60 learners who do not offer Mathematics or Accounting.
Calculate the probability that a randomly chosen learner:

1.3.1 offers Mathematics only
1.3.2 offers Accounting only
1.3.3 offers Mathematics and Accounting.

1.4 A and C are mutually exclusive events. If P (A) = 0,25 and P (A or C) = 0,41, determine P (C).

2. SUMMARY OF GRADE 11 PROBABILITY:

Notes to remember:

- Probability is a chance for something to happen; it ranges from 0 – 1 or 0 – 100% as shown below:
  - ✓ $0 \leq P(A) \leq 1$
- Notations
  - ✓ P (A or B) = P $(A \cup B)$
  - ✓ P (A and B)=P $(A \cap B)$
  - ✓ P (not A) = P $(A^{/})$
- Addition rule
  - ✓ P (A or B) = P (A) + P (B) – P (A and B)
- Mutually exclusive events
  - ✓ P (A and B) = 0
- Independent events
  - ✓ P (A and B) = P (A) $\times$ P (B)
- Complementary Events
  - ✓ P (A) +P (B) = 1
  - ✓ $\therefore$ P (not A) = 1 − P (A)
- Venn diagrams of Two events and at most THREE Events
  - ✓ Use numbers and probabilities (**decimals and percentages diagrams)**
  - ✓ Mutually exclusive
  - ✓ Inclusive events
- Tree diagrams
  - ✓ Use numbers and probabilities **(decimals and percentages diagrams)**
  - ✓ Emphasise the rule for independency
- Two way contingency tables
  - ✓ Use numbers and probabilities **(decimals and percentages diagrams)**
  - ✓ Emphasize independent rule

3. VENN DIAGRAMS:

Example no. 1:

The manager of a hotel in Pretoria recorded the number of guests sitting down for breakfast, lunch and supper on a particular day. Of the 300 guests, 29 did not arrive for any of the three meals.

153 were at breakfast
161 were at lunch
145 were at supper
95 were at breakfast and lunch
80 were at lunch and supper
52 were at supper, but did not arrive for the other two meals
70 were at all three meals.

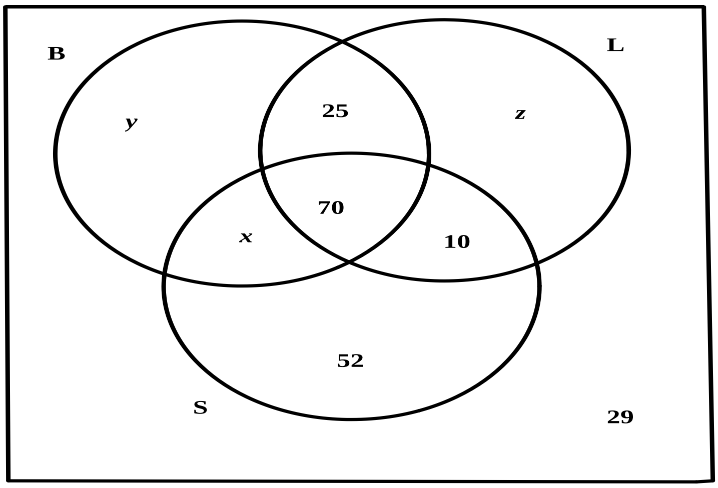
1.1 Draw a Venn diagram to illustrate the above information.

1.2 Calculate the probability that a guest chosen at random will have:

- 1.2.1 Been at both breakfast and lunch, but not supper
- 1.2.2 Been at both breakfast and lunch
- 1.2.3 Been at breakfast only
- 1.2.4 Been at one or more of the meals

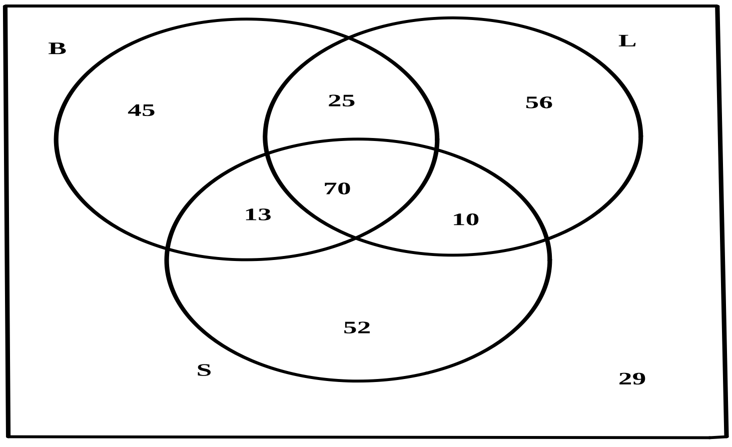
Solution:

1.1



Note : All unknown sample spaces should be indicated using variables.

For supper : $x + 70 + 10 + 52 = 145 \ldots \ldots . . x = 13$
For breakfast : $25 + 70 + 13 + y = 153 \ldots . . y = 45$
For lunch : $25 + 70 + 10 + z = 161 \ldots \ldots \ldots . . z = 56$



1.2

1.2.1 $P(B \text{ and } L \text{ but not } S) = \frac{25}{300}$

1.2.2 $P(B \text{ and } L) = \frac{95}{300}$

1.2.3 $P(B \text{ only}) = \frac{45}{300}$

1.2.4 $P(B \text{ or } L \text{ or } S) = \frac{271}{300}$

Example no. 2:

It is given that P (A) = 0.35, P (B) = 0.8 and P (A and B)= 0.25.
Use a Venn diagram and a formula to determine:

2.1 P (A or B) 2.2 P (not A and B) 2.3 P (not A or B)
2.4 P (not(A and B)) 2.5 P (not(A or B))

Solution:

2.1 0,9 2.2 0,55 2.3 $0{,}55+0{,}1+0{,}21=0{,}9$
2.4 $0{,}1+0{,}55+0{,}1=0{,}75$ 2.5 $P(A\cup B)^{/}=0{,}1$

Exercise on Venn diagrams:

3.1 In a group of 120 Grade 11 learners, each takes at least one of the following three subjects, namely, Mathematics, Physical Sciences and Accounting. 13 learners take all three subjects, 31 take Mathematics and Physical Sciences, 23 take Mathematics and Accounting, 16 take Physical Sciences and Accounting, 35 take Mathematics only, 22 take only Physical Sciences and 19 take only Accounting.

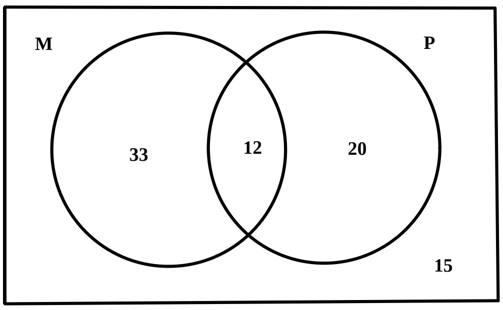
Calculate the probability that a learner chosen at random takes:

3.1.1 Mathematics
3.1.2 Physical Sciences
3.1.3 Mathematics and Physical Sciences but not Accounting
3.1.4 Mathematics or Physical Sciences
3.1.5 Mathematics or Accounting

3.2 At Mbaza High there are 126 learners in grade 11. Of these 44 offer Accounting, 112 offer Mathematics and 90 offer Economics. All learners who offer Accounting also offer Mathematics, 80 learners offer Mathematics and Economics and 30 learners offer Accounting and Economics.
If a learner is chosen at random, determine the probability that a learner:

3.2.1 offers none of these subjects.
3.2.2 offers Mathematics and Economics.
3.2.3 offers Mathematics or Economics.
3.2.4 does not offer Mathematics and Economics, i.e. determine P (not[M and E]).

3.3 Consider the following Venn diagram below.



3.3.1 Calculate:
(a) $P(M)$;
(b) $P(M\cup P)$;
(c) $P(M\cap P)$
(d) the probability that neither M nor P occurs;
(e) the probability that P occurs, but not M.

3.3.2 Show, by calculation, that
$P(M\cup P)=P(M)+P(P)-P(M\cap P)$

3.3.3 Are M and P mutually exclusive or not?
Justify your answer.

3.4 A and B are events such that P (A) = 0,35; P (B) = 0,25 and P (A or B) = 0,5.

Determine:

3.4.1 P (A and B); 3.4.2 P (not A) 3.4.3 P (not A or B)

3.5. Fifty learners were asked if they have ever broken an arm, a leg or their nose.

21 had broken a leg
28 had broken an arm
8 had broken their noses
9 had broken an arm and a leg
5 had broken an arm and their nose
6 had broken their nose and a leg
10 had not broken any of these

3.5.1 Display this information in a Venn diagram. Then determine how many learners had broken an arm, a leg and their nose.

3.5.2 Determine how many learners had only broken a leg.

3.5.3 A learner is randomly chosen from those surveyed.
Find the probability that :

(a) The learner had broken an arm only.

(b) The learner had not broken their nose.

(c) The learner had broken a nose and a leg.

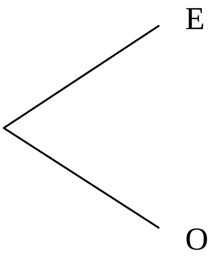
(d) The learner had broken an arm or a leg.

4. TREE DIAGRAMS

Tree diagrams can help to alleviate much of the confusion over when to add and when to multiply probabilities. In particular, the addition rule could be demonstrated, though all problems could be solved without recourse to the formula.

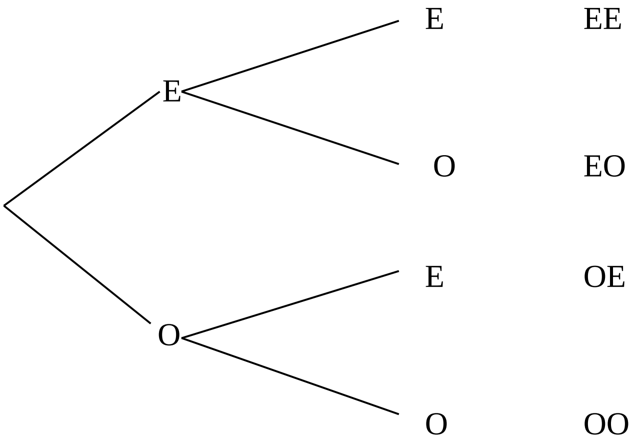
A tree diagram can be of great assistance to list all the possibilities of an event. Take for example, Suppose we are interested in whether we get an even number (E) or an odd number (O), when a die is rolled.

Outcome



If the die is rolled the second time the tree diagram grows as follows:

First roll **Second roll** **Combined outcomes**



Example:

Suppose the die is rolled 2 times.
What is the probability of getting even and odd numbers?

Solution: $P(Odd\ and\ Even) = \frac{n(O\ and\ E)}{n(S)} = \frac{2}{4} = \frac{1}{2}$

Suppose the die is rolled 3 times.
What is the probability of getting 1 even and 2 odd numbers?

Solution: This means that *n (A) = 8* and *n (1 even & 2 odd) = 3*
P (1 even & 2 odds) = $\frac{3}{8}$

Exercise on Tree Diagrams:

4.1 John, Tim, Dick, Mary and Helen are candidates for three positions on the Learner Representative Council. Assuming the election of any of them is equally likely, what is the probability that:
   4.1.1 Tim is elected?
   4.1.2 Mary and Helen are both elected?
   4.1.3 Dick is not elected?
   4.1.4 Either Mary or Helen (or both) is elected?

4.2 It is winter in England. The probability that there will be snow tomorrow if it is snowing today is 0,7 and the probability that it will snow tomorrow if it is clear today is 0,4. It is snowing on Saturday. What is the probability that it will snow on Monday?

4.3 Sipho and Mbali have entered a maths quiz. They are asked TWO questions. The probability that they get the first answer correct is 80%. If the first answer is correct, the probability of them getting the next answer correct is 70%.
However, if they get the first answer wrong, the probability of them getting the next answer correct is 40%.
Determine the probability that they get the second answer correct.

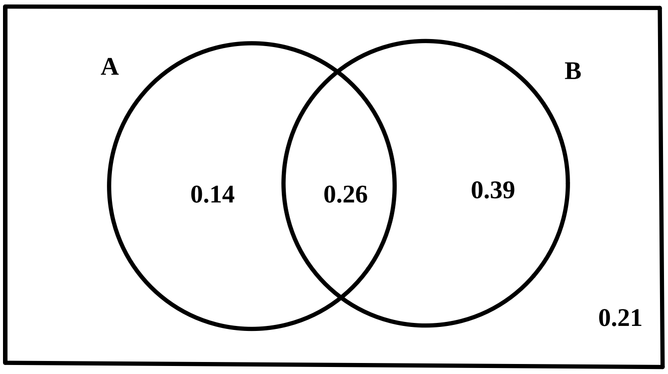
5. INDEPENDENT EVENTS AND VENN DIAGRAMS

Example:

The probability that event A will occur is 0.4 and the probability that event B will occur is 0.65.
The probability that both A and B will occur is 0.26.

1. Are events A and B mutually exclusive? Explain your answer.
2. Are evens A and B independent? Explain your answer.

Solution**:**



1. No. There is an intersection between A and B. Alternatively: events A and B occur simultaneously.

2. Yes. $P(A\ and\ B) = 0{,}26$ while $P(A) \times P(B) = 0.4 \times 0.65 = 0.26$

   $P(A\ and\ B) = P(A) \times P(B)$; $\therefore$ the events are independent.

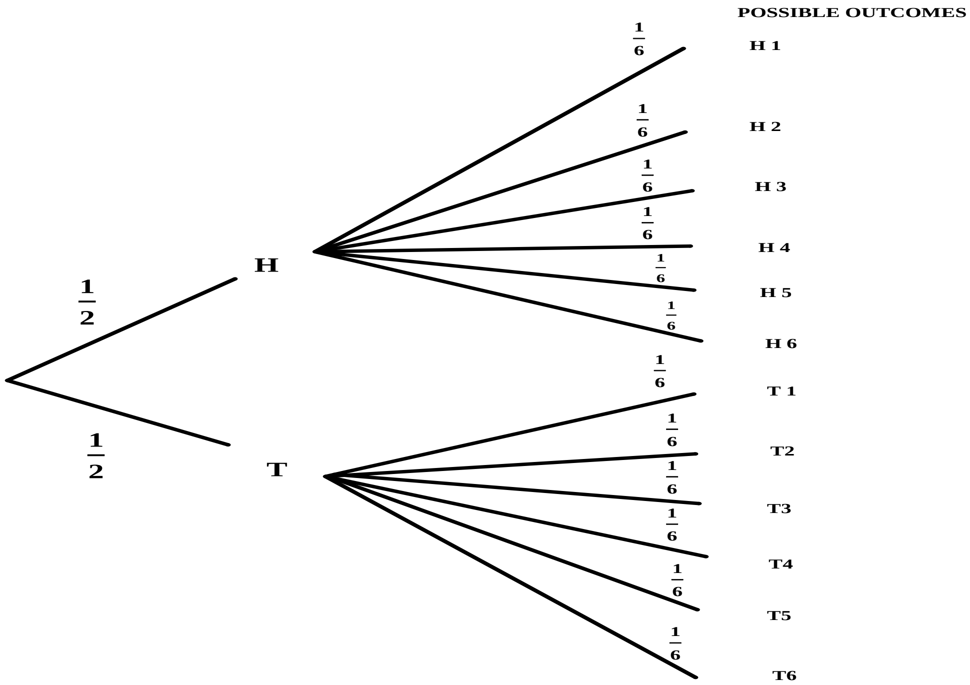
6. INDEPENDENT EVENTS AND TREE DIAGRAMS

The multiplication law can be used when you draw tree diagrams of independent events.

Example:

A tree diagram allows us to use both the **Addition law** of probabilities (for mutually exclusive events) and the **Multiplication law** (for independent events)

Suppose that a coin is tossed and then a die is thrown. We can represent the possible outcomes in a tree diagram:



There are twelve possible outcomes when event A (tossing a coin) is followed by even B (rolling a die). The chance of getting the number 1, 2, 3, 4, 5 or 6 is independent of whether you previously first got a Head or a Tail. Event A and event B are independent of each other.

If we now consider these events in terms of probabilities, some interesting information emerges.

With tossing a coin : $P(H) = \frac{1}{2}$ and $P(T) = \frac{1}{2}$

With throwing a die $P(any\ number\ from\ 1\ to\ 6) = \frac{1}{6}$

Probability of getting H and 4 (using the branches on the tree diagram)

$P(H\ and\ 4) = P(H) \times P(4) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

Also : There is one H4 out of twelve possible outcomes hence, $P(H4) = \frac{1}{12}$

Therefore the two events are **independent,** because $P(H \text{ and } 4) = P(H) \times P(4)$

Exercise on Independent events and Tree diagrams:

6.1 Calculate the following probabilities in a family of three children:

6.1.1 First is a boy and the next two are girls.
6.1.2 First two are boys and the third is a girl.
6.1.3 First two are boys or the third is a boy.

6.2 A bag contains six black balls, four red balls, and five white balls. Three balls are drawn in succession, each one being replaced before the next is drawn.
Calculate the following probabilities.

6.2.1 First is black, second is red and third is white.

6.2.2 All three are black.

6.2.3 All three are white.

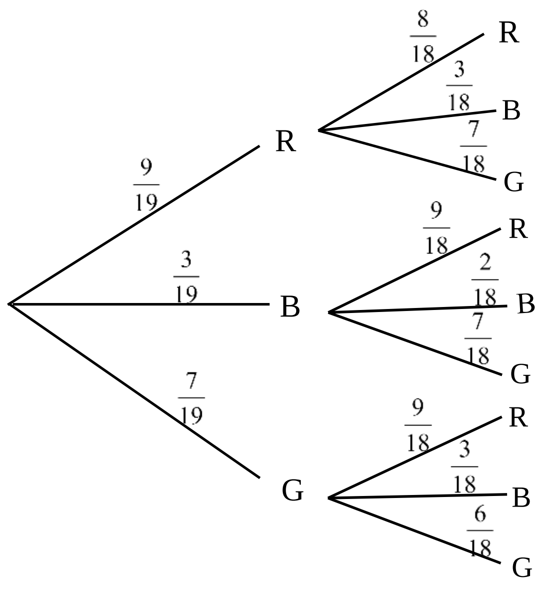
6.2.4 First two are black and the third is red.

Events that are not independent

Example:

In this example the outcome of the first event has an influence on the outcome of the second event.
A box contains 3 Blue, 9 Red and 7 Green cards. In a game a card is drawn from the box and not replaced. If another card is drawn, determine the probability that:
(a) The two cards are of the same colour
(b) The two cards are of different colours
(c) The first card was red

Solution:



(a) P (RR) + P (BB)+ P (GG) = $(\frac{3}{19} \times \frac{2}{18}) + (\frac{9}{19} \times \frac{8}{18}) + (\frac{7}{19} \times \frac{6}{18}) = \frac{20}{57}$

(b) P (cards of different colour) = $1 - \frac{20}{57} = \frac{37}{57}$

(c) P (first card is red) = $\frac{3}{19}$

7. CONTINGENCY TABLES:

Besides a Venn diagram and a tree diagram, the combination of events A and B can also be illustrated by means of a two-way table consisting of rows and columns. We call a table like this a contingency table.
A **contingency table** provides a different way of calculating probabilities.

Example no. 1:

1. Suppose a study of speeding violations and drivers who use cell phones while driving, produced the following data:

	Speeding violation	**No speeding violation**	**Total**
Cell phone user	25	280	**305**
Not cell phone user	45	405	**450**
Total	**70**	**685**	**755**

The total number of people in the sample space is 755. The row totals are 305 and 450.
The column totals are 70 and 685. Notice that 305 + 450 = 755 and 70 + 685 = 755.

Use the table to calculate the following probabilities:

1.1 P (driver is a cell phone user)
1.2 P (driver has no violation)
1.3 P (driver is a cell phone user AND driver has no violation)
1.4 P (driver is a cell phone user OR driver has no violation)

Solution:

1.1 P (driver is a cell phone user) $= \frac{305}{755}$

1.2 P (driver has no violation) $= \frac{685}{755}$

1.3 P (driver is a cell phone user AND driver has no violation) $= \frac{280}{755}$

1.4 P (driver is a cell phone user OR driver has no violation) $= \frac{305}{755} + \frac{685}{755} - \frac{280}{755} = \frac{710}{755}$

Example no. 2:

2. The following table show a random sample of 100 hikers and the preferred hiking spots.

Gender	**Near shopping centre**	**Cross roads**	**Near taxi-rank**	**Total**
Female	18	16	....	45
Male	....	....	14	55
Total	....	41	....	....

2.1 Complete the table.

2.2 Are the events "being female" and "prefer near shopping centre" independent events?
Let F = being female and C = prefer near shopping centre
Calculate P (F and C)
Calculate P (F) × P (C)
Are these two numbers the same? If they are, then F and C are independent.
If they are not, then F and C are not independent.

QUESTIONS FROM PAST PAPERS ON PROBABILITY

QUESTION 1

A packet of sweets contains 3 pink, 2 green and 5 blue sweets. Two sweets are remove in succession from the packet without replacing them.

1.1 Draw a tree diagram to determine all possible outcome. (6)

1.2 Determine the probability that: (Round off your answers to THREE decimal places)

1.2.1 Both sweets are blue (2)

1.2.2 A green and a pink sweets are selected (5)

1.3 A survey was conducted amongst 60 boys and 60 girls in grade 8 relating to their participation in sport. 20 girls did not participate in any sport and 50 boys did participate in sport.

1.3.1 Complete a two way contingency table for the above survey.

1.3.2 If a learner is selected at random, calculate the probability that it will be

(a) a girl who participates in sport. (1)

(b) a learner who does not participate in sport and is not female. (1)

[20]

QUESTION 8 (GR. 11 DBE NOVEMBER 2018)

A bag contains 6 red balls, 8 green balls and an unknown number of yellow balls.
The probability of randomly choosing a green ball from the bag is 25%.

8.1 Show that there are 32 balls in the bag (1)

8.2 A ball is drawn from the bag, the colour is recorded and not returned to the bag. Thereafter another ball is drawn from the bag, the colour is recorded and it is also not returned to the bag.
Draw a tree diagram to represent all the possible ways in which the two balls could have been drawn from the bag. Show the probabilities associated with each branch, as well as the combined outcomes. (4)

8.3 Calculate the probability that the two balls drawn from the bag will have same colour. (4)

QUESTION 9 (GR. 11 DBE NOVEMBER 2018)

9.1 On a flight, passengers could choose between a vegetarian snack and a chicken snack.
The snacks selected by the passengers were recorded.
The results are shown in the table below:

Snack	Male	Female	Total
Vegetarian	12	20	32
Chicken	55	63	118
Total	67	83	150

Was the choice of snack on this flight independent of gender? Motivate your answer with the necessary calculations.

9.2 For any two events, A and B, it is given that P (A and B) = 0,12 , P (A or B) = 0,83 and P (B) = 4P (A).

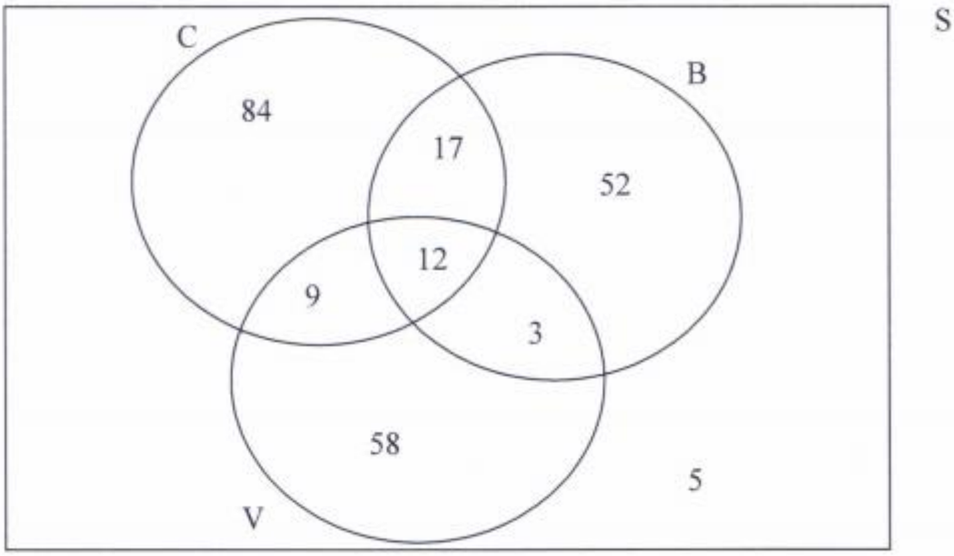
9.2.1 Are A and B mutually exclusive? Justify your answer. (2)

9.2.2 Calculate P (B) (4)

9.2.3 Calculate P (not A) (2)

QUESTION 10 (GR. 11 DBE NOVEMBER 2014)

A survey was carried out with 240 customers who bought food from a fast food outlet on a particular day. The outlet sells cheese burgers (C), bacon burgers (B) and vegetarian burgers (V). The Venn diagram below shows the number of customers who bought different types of burgers on a day.



10.1	How many customers did not buy burgers on the day?	(1)
10.2	Are events B and C mutually exclusive? Give a reason for your answer.	(2)
10.3	If a customer from this group is selected at random, determine the probability that he/she:	
	10.3.1 Bought only a vegetarian burger	(1)
	10.3.2 Bought a cheese burger and a bacon burger	(1)
	10.3.3 Did not buy a cheese burger	(3)
	10.3.4 Bought at least one of the burgers	(4)
		[12]

QUESTION 8

8.1 A bag contains 3 blue marbles and 2 red marbles. A marble is taken from the bag, the colour is recorded and the marble is put aside. A second marble is taken from the bag, the colour is recorded and then put aside.

8.1.1 Draw a tree diagram to represent the information above. Show the probabilities associated with EACH branch, as well as the possible outcomes. (3)

8.1.2 Determine the probability of first taking a red marble and then taking a blue marble, in that order. (2)

8.2 A and B are two events. The probability that event A will occur is 0,4 and the probability that event B will occur is 0,3. The probability that either event A or event B will occur is 0,58.

8.2.1 Are events A and B mutually exclusive?
Justify your answer with appropriate calculations. (3)

8.2.2 Are events A and B independent?
Justify your answer with appropriate calculations. (3)

QUESTION 9 (GR. 11 DBE NOVEMBER 2017)

A survey was done among 80 learners on their favourite sport.
The results are shown below.

- 52 learners like rugby (R)
- 42 learners like volleyball (V)
- 5 learners like chess (C) only
- 14 learners like rugby and volleyball but not chess
- 12 learners like rugby and chess but not volleyball
- 15 learners like volleyball and chess but not rugby
- x learners like all 3 types of sport
- 3 learners did not like any sport

9.1 Draw a Venn diagram to represent the information above. (5)

9.2 Show that $x = 8$. (2)

9.3 How many learners like only rugby? (1)

9.4 Calculate the probability that a learner, chosen randomly, likes at least TWO different types of sport. (3)

[11]

QUESTION 9 (GR. 11 DBE NOVEMBER 2016)

9.1 Given: P(A) = 0,2
P(B) = 0,5
P (A or B) = 0, 6 where A and B are two different events

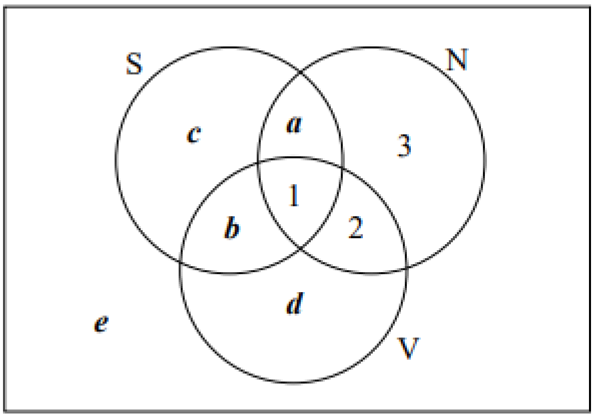
9.1.1 Calculate P(A and B). (2)

9.1.2 Are the events A and B independent ? Show your calculations. (3)

9.2 A survey was conducted amongst 100 learners at a school to establish their involvement in three codes of sport, soccer, netball and volleyball. The results are shown below.

- 55 learners play soccer (S)
- 21 learners play netball (N)
- 7 learners play volleyball (V)
- 3 learners play netball only
- 2 learners play soccer and volleyball
- 1 learner plays all 3 sports

The Venn diagram below shows the information above.



9.2.1 Determine the values of a, b, c, d and e. (5)

9.2.2 What is the probability that one of the learners chosen at random from this group plays netball or volleyball? (2)

9.3 The probability that the first answer in a maths quiz competition will be correct is 0,4. If the first answer is correct, the probability of getting the next answer correct rises to 0,5. However, if the first answer is wrong, the probability of getting the next answer correct is only 0,3.

9.3.1 Represent the information on a tree diagram. Show the probabilities associated with each branch as well as the possible outcomes. (3)

9.3.2 Calculate the probability of getting the second answer correct. (3)
[18]

QUESTION 12 (NOVEMBER 2013 EC)

12.1 It is given that A and B are independent events. P(A) = 0,4 and P(B) = 0,5.

Use a Venn diagram and calculate:

12.1.1 P(A or B) (4)

12.1.2 P(neither A or B) (1)

12.2 During a survey, 25 out of the 40 learners in a class indicated that they own a cellphone. Two learners are selected at random from the class, the first not being replaced before the second one is selected.

12.2.1 Draw a tree diagram that shows the possible outcomes of the situation. Write the probabilities on the relevant branches. (7)

12.2.2 What is the probability that of the two learners selected, one will own a cellphone and the other one not? (3)
[15]

QUESTION 12.1 (GR. 11 DBE EXEMPLAR 2013)

12.2 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy.

At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at the school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics. (6)

REFERENCES:

1. 2017 and 2018 JIT Gr. 11 Documents
2. DBE Mathematics Question Papers Gr. 11 and Gr. 12
3. Platinum Mathematics
4. Mathematics Handbook and Study Guide
5. Maritzburg High Maths Resources