COUNTING & PROBABILITY

TERMINOLOGY RECAP

- Probability lies between 0 (impossible event) and 1 (certain event)

- If all events are equally likely, then the probability of the event is:
$$P(Event) = \frac{P(Successful\ Event)}{Total\ number\ of\ events}$$

- Sample space is made up of all the possible events

TERMINOLOGY RECAP

- Set notation is often used, where

  ∪ is the union of sets

  -> operation is OR

  ∩ is the intersection of sets

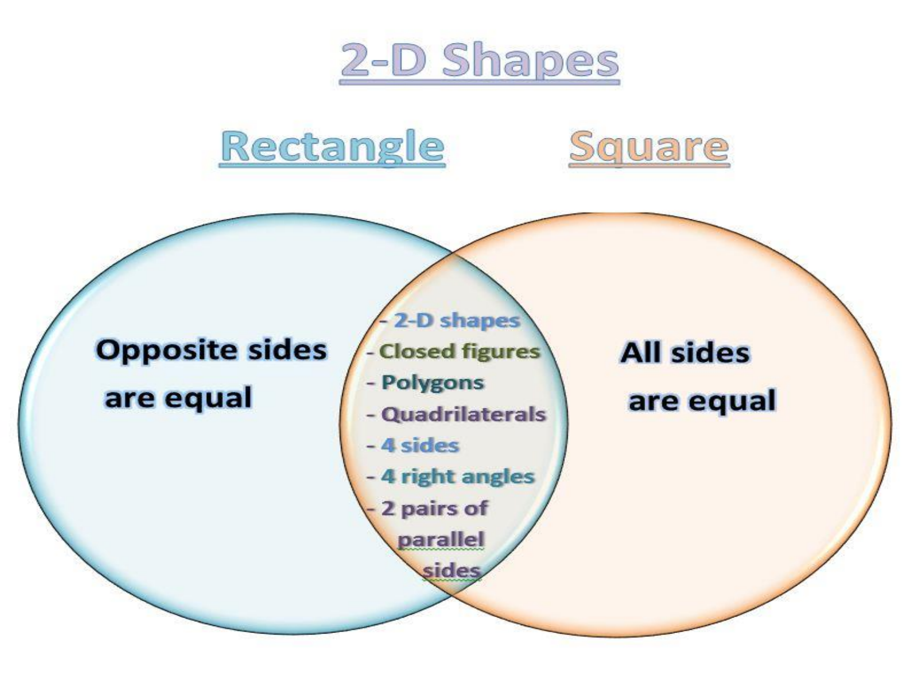
  -> operation is AND

Intersection and Union of Sets

Practicing Basic Set Notation

TOOLS USED IN PROBABILITY

- Venn Diagrams
  Show all possible logical relations between a finite collection of sets



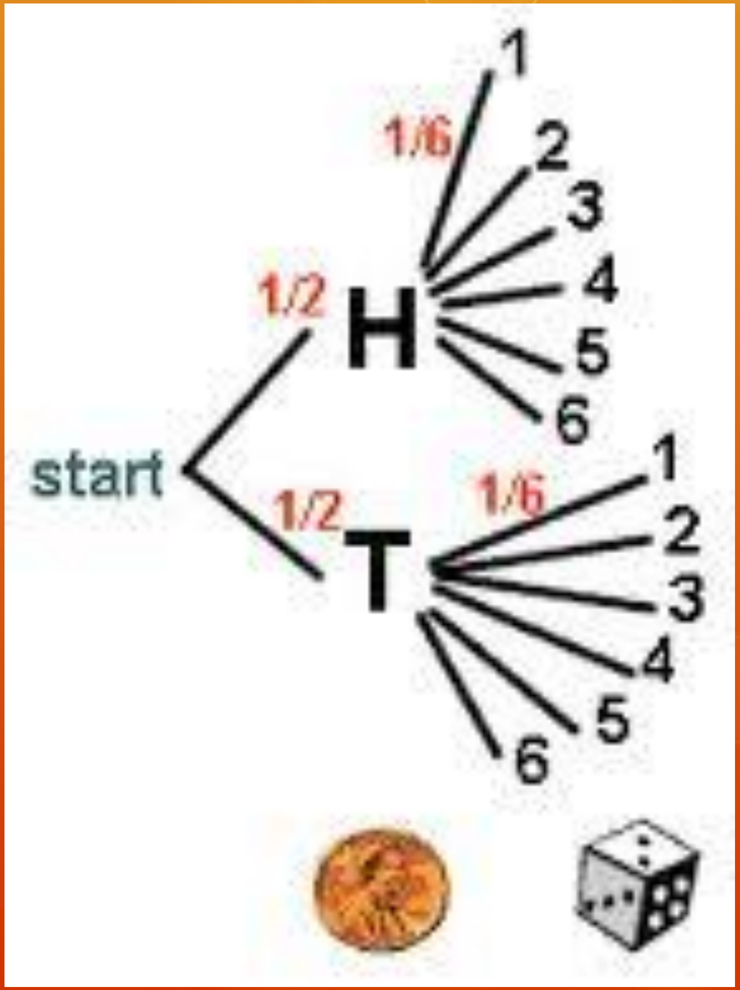
Practicing Venn Diagrams with Set Notation

Practicing Venn Diagrams with Actual Numbers

TOOLS USED IN PROBABILITY

- Tree Diagrams

  Have branches which show the possible outcomes of multiple types of events



Tree Diagrams

TOOLS USED IN PROBABILITY

- Contingency Table

  Is a two-way table consisting of 2 variables to summarize data and compare the variables.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Contingency Table for Die

Using Contingency Tables in Probabilities with Monopoly

INDEPENDENT EVENTS

- The outcome of the 1st event has NO effect on the outcome of the 2nd event
- Product Rule for Independent Events:

$$P(A\ and\ B) = P(A) \times P(B)$$

$$P(A \cap B) = P(A) \times P(B)$$

- E.g. What is the probability of flipping a coin twice and getting Heads twice?

$$P(H\ and\ H) = P(H) \times P(H)$$

$$= \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{4}$$

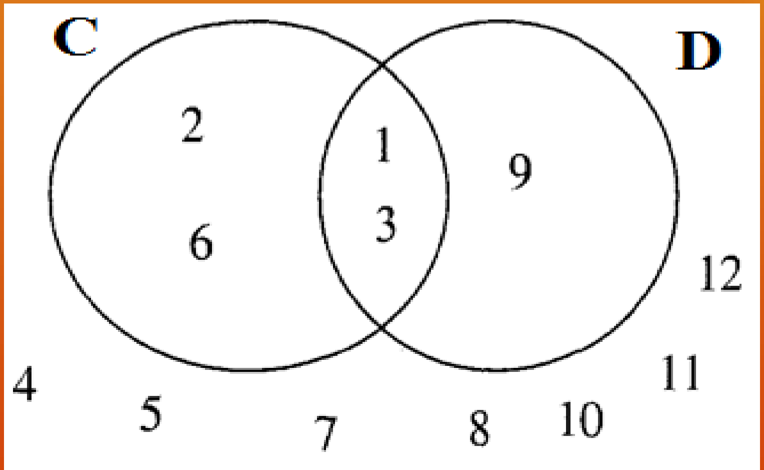
The Coin Toss

Independent Events Venn Diagram

Twelve numbers are dropped into a hat:

Event C = $\{factors\ of\ 6\}$

Event D = $\{factors\ of\ 9\}$

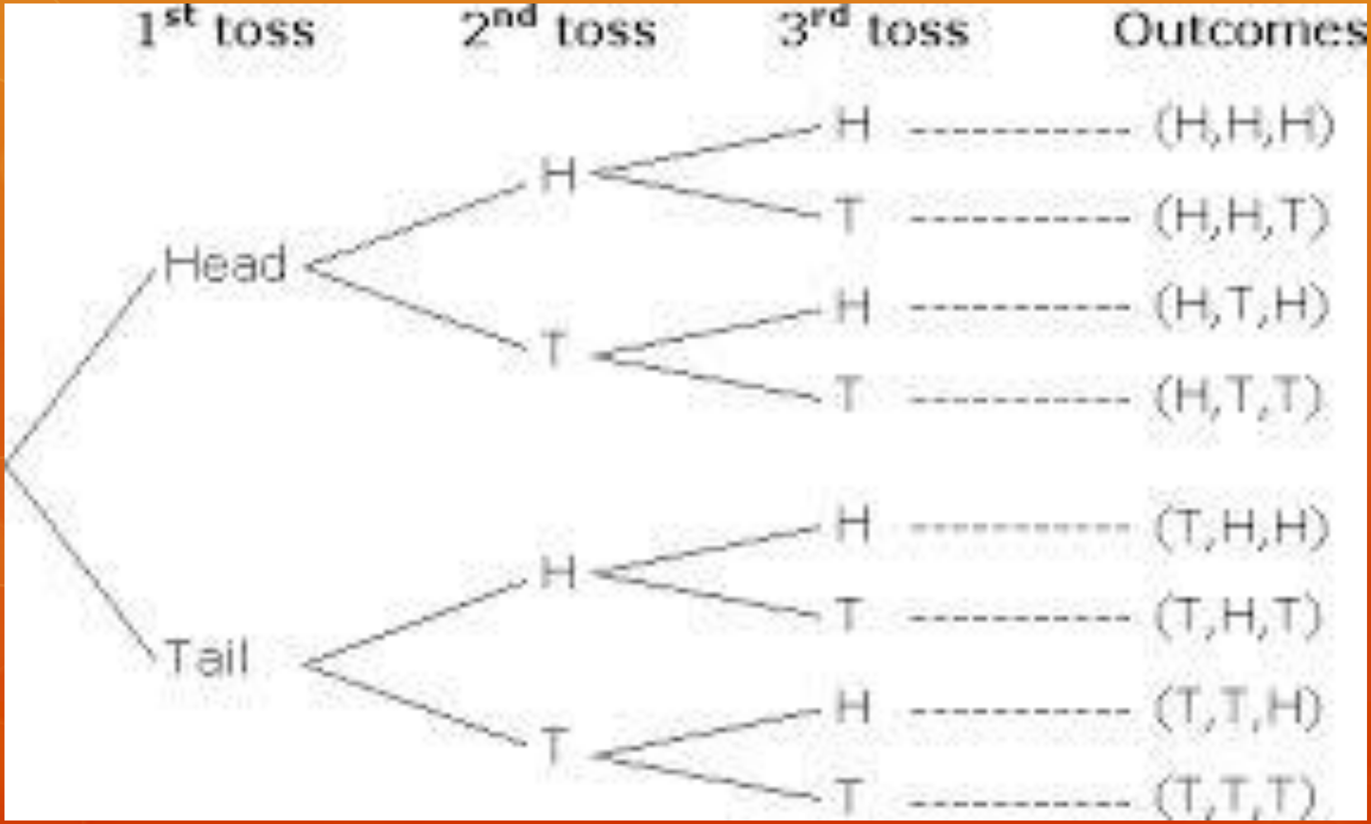


Basic Independent Test Example

$$P(C \cap D) = P(C) \times P(D)$$

Independent Events Tree Diagram

Use a tree diagram to determine the probability of getting two Heads and one Tail when tossing a fair coin three times?



Answer:

$\frac{3}{8}$

Independent Tree Diagram Example

Independent Events Question:

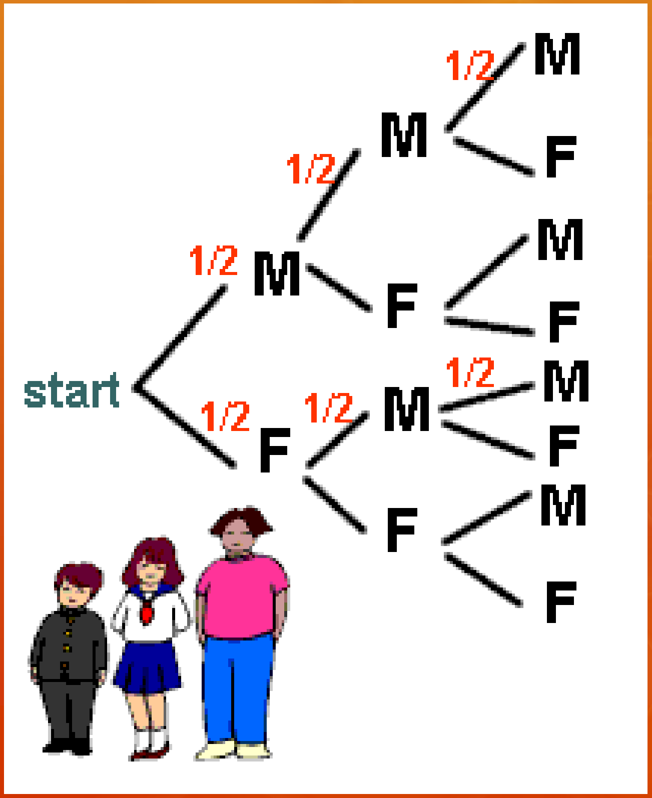
What is the probability of a couple having 3 male children?

$$P(B\ and\ B\ and\ B)$$

$$= P(B) \times P(B) \times P(B)$$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$



Independent Events Question:

If the probability of having an accident is 0,25 and the probability of using a taxi is 0,64, then determine the probability of having an accident while using a taxi.

$$P(accident\ and\ using\ taxi)$$
$$= P(accident) \times P(using\ a\ taxi)$$
$$= 0{,}25 \times 0{,}64$$
$$= 0{,}16$$

<u>Independent Probability Questions</u>

DEPENDENT EVENTS

- The outcome of the 1st event effects the outcome of the 2nd event
- Can use the Product Rule:

$$P(A\ and\ B) = P(A) \times P(B)$$

$$P(A \cap B) = P(A) \times P(B)$$

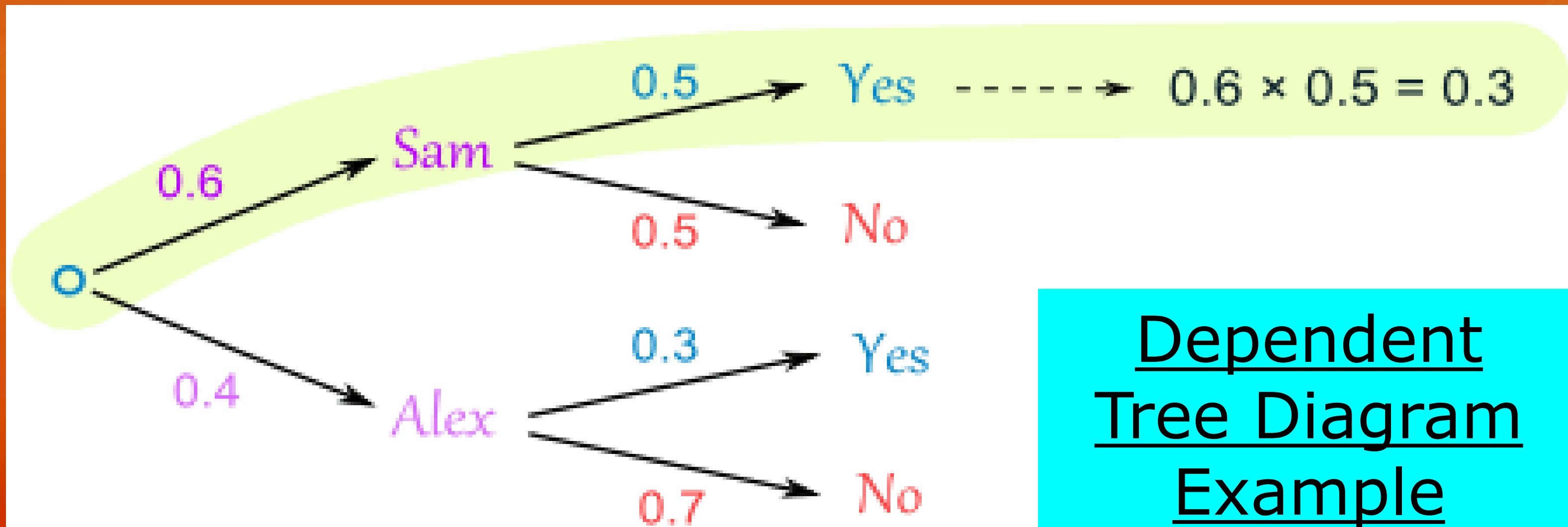
- E.g. What is the probability of drawing two Aces from a standard deck of cards, if the first card drawn is an Ace and is not replaced?

$$P(Ace) \times P(Ace) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$$

A Pack of Cards

Dependent Events Tree Diagram

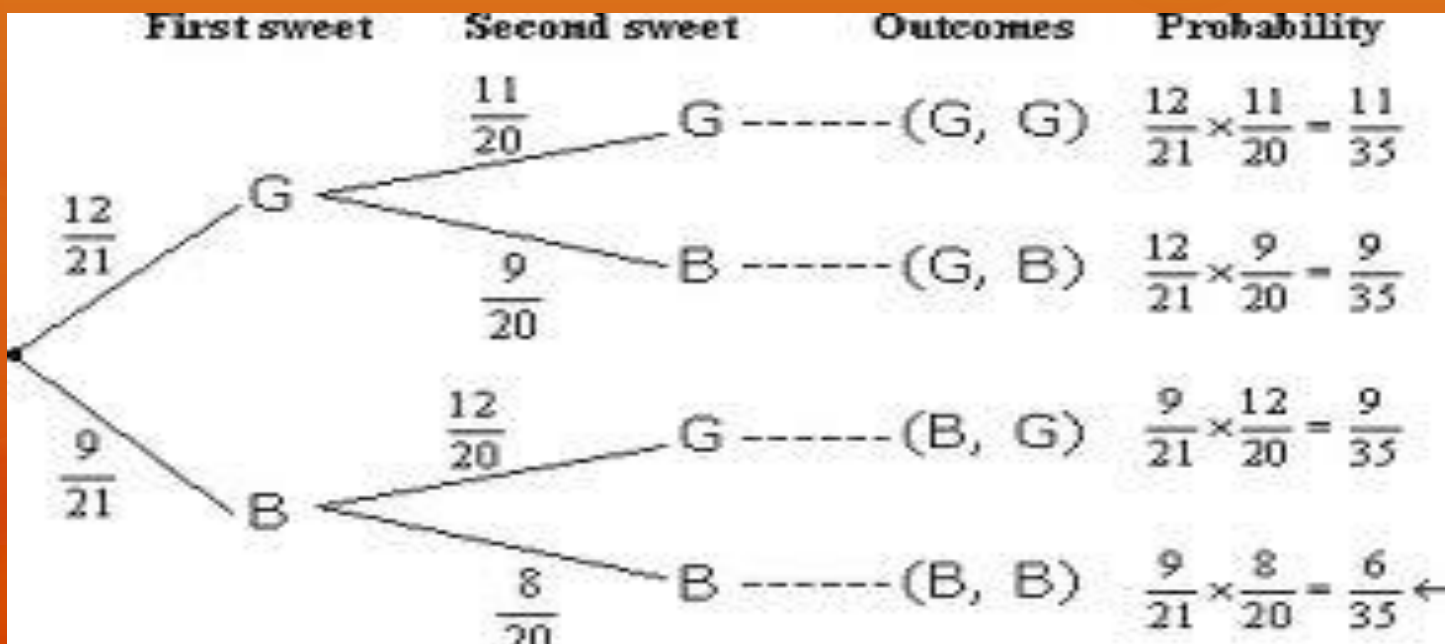
Sam and Alex work together. Sam has 0,6 chance of being picked to relocate. If Alex gets picked to relocate, the chance that he says yes is 0,3. Use a tree diagram to determine the probability of the boss picking Sam and Sam saying "yes" to the relocation.



[Dependent Tree Diagram Example]

Dependent Events Question:

Use a tree diagram to determine the probability of choosing two blue sweets (without replacement), if there are 12 Green sweets and 9 Blue sweets.



Dependent Events Question:

What is the probability of picking two blue marbles from a hat containing 4 blue; 8 red and 6 green marbles, if the first marble isn't replaced?

$$P(B \text{ and } B) = P(B) + P(B)$$

$$= \frac{4}{18} + \frac{3}{17}$$

$$= \frac{61}{153}$$

Dependent Events Questions

Identifying Dependent vs Independent Events

MUTUALLY EXCLUSIVE EVENTS

- Events that cannot happen at the same time are mutually exclusive events
- Sum Rule for Mutually Exclusive Events:

$$P(A\ or\ B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

- E.g. What is the probability of getting a 1 or a 5 when rolling a dice?

$$P(1\ or\ 5) = P(1) + P(5)$$

$$= \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6} = \frac{1}{2}$$

MUTUALLY EXCLUSIVE EVENTS

- Note!

$$P(A \; and \; B) = 0$$

$$P(A \cap B) = 0$$

- E.g. What is the probability of getting a 1 and a 5 when rolling a dice?

$$P(1 \; and \; 5) = 0$$

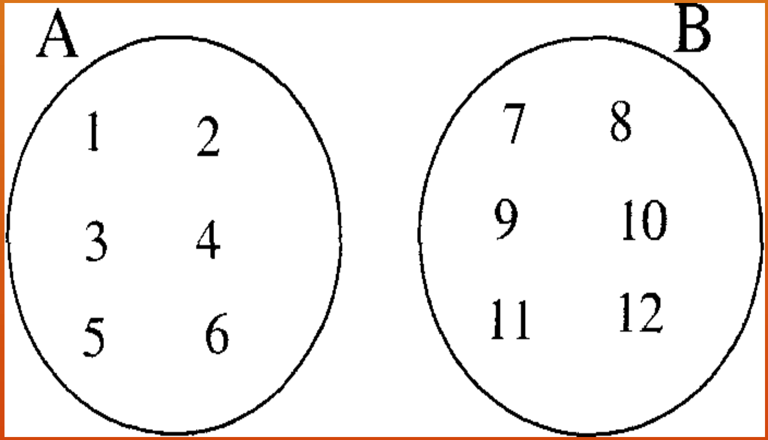
[Practical Examples of Mutually Exclusive Events]

Mutually Exclusive Venn Diagram

Twelve numbers are dropped into a hat:

Event A = $\{numbers \leq 6\}$

Event B = $\{numbers > 6\}$



$a)\ P(A \cup B)$

$= P(A) + P(B)$

$b)\ P(A \cap B)$

$= 0$

Mutually Exclusive Events Question:

What is the probability of picking a red or blue ball from a bag containing 3 red; 7 blue; and 5 green balls?

$$
\begin{aligned}
P(R\ or\ B) &= P(R) + P(B) \\
&= \frac{3}{15} + \frac{7}{15} \\
&= \frac{2}{3}
\end{aligned}
$$

COMBINED EVENTS

- If the combined events are NOT mutually exclusive, then

$$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- E.g. What is the probability of getting an even number or the number 4 when rolling a fair die?

$$P(E\ or\ 4) = P(E) + P(4) - P(E\ and\ 4)$$

$$= \frac{3}{6} + \frac{1}{6} - \left(\frac{3}{6} \times \frac{1}{4}\right)$$

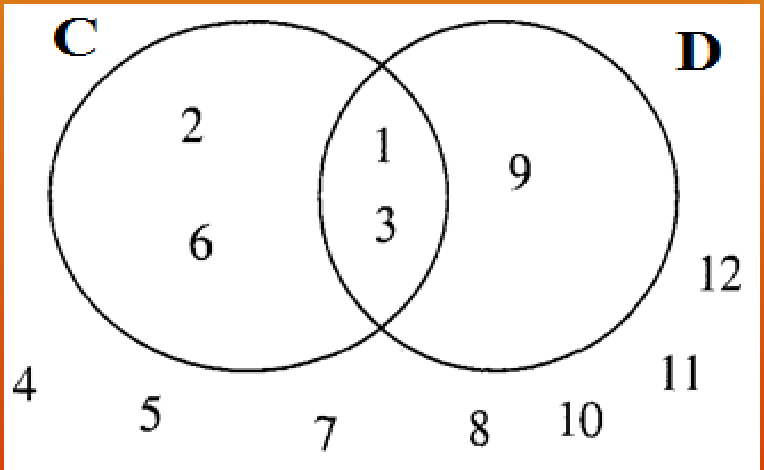
$$= \frac{13}{24}$$

Combined Events Venn Diagram

Twelve numbers are dropped into a hat:

Event C = $\{factors\ of\ 6\}$

Event D = $\{factors\ of\ 9\}$



Combined Events Card Example

Combined Events Shape & Colour Example

$$P(C \cup D) = P(C) + P(D) - P(C \cap D)$$

Combined Events Question:

What is the probability of drawing a King or a Red card from a standard deck of cards?

$$P(K\ or\ R)\ =\ P(K)\ +\ P(R) - P(K\ and\ R)$$

$$= \frac{4}{52} + \frac{26}{52} - \left(\frac{4}{52} \times \frac{26}{52}\right)$$

$$= \frac{7}{13}$$

Combined Events Question:

The probability of a wife going to the shops is 0,92; the probability of the husband going to the shops is 0,14; and the probability that both husband and wife will go to the shops is 0,45. Determine the probability of either the husband or the wife going to the shops.

$$P(W \text{ or } H) = P(W) + P(H) - P(W \text{ and } H)$$

$$= 0{,}92 + 0{,}14 - 0{,}45$$

$$= 0{,}61$$

Combined Events Questions

COMPLEMENTARY EVENTS

- Given an event "G", a complementary event is an event that is not "G"
- Complementary Rule:

$$P(not\ A) = 1 - P(A)$$

$$P(A') = 1 - P(A)$$

- E.g. What is the probability of not getting a 6 when rolling a fair die?

$$P(6') = 1 - P(6)$$

$$= 1 - \frac{1}{6}$$

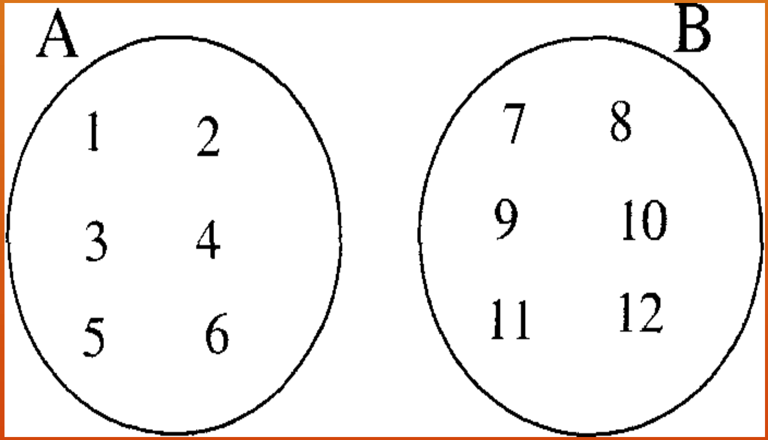
$$= \frac{5}{6}$$

Complementary Events Venn Diagram

Twelve numbers are dropped into a hat:

Event A = $\{numbers \leq 6\}$

Event B = $\{numbers > 6\}$



$$P(A') = 1 - P(A)$$

Complementary Events Question:

If you throw two dice, what is the probability of getting two different numbers?

$$P(not\ same\ numbers) = 1 - (same\ numbers)$$
$$= 1 - \frac{6}{36}$$
$$= \frac{5}{6}$$

[Rolling Doubles]

[Probability Laws Summary]

FUNDAMENTAL COUNTING PRINCIPLE

- If successive choices are made from $m_1, m_2, m_3, \ldots m_n$; then the number of combined options is the product thereof:

$$m_1 \times m_2 \times m_3 \times \cdots \times m_n$$

- E.g. How many different outfits can I wear if I have 5 shirts, 3 pants and 4 shoes to wear?

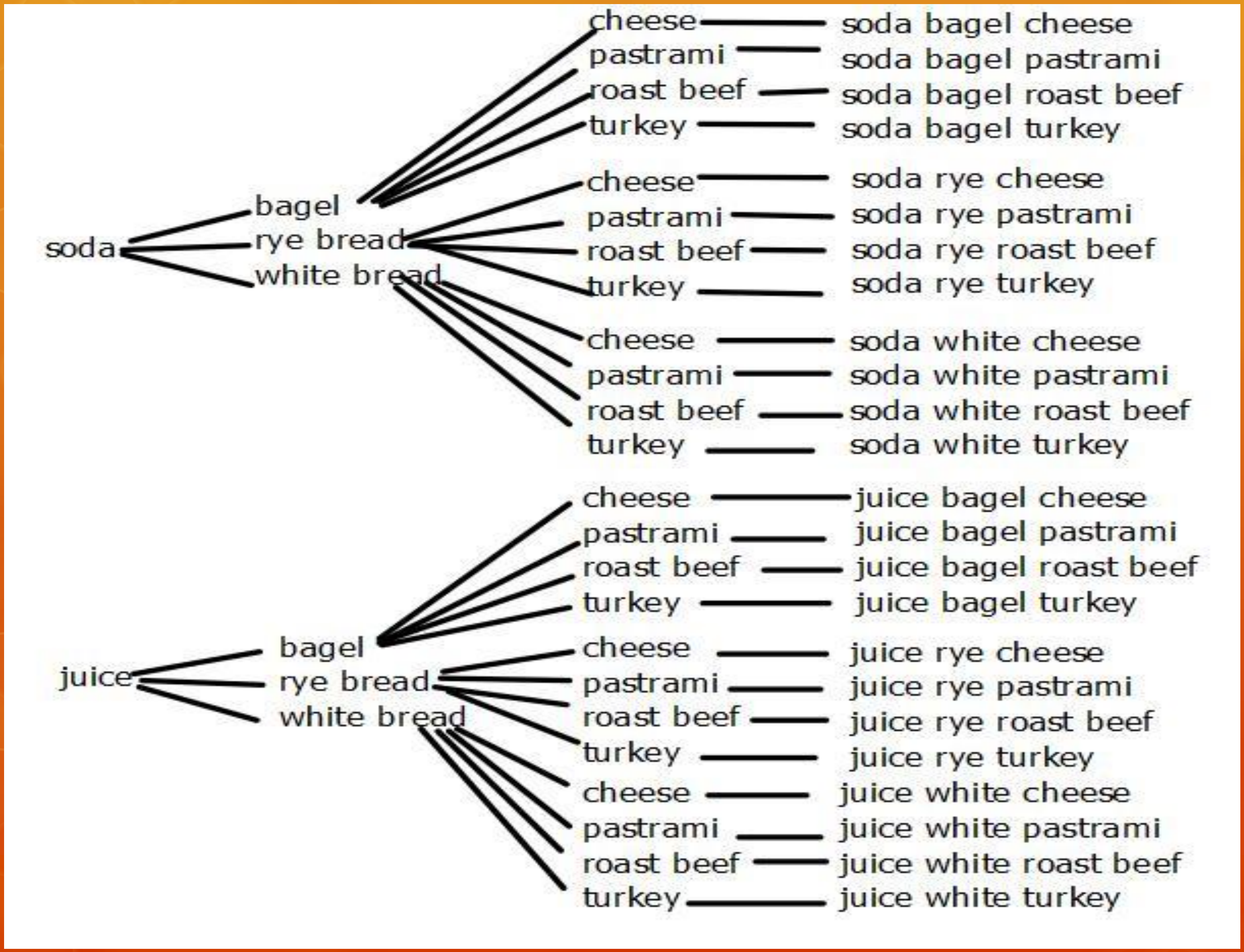
$$5 \times 3 \times 4 = 60\ different\ outfits$$

Fundamental Counting Principle illustrated by a Tree Diagram

How many different combinations of meals could I order if I could choose from the following:

- a soda or juice
- a sandwich on a bagel, rye bread or white bread
- with cheese, pastrami, roast beef or turkey as a filling?

Fundamental Counting Principle illustrated by a Tree Diagram



$2 \times 3 \times 4$

$= 24$

Different meals

Ways to Pick Officers Example

Fundamental Counting Principle Question:

How many different party packs can be chosen from each of the following:

- BarOne; Kit-Kat; Smarties; Tex; Aero; Nosh; or M&M's
- Coke; Fanta; Cream Soda or Sprite
- Nik-Naks; Lays; or Simba
- Peanut butter or Marmite sandwich

$$7 \times 4 \times 3 \times 2 = 168\ different\ party\ packs$$

Colour Combinations Question

Fundamental Counting Principle Question:

How many possible letter arrangements can be made from the word "MATHS", if:

a) the letters may be repeated?

$$5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 3125$$

b) the letters may NOT be repeated?

$$5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$$

FACTORIAL NOTATION

Probability Summary Examples